



Monetary Policy and the Evolution of Wealth Disparity: An Assessment Using US Survey of Consumer Finance Data

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Abstract

In the context of the studies of income and wealth distribution, this paper explores evidence to what extent recent monetary policy had distribution effects. This research utilizes the Federal Reserve Board's Survey of Consumer Finances to identify at least two sub populations and follow their dynamics as monetary policy moved from conventional interest rate setting to unconventional Quantitative Easing policy. This is achieved through use of Weighted Expectation Maximization Cluster Modeling, and the Malhalanobis Distance between cluster means. This approach is used to study volatility and bifurcation in distributions of capital gains and incomes to U.S. households through the Financial Crisis and Sars-CoV-2 pandemic.

Keywords Wealth disparity · Quantitative easing · Weighted expectation maximization · Cluster modeling · Malhalanobis distance · Unsupervised learning

1 Introduction

Monetary policy after the financial market crisis of 2008 has switched from conventional to unconventional monetary policy (UMP). This paper addresses the issue if there are shifting wealth shares over time in the US economy after the UMP and Quantitative Easing (QE) has been applied. This is in particular to be compared with

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other upswings in the US economy where such a policy was not applied but rather only the policy rate was used to support a new upswing.

The question has been raised whether the UMP had an overly strong effect on adverse income and wealth distribution in the US and Euro area, favoring asset holders more than non-asset holders. Yet, central banks argue that as compared to protracted period of recessions (as in the 1930's), the UMP helped to create new jobs, increased employment and allowed wage income to move up again- though it took a while to do so.

One notable issue regarding UMP is that, asset holders (real and financial assets), due to lower interest rates and a decrease of risk premiums, experienced an overall de-risking of their capital assets and investments. This was accomplished via lower capital cost through falling bond yields for real capital investors as well as capital gains for financial wealth holders (due to rising equity prices, bond prices and real estate property prices). Hence, although wealth holders also saw labor income rise, they, as a wealth holder group, may also gain less from labor income rather than from capital incomes, through rising asset prices.

We explore these two effects, the income and wealth effects for the two groups, first, then secondly evaluate the motivation for models presented in recent literature empirically, using SCF data from the year 1995 through 2020. The number of clusters chosen for the unsupervised learning approach utilized for the empirical section matches simple heterogeneous household models. One of the potential drivers of channels for inequality is the process of de-risking when the UMP sets in and the re-risking (of income and asset values) through the exit from UMP thus the boom-bust cycles of asset prices that are driven up in booms through borrowing and then rapidly decline through busts.

The literature review follows in Sect. 2, while the motivation and theoretical framework is presented in Sect. 3. The empirical Sect. 4, specifically Sects. 4.2, 4.3 and 4.4, describes the SCF data and explain how the findings bolster the presented dynamic heterogeneous households framework in the literature. The issue of causality is addressed in Sect. 4.5. In this context we should lastly note that high risk assets are a larger fraction of net wealth as income rises while low return assets are a dominant short-term financial vehicle for lower income brackets. Finally, Sect. 5 concludes the paper.

2 Literature Review

Recent literature has focused on a number of channels through which monetary policy affects measures of inequality. This paper serves as a complement to the established literature in that we do not dispute the beneficial effects of monetary policy intervention and QE on output following economic shocks, but yet, with regard to empirics, we demonstrate alternative data driven methodologies for measuring the lagged effects of such interventions on income and wealth accumulation for heterogeneous households.

The theoretical framework for analysis, preceding empirical results, follows the approach of recent researchers, see (Aguiar et al., 2021), where the stochastic

components for heterogeneous households and wealth holders are lumped together as one type of household (all optimizing), but having two stochastic sources of income from labor and assets generated by firms. Yet we further explore two types of households. These households can be viewed as adapters, driven for example by conspicuous consumption in the sense of Veblen (1899), or upward looking households, as presented in Drechsel-Grau and Schmid (2014). They are adapters, distinctively different from the second type of households that have dominantly capital income and are commonly called consumption smoothers. The two types of households may be differentiated with respect to saving rates, employment shocks, labor income, returns to assets and borrowing constraints. This paper thus considers those two types of households by undertaking an empirical study, see also (Cynamon & Fazzari, 2016).

Regarding all subsequent analytics, our main period of reference is the period before the great recession 2008 and the period thereafter. We use US multiply-imputed SCF data, in which all implicates (which are imputed for missing data) are considered so as to exclude consideration of only one implicate result, which would clearly ignore uncertainty, when studying the issue empirically. Our theoretical framework is motivated and closely related to Brunnermeier and Sannikov (2014); Stein (2012) and Schleer and Semmler (2015), but is also related to the work by Kaldor (1956) and Pasinetti (1962) of the 1950s and 60s. The motivation herein can therefore be viewed within the simplified context of the more complex asset accumulation models presented in Chappe and Semmler (2016), and Semmler and Parker (2017) yet stressing empirical methods not developed in the latter papers.

As known in the literature on asset prices and returns, the dynamics of capital gains greatly affect wealth distributions.¹ In fact, some research takes not gross wealth but net wealth as a benchmark and shows how credit expansion, asset prices and returns impact the net wealth distributions.² For example, the large asset price meltdown in the years 2007–9 (in particular the meltdown of housing prices) led to a declining share of the 80 or 90% wealth owners, see (Wolff, 2012). So it was both the increasing mortgage and consumption borrowing, before the 2007–9 crisis, as well as the asset price meltdown, that produced the adverse effect on the distribution of net wealth. Though there is borrowing to consider, in this paper, we do not explicitly consider the debt dynamics to study net wealth aside from the utilization of net wealth as a variable.

If the result on time varying wealth distributions holds more generally over a long time period, then also the Kaldor (1956) and Pasinetti (1962) debate has to be looked at in a new light. In Pasinetti's response to Kaldor, he assumes, as Kaldor does, that savings from wages are lower than that from capital income, but then Pasinetti also assumes that the asset returns from labor savings are the same as from capital income. This keeps the wealth shares constant. Yet if this does not hold empirically, the wealth shares are shifting over time.

¹ Gordon writes in his book: "If the stock market continues to advance, we know that the inequality will increase, for capital gains on equity accrues disproportionately to the top income brackets" (Gordon, 2016): 23).

² See Wolff (2012); Cynamon and Fazzari (2016), and Semmler and Parker (2017).

Regarding the empirical analysis, the various direct and indirect channels through which policy intervention can be shown to affect households has been well documented, see (Coibion et al., 2017; Cui & Sterk, 2021; Parker, 1999), and various researchers have used an assortment of measures of inequality to document these effects. However, unlike recent literature, see (Andersen et al., 2020; Holm et al., 2021; Coibion et al., 2017), we utilize statistical learning methods through which to study the effects of recent monetary policy on inequality. Hence, also unlike such studies, we do not order households according to their liquid assets and/or income percentiles measuring inequality via the Gini coefficient or the specified cross-sectional standard deviations of log values. Rather, we study such effects through a measure of inequality defined by calculation of the Mahalanobis Distance between the log values of ‘k’ cluster means (in this case $k = 2$) measured by the full sample covariance matrix, not conditioned on any cluster center. This is independent of any run of the EM algorithm. We therefore also utilize benefits of log values such as removing the effects of outliers, but conversely utilize our own machine learning algorithm to study the effects through time on various distributions with regard to both capital gains and the income channel.

In regards to data sources, recent literature in the field has utilized sources such as the Consumer Expenditure Survey (CEX), see (Coibion et al., 2017), or administrative micro-data, see (Andersen et al., 2020). Conversely, we use US Survey of Consumer Finance (SCF) so as to have better access to the top 1% of the distribution for analysis as well as to avoid reliance on the use of ex ante market values, as we include capital gains, as reported in IRS tax filings. However, as recent literature has demonstrated, see (Andersen et al., 2020; Holm et al., 2021), the effects of monetary policy shocks on the income gradient can be seen for periods of approximately two years and persistent effects lasting from three to as long as five years respectively. Hence, our use of triennial data for studying our measure of economic inequality is well substantiated in the literature. As such the empirical findings of statistically significant effects for multiple lags lasting many years are fully corroborated with established findings.

The empirical sections contained herein are designed with respect to the motivation presented in various theoretical frameworks for a heterogeneous households model. This paper primarily focuses on the change of the wealth and income distributions before and after the great recession of 2008 as well as the SARS-CoV-2 pandemic and how it is impacted by monetary policy. Further details regarding the motivation are provided in the following sections.

3 Motivation—Framework for Analysis

Our motivation for the selected empirical methodology follows from and supports heterogeneous households that obtain income from labor and wealth, consume, save and invest. This builds on a dynamic portfolio approach. As there are two households, both households may have a labor and wealth income, though their labor income and asset income may be different and thus their wealth dynamics will be

different. Therefore, we refer to wealth, as built up by both labor income and asset income.³

The following section discusses the specific topic of heterogeneous household models of inequality within the literature that is supported by the empirical results that follow. We further discuss some future areas of research into the topic, important questions worth consideration and our motivation for the empirical model presented herein.

3.1 Heterogeneous Households⁴

We are considering herein the social interaction theory of consumption spending, starting with two households in the sense of Veblen (1899), and further explored by Drechsel-Grau and Schmid (2014). The first household is likely a high wealth household and has higher returns from capital and capital gains, but also labor income, and can borrow to invest into assets. This type of household may behave as a consumption smoother, in the sense of Cynamon and Fazzari (2016) and undertakes optimal consumption-saving decisions. With high wealth the household can borrow to invest which means, using a dynamic portfolio approach, the household can also use short sales of the risk free asset.

The second household likely also has initial wealth, though possibly less than the first household, but has lower returns on assets, lower capital gains, lower labor income, and obtains income from a risk free rate, for example from bank accounts.⁵ Whereas the first household may act as a consumption smoother, the second household behaves deceptively in its consumption spending. For this household then consumption decisions are not necessarily optimal, they are likely to be different from optimal consumption decisions.⁶ Yet, note that both households can accumulate capital.

Eventually, to motivate consideration of such a theoretical framework, we present a two cluster weighted expectation maximization (WEM) model, utilized for empirical validation of two heterogeneous households. Technically, the capital shares could evolve as Brunnermeier and Sannikov (2014) predict. Studying evolving asset shares was already studied in Pasinetti (1962). But it should likely also be considered that the shares can change, due to a difference in: asset returns, saving rates, labor income and borrowing constraints of the two groups of households. In our empirical analysis we study whether the second type of household groups may be

³ In fact the motivation for our modeling approach can be viewed as further exploring recent literature where the dynamics of assets where to include retirement funds the wealth will be higher, see recent (Cynamon & Fazzari, 2016).

⁴ For a formal presentation of the following, see Semmler and Parker (2021).

⁵ For EU countries, for example it is estimated that “The most prevalent financial assets are deposits (sight or saving accounts), which are owned by 96.4% of households; voluntary private pensions/whole life insurance is held by 33.0% of households; all other financial assets are owned by less than 15% of households”.

⁶ There is a large body of literature that starts with Veblen (1899) and his theory of conspicuous consumption, then Duesenberry (1949) and recent work that shows that consumption of the latter group arises from some social interaction and adaptive behavior of households.

disadvantaged with respect to income from the assets, capital gains, labor income, and stricter borrowing constraints.⁷

The essential features of models we consider, and discuss, as motivation for our empirical framework of analysis can be found in the Kaldor–Pasinetti debate and in modern portfolio models of heterogeneous asset holders, see for example in Guvenen (2011). We also consider that our findings may support, and motivate, models utilizing shocks as in Brunnermeier and Sannikov (2014) and in Stein (2012).⁸ The former have explicitly the two types of households in their model, resulting in our use of a two cluster model for empirical research. Both papers make their specifications stochastic, but they do not explicitly model further macro feedback loops though they could likely abstract from the macro feedback effects as discussed in Mitnik and Semmler (2018), and Gross et al. (2017). For this reason we herein further empirically examine issues of causality with regard to both conventional and unconventional monetary policy.

As stated, in Sect. 2, there may be consumption smoothing households as in Cynamon and Fazzari (2016), while the second household is likely driven by behavioral variables such as observation of others and interaction effects with others. There may also be a tax levied on the first type of household and transferred back to the second household. As the following sections demonstrate, our empirical results support these various avenues of research.

3.2 Solution Method and UMP

As evidenced in the literature, to solve such a low dimensional but still complex stochastic model with interaction effects may employ the use of Nonlinear Model Predictive Control (NMPC) as suggested in Grune et al. (2015).⁹ Additionally, when we begin considering variables of interest in the following sections, we note that as in Brunnermeier and Sannikov (2014) capital returns are—due to capital gains—stochastic. Brunnermeier and Sannikov (2014) start with a model where only the capital return is stochastic, and they add a stochastic interest rate later by referring to time varying borrowing cost, reflecting the cost of screening and monitoring.¹⁰

We primarily empirically consider our findings to motivate models that move away from modeling homogeneous households in the portfolio approach, see (Guvenen, 2011).¹¹ We are also curious to question, in our future research, whether toward the end of booms the asset prices are likely to fall stronger than wage income and thus the capital assets declining faster for low income households than for higher

⁷ For details of roughly zero or negative saving rates of low income households, see (Dynan et al., 2004) and on differences in asset return, see (Chappe & Semmler, 2016).

⁸ For further details of the Stein model and the derivation of this, see Gross et al (2017).

⁹ Additionally, the modeling framework, see (Semmler & Parker, 2021) is described within the following URL: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3917625.

¹⁰ As presented, Stein (2012) adds something important to this type of model by providing a static solution for the model in terms of a mean variance approach that allows to measure excess leveraging, as per portfolio theory.

¹¹ There the state equation is defined for homogeneous households.

income households. Then after a recovery takes shape the asset income may recover faster than the wage income and thus the higher income households are faster recovering their asset losses.

The next question of interest our findings motivate, may be how the share of a country's wealth will evolve and the effects on wealth distributions. This is also an imminent issue in the Kaldor–Pasinetti debate. Therefore, we are further interested to see if the share of high net worth households will rise which seems to be supported by empirical evidence, see Cynamon and Fazzari (2015).¹²

If we bring in the topic of UMP, we need to consider if the above process is magnified by the periods following UMP. Yet, central banks (the Fed and ECB) argue that as compared to a protracted period of recessions (as in the 1930) the UMP indeed helped to create new jobs, increased employment and allowed wage income to move up again- though it took a while to do so.

Although wealthy asset holders had some income rise as well, they, as the wealth holder group, may appear to gain less in income than from capital gains, through rising asset prices. This at least holds for firms in the faster upswing periods, since 2008, particularly generated through UMP since the start of the QE.

Our following empirical analysis supports researching whether one of the drivers of the two processes, stylized above, may occur through the process of de-risking when the UMP and QE sets in. The de-risking means yields on long term bonds and risky corporate bonds—and thus risk is driven down what one can call de-risking. Therefore, capital cost is driven down. Though this capital cost of firms are moving down, stock prices move up. Then there are low returns for asset holders in the bond and credit markets but asset prices, in particular stock prices, are driven up with expected return rising. This may change the disparity of wealth distribution by harming the lower income households. But the wealth disparity could be corrected again, in particular if distributional concerns—as our dispersion measure described in the following sections indicates—lead to reducing wealth disparity using the above policy goal. The spending of the adapting consumers might rise as well.

Yet on the other hand there could be re-risking (of income and asset values) if the exit from UMP takes its course through the bust period of cycles of asset prices—driven up in booms through borrowing and then rapidly declining income and asset prices through busts and insolvencies.

Note that the leveraging of two heterogeneous households, which is not explicitly analyzed herein, could further entail, as in Brunnermeier and Sannikov (2014), a macro feedback loop where the deviations can be unstable downward and not mean reverting as in Bernanke et al. (1999). In Brunnermeier and Sannikov (2014), feedback loops arise from large shocks below some steady state, triggering fire sales of assets, fall of asset prices and fall in net worth, generating a downward spiral.¹³ What the shocks might trigger, in the sense of Brunnermeier and Sannikov (2014) is not discussed here. Moreover, the issue could be the macro feedback effects arising from wealth disparities, not only with respect to long run growth Gordon (2016), but

¹² Brunnermeier and Sannikov (2014) have devoted a considerable part of their study to this issue.

¹³ In Stein (2012) the vulnerabilities and possibly adverse feedback loops are triggered by over leveraging, capital losses and rising borrowing cost.

also for medium run macroeconomics, pointing towards potential further research into the phenomena.

The increasing inequality of wealth distribution using SCF data is additionally observed in the empirical analysis in Semmler and Parker (2017). An observable trend is indeed that first, the share of high yielding assets owned by high income earners is substantially larger than that of lower income households. Stocks and bonds become an increasing component of the net worth of such households while lower income earners may rely more upon pension funds or simply checking accounts. The incorporation of a two cluster unsupervised learning algorithm for empirical focus is in alignment with the two types of heterogeneous households previously considered. Further discussion regarding the dimensions upon which wealth disparity can be analyzed within SCF data is undertaken below.

4 Empirical Methodological Approach

The following subsections provide empirical support and motivation for the theoretical framework of heterogeneous households and inequality as previously discussed.

4.1 The Survey of Consumer Finances Data

The data used herein originates from the Federal Reserve Boards' Survey of Consumer Finances (SCF) data sets spanning the years 1989 through 2020.¹⁴ The SCF data sets are one of the most widely accepted sources of household finance data within the United States. The data has been used throughout numerous studies along with similar PSID data regarding examinations of differences in wealth, income and savings capabilities for households of differing financial means, see Dynan et al. (2004); Wolff (2016).

The data set is designed in conjunction with the census such that weighted metrics may be evaluated as representative of the tax paying portion of the population in IRS tax filings. When analyzing inequality at the tails, IRS tax data is frequently used, see Piketty and Zucman (2014). However, the SCF data sets include additional demographic classifiers of interest along with assessed asset values as opposed to strictly tax filing related data. Hence, for this paper the SCF data is the primary data of choice in order to examine the effects of recent monetary policies on the wealth and income disparity in the United States.

The SCF data is multiply imputed five times to account for missing data. The end result is that the data is provided such that there are five (5) implicates (for example, replicates) corresponding to the five (5) times in which missing values were imputed. Hence, implicate one, refers to the first imputed data set. Many modern economic surveys have begun to use multiply imputed data and weighting in order

¹⁴ Survey data and related variable descriptions may be found at the following Federal Reserve URL: <https://www.federalreserve.gov/econres/scfindex.htm>. Similarly source codes for downloading data within.rda file extension formats may be found at the URL: <http://asdfree.com/survey-of-consumer-finance-scf.html>.

to refine how missing values are managed in calculations, see Montalto and Sung (1996). Due to lack of response and the complexity in the distributions of variables of interest that may not be well defined, multiple imputation gives a more reasonable understanding of the lack of precise information on missing values, see Kennickell et al. (1996). The SCF multiple imputation methodology is based upon the Gibbs Sampling implementation in the FRITZ software package developed at the Fed and described in detailed publications, see Kennickell (1998).¹⁵ The weighting schematic was determined by the Federal Reserve Board, which corrects for over and under sampling of sub-populations, and herein is taken as given.

The multiply imputed nature of the data requires running all analyses that follow across all five implicates. However, results for final analysis are taken in conjunction, not separately implicate-by-implicate (thereby not emphasizing the results of any one implicate separate from the rest). As a result, there are five model results frequently demonstrated for any given metrics that follow in subsequent portions of the empirical writings herein. Specifically, this refers to the WEM cluster model output in Sect. 4.3, as well as calculations of the Mahalanobis distance in Sect. 4.4 and Appendices A through E. The benefit of this approach thereby does not necessitate alternative approaches such as Repeat Imputation Inference (RII) Techniques, which turns out to be essentially a synonym for an averaging of imputation values, see Montalto and Sung (1996).¹⁶

The Federal Reserve publishes findings tracking changes in wealth and income distribution trends following each triennial SCF data release. The 2016 survey found decreasing median incomes accompanied by increasing mean incomes, see Bricker et al. (2017), confirming the finding of growing inequality herein.

Yet interestingly, home ownership of primary residence appears to have increased to approximately 65% in 2019, Bricker et al. (2020). Furthermore, our results show that in 2019 the trend towards a widening wealth disparity has continued to increase beyond previous peaks. Most notably, the lowest quartile actually saw a loss in mean net worth, Bricker et al. (2020), with an increase in negative mean net worth in 2019 relative to 2016.

According to Federal Reserve findings holdings of financial assets rose for the top income groups with 93.6% holding publicly traded stocks, see Bricker et al. (2017). Furthermore, the mean and median values of stock holdings were significantly higher in 2019 than in 2016, Bricker et al. (2020). It is for such reasons that we focus our empirics here upon the effects of unconventional monetary policy, overlapping substantially with periods of low interest rates, as the evidence may suggest a disproportionate benefit of such policies for the higher end of the wealth and income distributions in the U.S. as opposed to what may be seen in Europe where financial assets are less commonly held assets.

Although accumulated wealth is an important topic, as inequality in wealth is greater than that found in income distributions, see Piketty (2014), this paper focuses

¹⁵ The details may be sourced for further clarity at the following URL: <https://www.federalreserve.gov/econresdata/scf/files/impute98.pdf>.

¹⁶ The details of RII Techniques are further detailed in a programming script from Duke University: <https://scholars.duke.edu/publication/860554>.

predominantly on cash flows from assets resulting in capital gains as they relate to the unconventional monetary policies adopted by the Federal Reserve in response to the financial crisis of 2008. After all, the treatment of capital gains in taxation policy has a great effect on the distribution of cash flows and incomes from related assets, see Hungerford (2013). Much of the literature on the topic of economic inequality shows that it is the higher end of wealth and income distributions that benefit from recent changes in taxation policy, see Hungerford (2009), which, taken in conjunction with new unconventional monetary policy, makes such assets and related cash flows a subject of emphasis for the empirical validation of the Weighted Expectation Maximization Cluster model presented in Sect. 4.2. Some stylized facts from the Federal Reserve Board's SCF data help emphasize this fact.

Capital income, according to the early SCF questionnaires, generally consists of rents, dividends and interest. However, realized capital incomes, denoted by the variable 'kginc', became commonly reported in a consistently weighted manner by the 1989 survey.¹⁷ There are multiple market value estimates for additional financial and non-financial unrealized capital gains/loss variables available in SCF datasets (ex: stocks, bonds, mutual funds and real estate). Such variables were used herein, such that the data was a further subset of those households that reported positive capital gains income and have market estimates for the values of assets that encompass the total net worth of each household. Therefore net worth was also included for subsetting of the final data set.

It is worth noting that those variables that are listed as capital gains incomes correspond to actual data that would have been reported to the IRS by any given household. It was found that the normal income variable was only reported beginning in 1995. Hence, the data used for analysis herein were chosen based upon both the availability of any variables of interest along with the years corresponding to periods prior to the implementation of unconventional monetary policy and those years after the implementation of such policy tools ending with the most recent survey year available.

From Fig. 1, we can see a regression line through two dimensions with the log of capital gains on the y-axis and the log of normal income ('norminc' which includes labor income) on the x-axis.¹⁸ When analyzing these dimensions of the data, a large proportion of respondents reported zero or near zero capital gains income within the first impute (the first of the five imputed data sets). The diameter of the circles corresponds to the log of the weights of the data points, such that a larger circle represents a larger portion of the distribution of households. Log values are used because unlike the level data, the transformed data approaches a log normal distribution. The appearance of a "dog-leg" in the figure demonstrates that the data distributions would be more adequately modeled by a Gaussian mixture model than

¹⁷ The precise variable 'KGINC' used herein, may be defined as realized capital gains or losses in answer to question x5712 regarding the total family net annual income from gains or losses from mutual funds or from the sale of stocks, bonds or real estate in the previous year, before deductions for taxes and anything else. One may refer to 2015 IRS form 1040 line numbers 13 and 14 for additional clarification.

¹⁸ The normal income variable x7362, used herein is smoothed for unusual shocks so as to reflect usual income patterns.

a simple linear model. This is because the data is clearly a mixture of populations of varying size. Hence, the selection of an unsupervised statistical learning method was chosen for econometric modeling.

In the 2019 survey year, only a little over 17.50% of all respondents had any positive realized capital incomes, while over 99% had positive normal incomes, within the first implicate. Additionally, those that had normal incomes in excess of \$3.5 million, constitute approximately 4% of the total survey dataset. In total, of those households with both positive capital gains and normal incomes, approximately 80% reported normal incomes under \$3.5 million.

The subset of data utilized for this study includes only those households with more than a dollar across the five dimensions of normal income, net worth, realized capital gains, unrealized financial and non-financial capital gains ensured that the data under analysis constituted active market participants with holdings of various asset classes.¹⁹ A selection bias is therefore not problematic since the data was intentionally subsetting in this manner to study this portion of households participating in the market. The log transformations of variables were then utilized for scaling purposes.

The data restrictions utilized herein were determined such that we condition on households that engage in multifaceted investing in order to have a complete picture of investor household(s). On a practical level, since all of our analysis uses logarithms of money/value/wealth, a zero or negative wealth component would introduce more NAs into the data, which would necessitate a second round of imputation not done by the Federal Reserve and so is not part of our analyses.

In regards to negative variables, although We considered having a secondary model for predicting the binary event of losses, such a model would extend the analysis beyond unsupervised EM and introduce a supervised workflow for “loss prediction.” Therefore, we abandoned that line of analysis because it did not seem as though it would help clarify the goal, which was the identification of sub-populations of investing households.

As shown in Table 1, the proportion of the population reporting positive income and capital gains in years following financial crises decreased. The final column demonstrates the second column divided by the third column and is expressed as a percentage. Hence, the observed trend can be seen most notably in the 2004 survey year following the tech bubble of 2001, as well as in the survey years 2010, 2013 and 2016 following the 2008 financial crisis. Otherwise, the data utilized in this study represents between 10–15% of survey data where positive values were reported for the variables listed herein.

4.2 Cluster Analysis and the Weighted Expectation Maximization Algorithm

The Expectation Maximization (EM) algorithm is related to the well known method of K-Means. However, Expectation Maximization allows for the usage of

¹⁹ Technically, the variables used herein were taken from Federal Reserve Board’s net worth as well as capital gain/loss variables ‘kgstmf’, in the case of stocks and mutual funds, and cumulatively both ‘kgghouse’ and ‘kgore’ for the housing variables used for segmentation and cluster modeling.

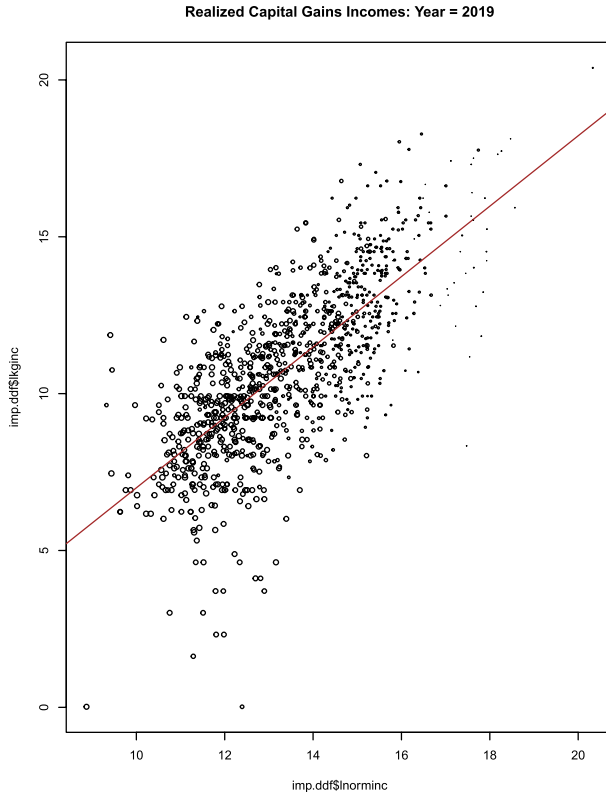


Fig. 1 Prevalence of Capital Incomes (U.S.): demonstrating the possibility of multiple distributions of households with differing capital gains and income dynamics within a given year

probabilities whereby an observation may possess attributes of multiple distributions such that the degree to which an observation fits more within one distribution than another may also be analyzed. The algorithm utilizes an iterative process by applying an expectation step (E-step) to create a function for the expectation of the log-likelihood, evaluated at the current parameter estimate, and a maximizing step (M-step) which calculates parameters that maximize the expected log-likelihood from the E-step.

This paper provides specification clarification to WEM modeling methods found in the literature on cluster analysis. Clustering methods have garnered attention in recent years with a proliferation of approaches for use in programming languages, such as R and C++ (among others), see James et al. (2013). An approach providing application to weighted data, see Gebru et al. (2016), is of substantial relevance for multiply imputed surveys of household wealth produced by various central banks. Correct specification is particularly useful for determining the distribution of latent variables during the repetition towards convergence, as presented herein.

Table 1 Number of observations utilized in sub-selected data from SCF data (Implicate 1 of 5 depicted): demonstrates the approximate percentage of survey household respondents that were included in the datasets used in this study (column 3) for a given year

Year	Observations used	Total survey respondents	Percent of data used (%)
1995	488	4299	11.35
1998	611	4305	14.19
2001	505	4442	11.37
2004	422	4519	9.34
2007	626	4417	14.17
2010	219	6482	3.38
2013	419	6015	6.97
2016	580	6248	9.28
2019	636	5777	11.01

Utilizing the observations of the variable x , the density for x is obtained over the latent variable z :

$$p(x; \theta) = \sum_z p(x, z; \theta) \quad (1)$$

such that $\theta = \{(\mu_1, \Sigma_1, \dots, \mu_K, \Sigma_K)\}$ where μ_k = the Gaussian distribution mean and Σ_k = the covariance matrix of the k th mixture component. The density at x_i due to mixture component k and the presumed multiplicity weight ω_i is given by:

$$\hat{p}(x_i; \Theta, \omega_i, k) = [N(x_i, \mu_k, \Sigma_k)]^{\omega_i} = A_i \mathcal{N}\left(x_i, \mu_k, \frac{1}{\omega_i} \Sigma_k\right) \quad (2)$$

where the proportionality constant $A_i = (2\pi)^{-d/2} |\Sigma|^{-\omega_i/2}$ depends on both dimension d and weight w_i , but is not a normalization constant. Including all mixture components in the full density representation, we have:

$$\tilde{p}(x_i; \Theta, \omega_i) = \sum_{k=1}^k \pi_k A_i \mathcal{N}\left(x_i, \mu_k, \frac{1}{\omega_i} \Sigma_k\right) \quad (3)$$

where π_k is the prior proportion of component k , viz., $\sum \pi_k = 1$.

Note that increasing ω_i effectively increases the precisions of all components at point i . This is a bit counter intuitive. How can one measurement increase precision (reduce variance) of *all* component estimates? It forces all covariances to adjust accordingly, increasing variances if necessary to maximize the observed data likelihood. In any event, it makes high weight points much more relevant in calculations for the moments, both means and covariances.

Maximizing the log-likelihood of the data to fit the mixture parameters where $\Theta = (\pi_1, \dots, \pi_k, \theta_1, \dots, \theta_k)$ and $X = (x_1, \dots, x_n)$ and $W = (\omega_1, \dots, \omega_n)$:

$$\ell(\tilde{p}(X; \Omega, W)) = \sum_{i=1}^n \log \left(\sum_{k=1}^k \pi_k A_k \mathcal{N}(x_{(i)}, \mu_{(k)}; \frac{1}{\omega} \Sigma_k) \right) \quad (4)$$

Typically, this would be provided Q is a distribution over potential values z , such that $\sum_z Q(z) = 1; Q(z) \geq 0$, we then find a lower bound:

$$\log p(x; \theta) \geq \sum_z Q(z) \log \frac{p(x, z; \theta)}{Q(z)} \quad (5)$$

Yet, in this case we write the complete (c) expected log-likelihood:

$$Q_c(\Theta, \Theta^{(r)}) = E_{P(Z|X; W, \Theta^{(r)})}(\log P(X, Z; W, \Theta)) \quad (6)$$

Where the $(r + 1)$ -th Expectation Maximization iteration consists of both the E-step (evaluating the posterior with weights ω) and the M-step such that, the Q_i that attains equality in the algorithm must satisfy:

$$\Theta^{(r+1)} = \arg \max_{\Theta} Q_c(\Theta, \Theta^{(r)}) \quad (7)$$

The E-Step is then written where the posterior $\eta_{ik}^{(r+1)} = p(z_i = k | x_i; \omega_i, \Theta^{(r)})$ iterates:

$$\eta_{ik}^{(r+1)} = \frac{\pi_k^{(r)} \tilde{p}(x_i; \Theta_k^{(r)}, \omega_i)}{\tilde{p}(x_i; \Theta^{(r)}, \omega_i)} \quad (8)$$

Therefore, the M-Step, from (6) may be expressed in terms of Q_c , where after canceling, the mixture proportions, means and covariances are as follows:

$$Q_c(\Theta, \Theta^{(r)}) \stackrel{\Theta}{=} \sum_i^n \sum_{k=1}^k \eta_{ik}^{(r+1)} \left(\log \pi_k - \frac{w_i d}{2} \log(2\pi) - \frac{\omega_i}{2} \log |\Sigma_k| - \frac{\omega_i}{2} (x_i - \mu_k)^\top \Sigma_k^{-1} (x_i - \mu_k) \right) \quad (9)$$

$$\pi_k^{(r+1)} = \frac{1}{n} \sum_{i=1}^n \eta_{ik}^{(r+1)} \quad (10)$$

$$\mu_k^{(r+1)} = \frac{\sum_{i=1}^n \omega_i \eta_{ik}^{(r+1)} x_i}{\sum_{i=1}^n \omega_i \eta_{ik}^{(r+1)}} \quad (11)$$

$$\Sigma_k^{(r+1)} = \frac{\sum_{i=1}^n \omega_i \eta_{ik}^{(r+1)} \left(x_i - \mu_k^{(r+1)} \right) \left(x_i - \mu_k^{(r+1)} \right)^T}{\sum_{i=1}^n \omega_i \eta_{ik}^{(r+1)}} \quad (12)$$

The E-step and M-step are repeated until the algorithm converges. It should be noted technically and to emphasize the intuitive nature of EM, that π_k is the posterior probability of assignment of any one unweighted unknown to cluster k. Furthermore, on that same note, η_{ik} is the specific posterior probability of assignment of point of type i to cluster k, conditional on weight w_i as well as all parameters of all clusters.

We empathize with the need to develop intuition for mathematical processes such as Expectation Maximization. It has been our experience that EM is far better intuited by doing it than by discussing it, or deriving it. It is no surprise that EM was a well established method long before a formalism of complete data log likelihood maximization by missing data expectation was provided as a capstone to several generations of successful implementation.

Lastly, although R programming language provides a natural structure for working with the data, the processing time, due to compartmentalization of the scripts, was cut down dramatically through utilizing C++.

4.3 Expectation Maximization Cluster Mean Results

In regards to studying the effects of monetary policy on asset returns, the process of Expectation Maximization (EM) Cluster Modeling presents unique attributes as well as the ability to define and study the optimal number of clusters (or groupings of households) within a population or sample. The results initially focus on two particular dimensions of the data, the log of normal income and the log of realized capital gains variables from SCF datasets.

The results of this empirical analysis involve modeling across all five impicates. Hence, the subject of variance must be taken into account so that the reader does not infer that any one result is over-emphasized. Of course it should be noted that each impicate is a valid sample of the posterior of the distribution of the unobserved data. Consequently, the clusters derived from each impicate are, first of all, based on a valid sampling from posteriors and, secondarily, are in and of themselves credible instances of clusters. A direct side-by-side comparison of all of these clusters is precisely in the spirit of multiple imputation that emphasizes uncertainty in variance.

As demonstrated in the literature, see Montalto and Sung (1996), RII is one method for working within a multiple imputation framework. However, as demonstrated by earlier scholars on the topic, see Rubin (2004), RII is presented as a natural gesture for post-analysis of multiple imputed data sets, but is, by no means, presented as the only appropriate method of such analysis. Hence, it should further be noted that the following results never present a single impicate, but always emphasize results from all impicates together.

As can be seen in Fig. 2, the x-axis is the log of normal income, while the y-axis displays the log of the realized capital gains. The results demonstrate that within the population of households reporting both positive incomes and capital gains, the data ranges from under \$100, to well over several hundred million dollars. Hence the log transformations allow for scaling and data visualizations that encompass substantial variations in earnings and wealth accumulation.

The data is depicted in Figs. 2, 3 and 4, as circles of various size for the first implicate in the data set. The use of circles is due to the multiply imputed survey design structure of the data.²⁰ It would be incorrect to give each data point an equal weight in any model. Hence, the size of any given circle reflects the population size of any given observation or alternatively, the weight that any circle should be given based upon the survey design weights determined by the Federal Reserve Board. Variance amongst the five implicates, for a given year, is demonstrated in Appendix A (Figs. 6 and 7).

The two cluster model is utilized for this portion of the analysis due to the simple elegance of analyzing two distributions within the investor class that encompass the data set through subsetting along variables representing active market participation. This approach is also in alignment with the heterogeneous models discussed in Sect. 3. There is a notable spread between the two cluster means. The means of each cluster are denoted by highlighted plus symbols ('+'). The corresponding numerical findings depicted herein are summarized in Appendix B, Table 2 by year and implicate, whereby implicate one simply refers to the first of five data sets for which missing values were imputed. The remaining data points are simply denoted by color coded circles, whereby the size of a given circle is determined by the weight given to that observation in SCF data and the color red reflects the overall larger population (determined by the sum of weights), which help identify to which cluster grouping any one observation belongs. The first year for which the data was estimated is 1995, prior to Quantitative Easing.

From each of Figs. 2, 3 and 4, we can see how the clusters shift through the entire time horizon for the years 1995 through 2020. The years 2007 and onward clearly stand out as years with a substantial spread between the two groupings of households represented by the cluster means. This is in alignment with the peak of the 2007 market bubble, the subsequent market crash, the subsequent period of low interest rates and quantitative easing originally initiated by the Federal Reserve in response to the 2008 financial crisis.

The color coding of data points corresponds to placing any one given observation within one cluster or the other. In this case the color red was reserved for the distribution with the greater population, while the color blue was reserved for the smaller of the two clusters. The weights were used to calculate the overall population of households encompassing the so called 'investor class' studied herein and are included in the model output in Appendix B, Table 2. In this manner it is then worth noting periods when the color alternates between the upper and lower distribution.

²⁰ Please see Montalto and Sung (1996) for further explanation of the usage of such methods in SCF data. Additionally, see Kennickell (1998) for an explanation of how multiple imputation helps account for uncertainty in imputing missing values.

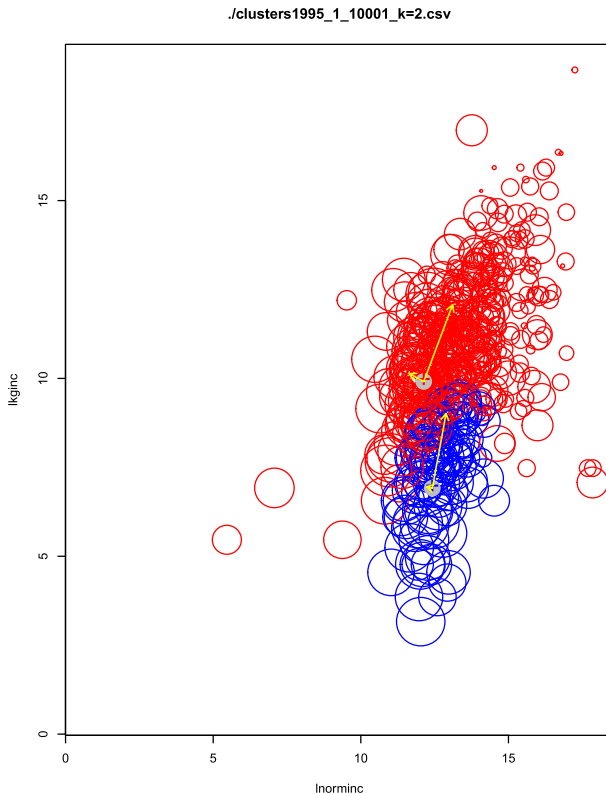


Fig. 2 Expectation maximization—two cluster model, 1995: depicts the preferable two cluster specification (as opposed to the one cluster model) and the principal components (in yellow) along which the data for each distribution is clustered, with log income on the x-axis and log capital gains income on the y-axis

The usage of Expectation Maximization also occasionally yields an observation that may probabilistically share attributes with either both households, or one household more than the other depending upon the specific observation. In this sense the model does not place observations only in one of two distributions in the manner of K-Means, but allows for some probability by which membership can vary. In theory the more dimensions added to the model, the greater the color variation that may be observed in the households present in the survey data.

The utility of the two cluster model is statistically evidenced by the lower Bayesian Information Criterion (BIC), or alternatively the higher log likelihood, of the two cluster model as opposed to a single cluster, demonstrated in Appendix B, Table 2. As such the delta in the BIC between the one and two cluster model demonstrates the preference for the two cluster specification. Additionally, it should be noted that, as opposed to emphasizing the optimal number of clusters, the hypothesis here is that the proposed distribution is a Gaussian Mixture model. Convergence was set to $1 * e^{-8}$ in the observed data log likelihood, as per eq. (4). The model was run with

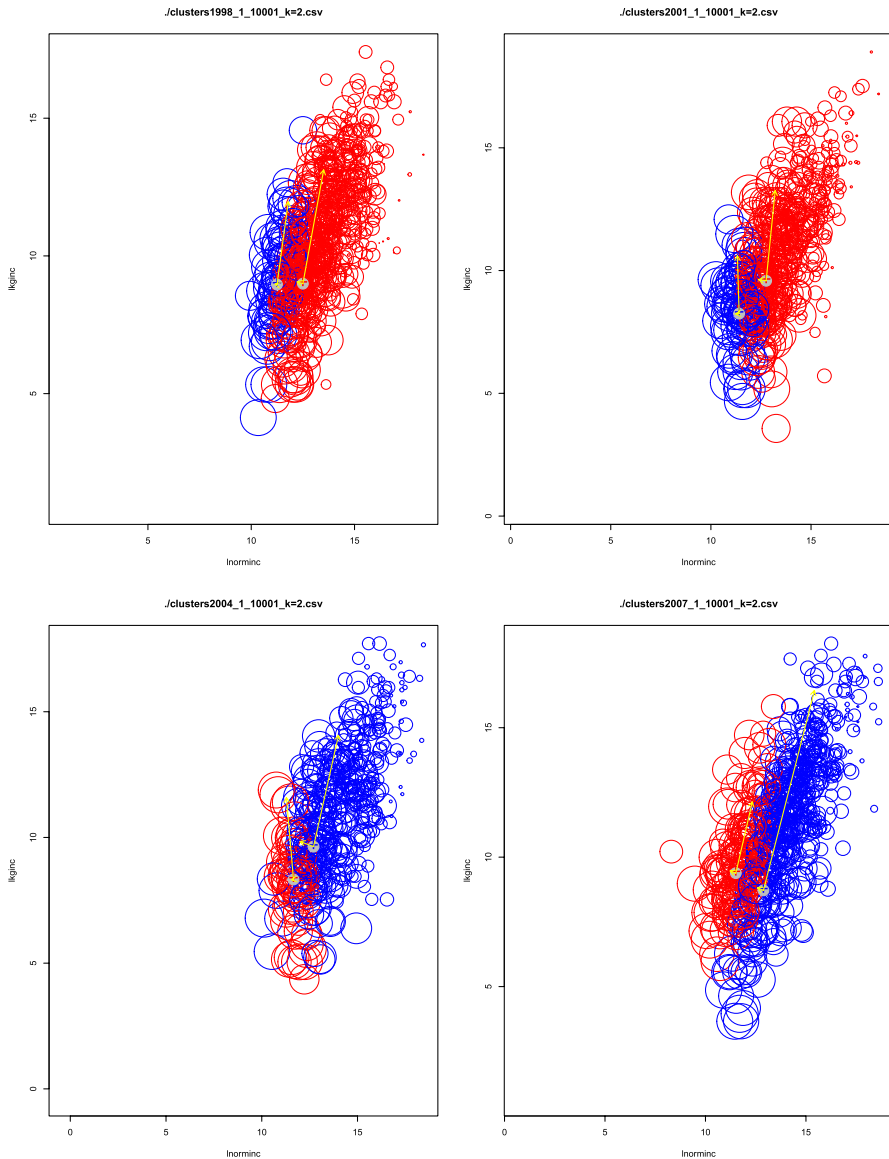


Fig. 3 Expectation maximization—two cluster model, 1998–2007: depicts two separate classifications through time with population size defined by color

various numbers of reps ranging from 20 to 500, to verify the stability of the results such that the findings could be readily reproduced.

Utilizing an unsupervised machine learning method and making use of the covariance matrices, the principal components, denoted by the mean centered yellow lines with arrows in Figs. 2, 3 and 4, presented allow for visual and numerical verification of orthogonality through calculation of the requisite eigenvalues/vectors of

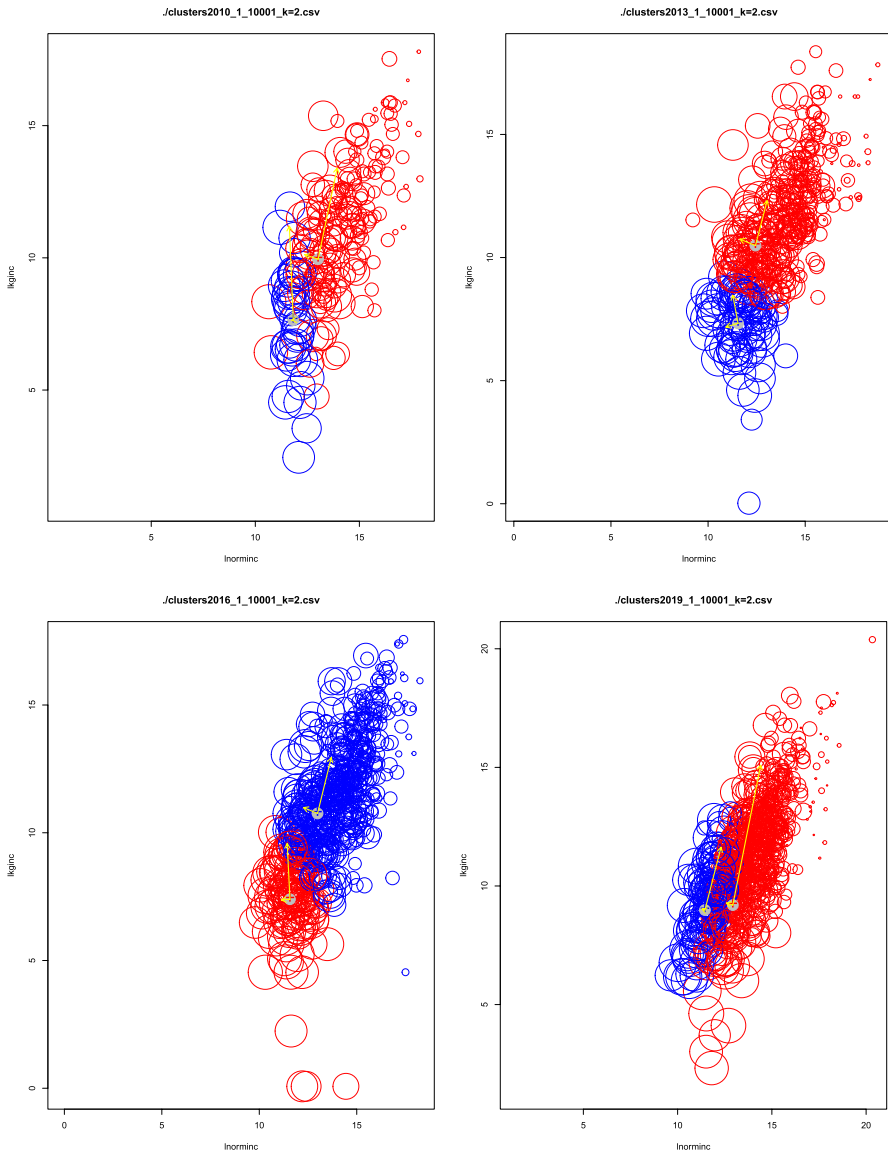


Fig. 4 Expectation maximization—two cluster model, 2010–2019: depicts a growing spread between two distributions through time, with cluster means depicted at the center of each cluster denoted by the ‘+’ symbol from which the first and second principal components (in yellow) converge

the covariance matrices.²¹ The eigen decomposition for the principal components of

²¹ It should be noted that in the Principal Component Analysis (PCA), we care about the mean centering within each cluster. However, with regard to the normalization and the standard deviation, we care about the scaling differences among logarithmic variables for economic reasons. Hence, we omit here normalized units of the standard deviation.

the two clusters was exported as a table and demonstrated in Appendix C, Table 3. To visually see this, the graphs can be inspected whereby the axes are modified geometrically to demonstrate a perfect square. As such the larger arrow represents the first principal component of the given cluster, such that the line passes through the cluster mean and minimizes the sum of squares of the distances from the points to the line. The smaller arrow represents the second principal component. In each case the principal components were calculated based upon each individual cluster of households using the corresponding covariance matrices.

The principal component analysis, a common dimensionality reduction technique in the presence of latent variables, helps clarify the differing slopes of the two clusters of households as well as the dimensions (variables) constituting the primary cause for differences (variance) in wealth in any given year. The eigen decomposition is readily visualized through the projection of the data onto the yellow lines with arrows such that the distance between such projected points is maximized along the line while the distance to the lines is minimized. From these results it appears the difference in most recent years is primarily attributable to 'kginc' as opposed to 'norminc'. As such all eigenvalues and vectors were calculated such that the greatest source of variance in wealth accumulation amongst households within each cluster could be verified through the first principal component, frequently appearing visually as an almost vertical line. The statistical spread between the cluster means is then measured through calculation of the Mahalanobis Distance (MD).

4.4 Calculating the Mahalanobis Distance

Once convergence was verified for the two cluster model, the cluster means were utilized to calculate the Mahalanobis distance, between the $k = 2$ means, for the two dimensional EM model. As such, the MD serves as a measure of inequality between the distributions. For each year in the dataset, there are five implicates, reflecting the five times missing values were imputed by the Federal Reserve Board to account for uncertainty. This, in turn, necessitates five calculations of the Mahalanobis Distance. The purpose is to measure the distance between the distributions in dimensional space not in Euclidian terms, but rather in relation to the standard deviations that one cluster mean is from the other distribution. The Mahalanobis distance can be measured in one of several possible frames of reference, that of the smaller cluster, the larger cluster or the full-sample covariance matrix Σ . As such all three measures of the Mahalanobis distance $D_m(\vec{\mu}_i, \vec{\mu}_j)$ were calculated from the EM Algorithm output.

$$D_M(\vec{\mu}_i, \vec{\mu}_j) = \sqrt{(\vec{\mu}_i - \vec{\mu}_j)^T \Sigma^{-1} (\vec{\mu}_i - \vec{\mu}_j)} \quad (13)$$

s.t. μ_i and $\mu_j \in \mathbb{R}^2$

In this particular case, the distance measures how many standard deviations one point is from the other point with n-dimensional data. Hence, if the points were identical, the distance would be zero and as one cluster mean μ_i moves away

from the other cluster mean μ_j along each principal component axis, the distance grows. As such, from the cluster distribution, $\vec{\mu}_i = (\mu_{i1}, \mu_{i2}, \mu_{i3}, \dots, \mu_{in})$, while $\vec{\mu}_j = (\mu_{j1}, \mu_{j2}, \mu_{j3}, \dots, \mu_{jn})$.

As shown in Fig. 5, the Malhalanobis Distance in this case was measured in terms of the full-sample covariance matrix. As previously stated, this could be set in terms of either the smaller or larger cluster as well. In such situations the scale would change accordingly. For example, the distance, measured in terms of the smaller cluster, would yield a larger spread while utilizing the larger cluster would conversely change the scale in the opposite direction. Although seemingly counter-intuitive, the logic should be clear from careful examination of the nature of the measurement. The full-sample covariance matrix was utilized, though the same trends are present regardless of the chosen covariance matrix.

Figure 5, demonstrates that the distance between the cluster means of the upper and lower distributions increased in the years since the initiation of the Federal Reserve Board's Policy of Quantitative Easing. It should be noted that the final year of the triennial survey included some data from 2020 when the Sars-CoV-2 pandemic began later requiring a re-initiation of Quantitative Easing, see Bricker et al. (2020).²² The data output from running 200 reps can be seen in Table 5, Appendix D.

Figure 5, confirms the findings from the WEM cluster model for heterogeneous households. It can be seen that the distance between the two households was clearly decreasing for all five implicates consistently, particularly in the years 1998 through 2004, until the year 2007. From the year 2007 through 2019/2020, the Malhalanobis Distance increased to approximately 1.64, across all five implicates, depicted on the y-axis of Fig. 5. This bifurcation is the growing divide between distributions confirming the theoretical prediction from the proposed model of heterogeneous households. In fact multiple implicates provide evidence for heterogeneous households having a smaller distance between them in regards to the two-dimensions under consideration ('kginc' and 'norminc') for 1995 to 2007. However, as evidenced by a Malhalanobis Distance greater than 1.64 (see Fig. 5; Table 5), in the year 2007 and on, a low interest rate environment and unconventional monetary policy are accompanied by growing wealth and income divides as evidenced by the dimensions demonstrated by the EM algorithm.

It is worth noting that the findings demonstrated in Fig. 5, confirm the theorized heterogeneous household models presented in Sect. 3. As such the WEM cluster model utilized for unsupervised learning in an econometric application, along with calculation of the Malhalanobis Distance provides empirical validation and confirms the theoretical framework of analysis implied herein.

²² Technically, it is specifically stated, as outlined, Bricker et al. (2017), that any data collected by the Federal Reserve Board for the 2020 Survey of Consumer Finances (SCF), does not include the dramatic effects of the stock market crash and subsequent Monetary Policy that was enacted. The most recent survey year still is utilized in this study to evaluate the longer lasting effects, specifically on the Financial Markets dimension of this model, from prolonged low interest rates and previous periods of Unconventional Monetary Policy.

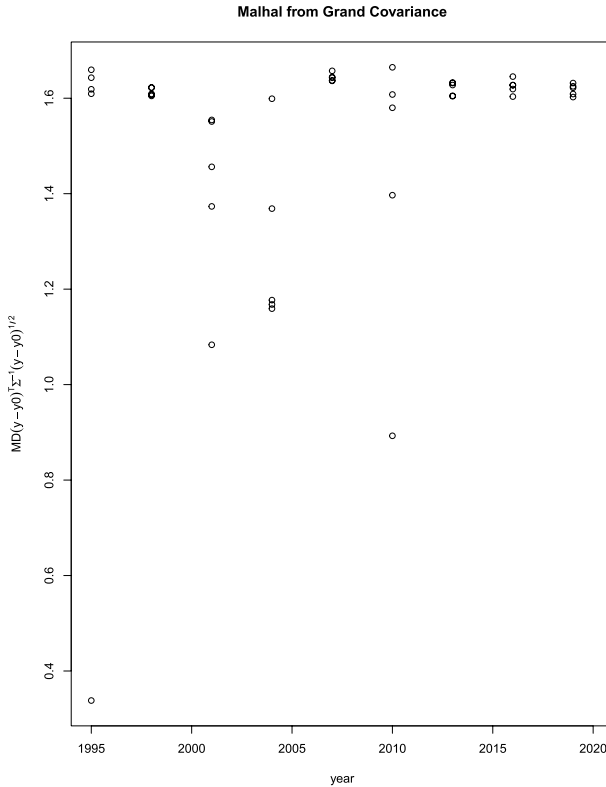


Fig. 5 Malhalanobis distance between two cluster means for log ‘norminc’ and log ‘kginc’: depicts a notable decrease in MD until 2007 (x-axis) when it rises to above 1.64 (y-axis), across all five implicates, and then remains elevated thereafter with a large outlier present in 1995 and then again in 2010, reflecting two years with greater uncertainty in one of the implicates for which missing data was multiply-imputed

4.5 Causality

We next turn to the matter of causality. Although our measure of economic inequality (i.e. the spread between the ‘k’ cluster means measured by the MD) has thus far been shown to be increasing, we utilize a simple directional model as supporting evidence in which it can be shown that the lagged effect of one variable upon another is not interchangeable. A common approach is to use granger causality and a simple expression of a restricted and unrestricted model specification is demonstrated below.

$$y_t = \alpha + \omega_1 y_{t-1} + \omega_2 y_{t-2} + \epsilon_t \quad (14)$$

$$y_t = \alpha + \omega_1 y_{t-1} + \omega_2 y_{t-2} + \phi x_{t-1} + \phi x_{t-2} + \epsilon_t \quad (15)$$

Equations (14) and (15) were applied to the mean MD (denoted by the variable ‘mimp’ in Appendix E) for all five implicates for a given year, as well as triennial variables in the form of a dummy variable for the years of economic shocks in which QE was implemented in the US and the three year average Federal Funds Rate, centered for the middle year of the triennial periods.²³

The results of statistical output of an F-test are displayed in Table 6 of Appendix E. As can be seen the effects of QE and the Federal Funds rate are quite persistent with two period lagged statistical significance at approximately the 8% and 4% levels for alpha, respectively. However, clearly this is not bi-directional in nature as evidenced by the lack of statistical significance when the autoregressive terms are switched for the converse specification. It should be noted that these findings are keeping up with the established literature of strong persistent effects of monetary policy on other researcher’s measures of inequality within the distribution, see Sect. 2.

5 Conclusion

This paper studied the impact of recent monetary policy on the wealth dynamics for heterogeneous households—considering the possibility for one group wherein wealth is dominantly generated from assets while for the other group wealth is dominantly generated from mainly labor income. This issue was pursued to what extent wealth dynamics and wealth disparity might have been impacted by monetary policy enacted after the great recession 2007–9 as well as in response to the Sars-CoV-2 pandemic. Empirically this has been a challenging issue. We apply unsupervised statistical learning methods to explore some dimensions of this issue taking the weighting schematics of wealth survey data sets into account in the empirical analysis.

Econometrically, utilizing unsupervised machine learning models from the data sciences, we are able to verify that the two cluster model validates many features of the theoretical models and assumptions presented in Sect. 3 for an economy with heterogeneous households. This is evidenced by a new normal in the bifurcation of cluster means within the investor class in the policy interventions after the years 2007 through the interventions undertaken by the Federal Reserve Board. Essentially, persistently low interest rates and unconventional monetary policy are shown to accompany a growing wealth disparity as visually verified from the graphical output contained herein and further confirmed by the output included in Appendices A through E.

Further research would benefit from studying capital gains as well as losses across various dimensions. Additionally, the restrictions on the number of clusters could be relaxed such that the data determines the optimal number of clusters in any given period. It would be of interest to see how the number of ‘k’ clusters changes through time as well as which distributions contain larger and smaller populations, accounting for weighting schematics.

²³ Federal Funds Rate data utilized may be found at the following Federal Reserve URL: <https://fred.stlouisfed.org/series/FEDFUNDS>.

A Cluster Variance Amongst Implicates

When analyzing the distance between cluster means for multiply imputed SCF data, it is necessary to consider the variance in estimates with regard to all implicates for given years (Figs. 6 and 7).

B Numerical Cluster Output and BIC—Calculated From Log Likelihood of One and Two Cluster Models

See Table 2.

Table 2 column heading definitions:

1. Year—refers to the survey year for which data was used
2. Implicate—refers to the specific implicate for which missing values were imputed
3. MinBIC—refers to the model replicate with the minimum BIC or maximum log likelihood
4. BICGC—refers to the corresponding BIC for the one cluster specification utilizing the grand covariance matrix
5. DeltaBIC—refers to the difference between the BICs for the one and two cluster model displaying evidence for the two cluster specification
6. Pop0—refers to the household population of the cluster after accounting for the weights
7. Pop1—refers to the household population of the second cluster also through utilization of the corresponding implicate weights
8. $\mu[0][0]$ —The ‘lnorminc’ value (log of normal income) for the cluster mean
9. $\mu[0][1]$ —The ‘lkginc’ value (log of capital gains/losses) for the cluster mean
10. $\mu[1][0]$ —The ‘lnorminc’ value (log of normal income) for the second cluster mean
11. $\mu[1][1]$ —The ‘lkginc’ value (log of capital gains/losses) for the second cluster mean

C Principal Component Analysis; Eigen decomposition

See Tables 3 and 4.

D Malhalanobis Distance

See Table 5.

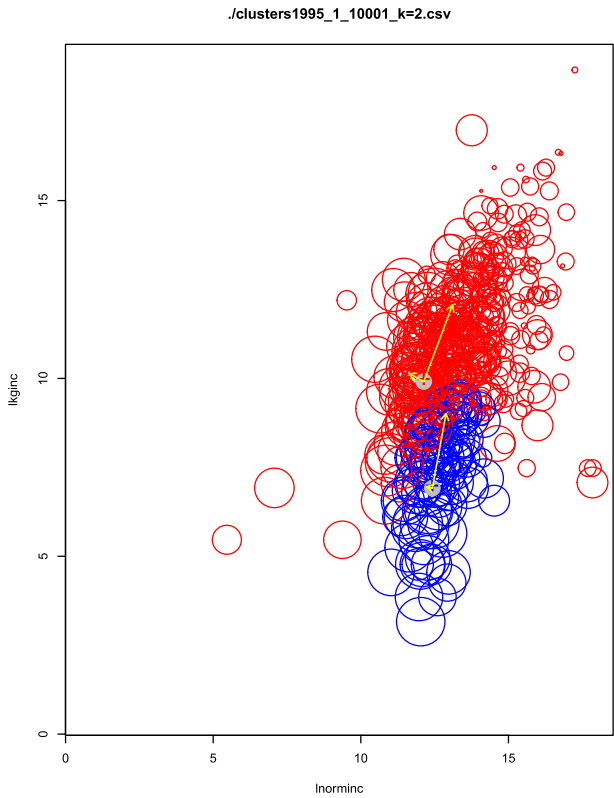


Fig. 6 WEM cluster variance—implicate 1

E Tests of Causality

See Table 6.

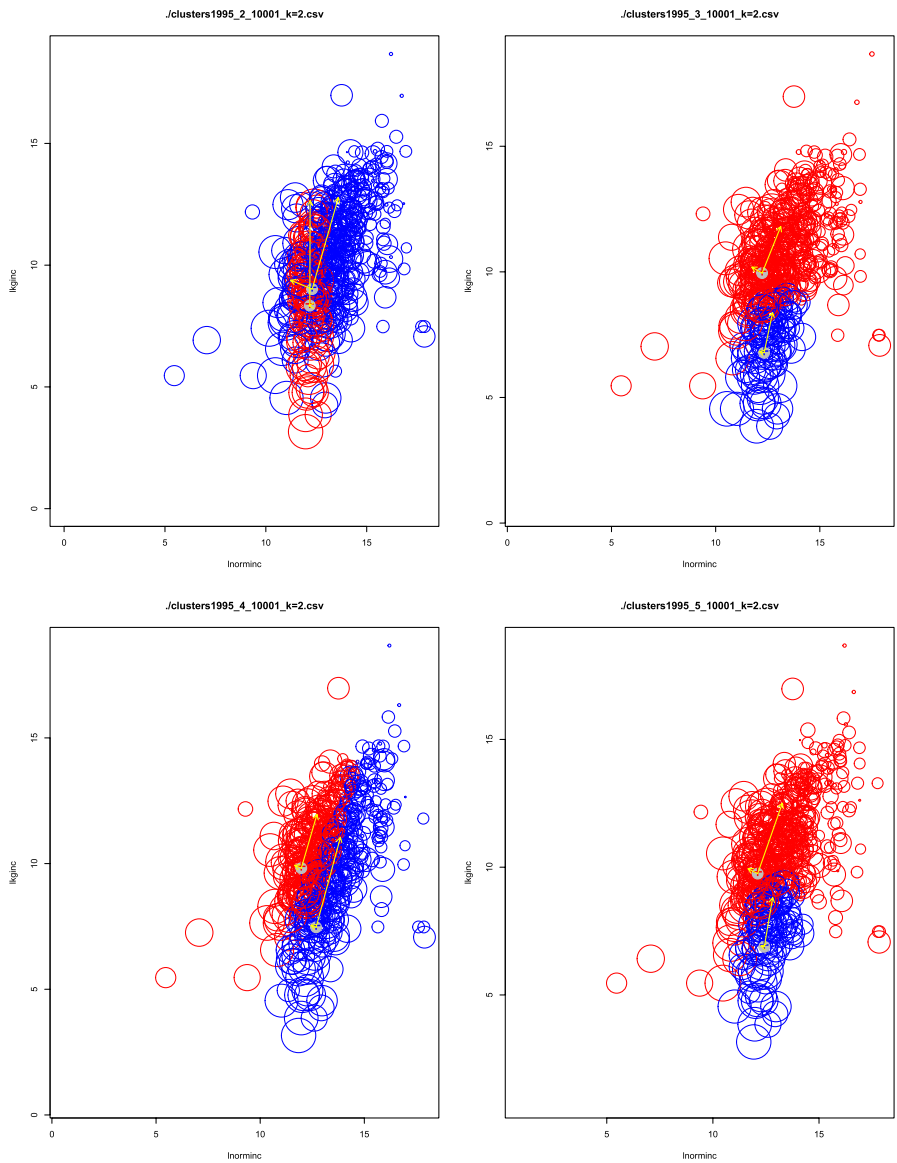


Fig. 7 WEM cluster variance—implicates 2–5

Table 2 WEM cluster output

Year	Implicate	MinBIC	BICGC	DeltaBIC	Pop0	Pop1	Mu[0][0]	Mu[0][1]	Mu[1][0]	Mu[1][1]
1995	1	3.28E+06	1.43E+08	- 1.40E+08	243,451	355,050	12.4216	6.91548	12.1319	9.91301
1995	2	3.21E+06	1.26E+08	- 1.23E+08	280,434	303,353	12.2951	9.0158	12.1788	8.3379
1995	3	3.17E+06	1.45E+08	- 1.41E+08	236,356	355,352	12.3243	6.77692	12.23	9.95161
1995	4	3.23E+06	1.31E+08	- 1.28E+08	261,526	325,323	12.6839	7.4652	11.9733	9.8183
1995	5	3.30E+06	1.43E+08	- 1.40E+08	233,780	370,884	12.4219	6.88521	12.1087	9.74831
1998	1	6.42E+06	2.42E+08	- 2.36E+08	462,962	747,636	11.2472	8.96035	12.4992	8.99184
1998	2	6.57E+06	2.26E+08	- 2.19E+08	578,271	639,052	11.3775	9.12577	12.6134	8.90335
1998	3	6.20E+06	2.36E+08	- 2.30E+08	563,193	614,061	12.4649	10.5825	11.5744	7.55571
1998	4	6.53E+06	2.44E+08	- 2.37E+08	601,796	633,490	11.5143	7.51984	12.5156	10.3641
1998	5	6.36E+06	2.24E+08	- 2.18E+08	470,203	724,104	11.2632	9.17119	12.4565	8.83905
2001	1	5.81E+06	2.12E+08	- 2.06E+08	536,466	538,247	11.4096	8.24517	12.758	9.58388
2001	2	5.49E+06	2.20E+08	- 2.14E+08	492,774	546,634	12.6407	9.04128	11.5908	8.99609
2001	3	5.36E+06	2.07E+08	- 2.01E+08	501,331	503,288	11.5266	8.97886	12.6393	8.84704
2001	4	5.72E+06	2.42E+08	- 2.37E+08	521,271	571,576	12.5319	8.93841	11.7202	8.91717
2001	5	5.54E+06	2.09E+08	- 2.03E+08	511,459	523,602	11.4015	8.1544	12.7365	9.54341
2004	1	4.13E+06	1.30E+08	- 1.26E+08	357,846	368,326	12.7025	9.63389	11.6566	8.32819
2004	2	4.30E+06	1.37E+08	- 1.33E+08	364,606	385,314	12.7125	9.61987	11.6535	8.31405
2004	3	4.11E+06	1.23E+08	- 1.19E+08	355,668	356,711	12.723	9.67656	11.6449	8.36435
2004	4	4.18E+06	1.36E+08	- 1.31E+08	314,460	422,927	12.5976	10.7673	11.8821	7.52376
2004	5	4.14E+06	1.32E+08	- 1.28E+08	337,090	388,383	12.5092	10.4889	11.9031	7.67306
2007	1	6.12E+06	1.34E+08	- 1.28E+08	404,620	626,748	12.8842	8.71816	11.5102	9.38198
2007	2	6.32E+06	1.33E+08	- 1.27E+08	417,614	642,194	12.9724	9.01059	11.4248	9.23231
2007	3	5.95E+06	1.25E+08	- 1.19E+08	386,856	610,201	12.9523	8.8329	11.4914	9.32607
2007	4	6.07E+06	1.33E+08	- 1.27E+08	417,521	609,842	12.9347	8.90061	11.4885	9.4284
2007	5	6.28E+06	1.31E+08	- 1.25E+08	435,264	619,622	12.8918	8.72806	11.4594	9.52971

Table 2 (continued)

Year	Implicate	MinBIC	BICGC	DeltaBIC	Pop0	Pop1	Mu[0][0]	Mu[0][1]	Mu[1][0]	Mu[1][1]
2010	1	1.55E+06	5.71E+07	- 5.56E+07	141,347	144,069	11.8231	7.64547	13.0165	9.9486
2010	2	1.55E+06	5.51E+07	- 5.36E+07	114,473	163,310	12.2229	9.45843	12.6321	8.38148
2010	3	1.57E+06	5.90E+07	- 5.75E+07	132,936	147,251	13.1826	9.55661	11.7761	7.96774
2010	4	1.61E+06	5.37E+07	- 5.21E+07	122,981	162,189	13.3056	10.1711	11.7763	7.78952
2010	5	1.53E+06	5.45E+07	- 5.30E+07	132,557	143,297	13.1412	10.0115	11.7399	7.71537
2013	1	3.74E+06	9.51E+07	- 9.14E+07	297,435	347,185	11.5437	7.32887	12.4388	10.4849
2013	2	3.70E+06	9.32E+07	- 8.95E+07	306,911	331,642	11.4616	9.80421	12.5426	8.35511
2013	3	3.66E+06	9.19E+07	- 8.83E+07	281,687	346,521	11.5626	7.29239	12.4403	10.4718
2013	4	3.65E+06	9.17E+07	- 8.81E+07	281,847	343,100	11.5021	9.99531	12.4678	8.31927
2013	5	3.68E+06	9.01E+07	- 8.64E+07	272,927	357,507	11.5472	10.1604	12.3787	8.22499
2016	1	5.06E+06	1.23E+08	- 1.18E+08	385,092	472,621	12.9888	10.7562	11.5723	7.39786
2016	2	5.04E+06	1.24E+08	- 1.19E+08	396,521	452,413	12.9663	10.6687	11.5613	7.38732
2016	3	5.03E+06	1.24E+08	- 1.19E+08	364,950	491,639	13.0186	10.7962	11.5905	7.45088
2016	4	5.14E+06	1.29E+08	- 1.24E+08	380,626	495,836	13.0269	10.715	11.5653	7.50603
2016	5	5.43E+06	1.27E+08	- 1.21E+08	384,543	524,235	13.0489	10.8349	11.5919	7.54024
2019	1	5.65E+06	1.45E+08	- 1.39E+08	480,949	515,653	11.4481	8.98513	12.9283	9.19429
2019	2	5.59E+06	1.39E+08	- 1.33E+08	482,139	513,576	11.3831	8.85895	12.9602	9.29933
2019	3	5.56E+06	1.36E+08	- 1.31E+08	484,518	502,575	11.402	8.91851	12.9632	9.29251
2019	4	5.57E+06	1.42E+08	- 1.36E+08	473,546	519,109	11.4156	8.97558	12.9354	9.27615
2019	5	5.60E+06	1.38E+08	- 1.33E+08	459,340	513,187	11.4036	8.8931	12.9245	9.26217

Table 3 Eigen decomposition: implicate 1 of 5 [,1]; [,2] refers to principal components 1 and 2 respectively

Implicate 1, Cluster 1		Implicate 1, Cluster 2	
Eigenvectors 1995 [,1]	Eigenvectors 1995 [,2]	Eigenvectors 1995 [,1]	Eigenvectors 1995 [,2]
0.1856386	-0.9826181	0.4312182	-0.9022477
0.9826181	0.1856386	0.9022477	0.4312182
Eigenvectors 1998 [,1]	Eigenvectors 1998 [,2]	Eigenvectors 1998 [,1]	Eigenvectors 1998 [,2]
0.1775838	-0.9841057	0.2320989	-0.9726922
0.9841057	0.1775838	0.9726922	0.2320989
Eigenvectors 2001 [,1]	Eigenvectors 2001 [,2]	Eigenvectors 2001 [,1]	Eigenvectors 2001 [,2]
-0.03672905	-0.99932526	0.1222083	-0.9925045
0.99932526	-0.03672905	0.9925045	0.1222083
Eigenvectors 2004 [,1]	Eigenvectors 2004 [,2]	Eigenvectors 2004 [,1]	Eigenvectors 2004 [,2]
0.2827346	-0.9591982	-0.1126528	-0.9936344
0.9591982	0.2827346	0.9936344	-0.1126528
Eigenvectors 2007 [,1]	Eigenvectors 2007 [,2]	Eigenvectors 2007 [,1]	Eigenvectors 2007 [,2]
0.3124177	-0.9499448	0.2855221	-0.9583721
0.9499448	0.3124177	0.9583721	0.2855221
Eigenvectors 2010 [,1]	Eigenvectors 2010 [,2]	Eigenvectors 2010 [,1]	Eigenvectors 2010 [,2]
0.1059049	-0.9943763	-0.03443698	-0.99940687
0.9943763	0.1059049	0.99940687	-0.03443698
Eigenvectors 2013 [,1]	Eigenvectors 2013 [,2]	Eigenvectors 2013 [,1]	Eigenvectors 2013 [,2]
-0.2325588	-0.9725823	0.3128055	-0.9498172
0.9725823	-0.2325588	0.9498172	0.3128055
Eigenvectors 2016 [,1]	Eigenvectors 2016 [,2]	Eigenvectors 2016 [,1]	Eigenvectors 2016 [,2]
0.2867112	-0.958017	-0.07082513	-0.99748875
0.958017	0.2867112	0.99748875	-0.07082513
Eigenvectors 2019 [,1]	Eigenvectors 2019 [,2]	Eigenvectors 2019 [,1]	Eigenvectors 2019 [,2]
0.3152862	-0.9489966	0.2452179	-0.969468
0.9489966	0.3152862	0.969468	0.2452179

Table 4 Eigendecomposition: in conjunction data displays orthogonality of principal components [1]; [2] refers to the Eigenvalues corresponding to the first and second axis respectively

Year (implicate 1)	Cluster 1		Cluster 2	
	Eigenvalues [1]	Eigenvalues [2]	Eigenvalues [1]	Eigenvalues [2]
1995	2.33	0.19	2.5	0.55
1998	2.73	0.12	4.6	0.28
2001	2.36	0.18	3.7	0.38
2004	4.61	0.72	3.28	0.1
2007	8.13	0.31	2.87	0.3
2010	3.1	0.44	3.4	0.16
2013	1.17	0.64	1.92	0.84
2016	2.38	0.75	2.22	0.42
2019	2.75	0.17	6.22	0.34

Table 5 Malhalanobis distance from 200 reps: demonstrates calculations from the grand covariance matrix for implicates 1 through 5, with mean and between-implicate standard deviation

Implicate	1995	1998	2001	2004	2007	2010	2013	2016	2019
1	1.62	1.62	1.55	1.16	1.64	1.40	1.61	1.62	1.61
2	0.34	1.60	1.37	1.18	1.64	0.89	1.63	1.60	1.63
3	1.66	1.62	1.46	1.17	1.64	1.58	1.60	1.65	1.62
4	1.64	1.60	1.08	1.59	1.64	1.66	1.63	1.63	1.63
5	1.60	1.60	1.55	1.37	1.66	1.61	1.63	1.63	1.60
Between Implicate means	1.37	1.61	1.40	1.29	1.64	1.43	1.62	1.63	1.62
Between Implicate std	0.52	0.01	0.17	0.17	0.01	0.28	0.01	0.02	0.01

Table 6 Demonstrates persistent directional effects with statistical significance

Granger tests of causality			
Restricted model $lm(mimp \sim mimp_lag1 + mimp_lag2)$		Restricted model $lm(mimp \sim mimp_lag1 + mimp_lag2)$	
Unrestricted model	$lm(mimp \sim mimp_lag1 + mimp_lag2 + QE_lag1 + QE_lag2)$	Unrestricted model	$lm(mimp \sim mimp_lag1 + mimp_lag2 + FFR_lag1 + FFR_lag2)$
f statistic	11.03	f statistic	35.53
p value	0.08	p value	0.03
Restricted model $lm(QE \sim QE_lag1 + QE_lag2)$		Restricted model $lm(FFR \sim FFR_lag1 + FFR_lag2)$	
Unrestricted model	$lm(QE \sim QE_lag1 + QE_lag2 + mimp_lag1 + mimp_lag2)$	Unrestricted model	$lm(FFR \sim FFR_lag1 + FFR_lag2 + mimp_lag1 + mimp_lag2)$
f statistic	0.75	f statistic	0.37
p value	0.57	p value	0.73

The variable 'mimp' refers to the mean difference in cluster means (Malhalanobis distance) among the five implicates; the variable 'QE' refers to the dummy variable for quantitative easing; the variable 'FFR' refers to the federal funds rate

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Code Availability All codes have been cited and although data downloads were enabled through original sources the algorithms used for analysis are of the authors' custom development in both R and C++.

Declarations

Conflicts of interest The working paper draft is searchable in SSRN.

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