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Monetary Policy and the Evolution of Wealth Disparity

An Assessment Using US Survey of Consumer Finance Data

*There is explicitly stated no corporate affiliation with regard to any research or content presented herein.
All content contained herein, is purely representative of independent academic scholarly research
and findings of the two afore mentioned authors listed on the introductory page of this presentation.

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PART I

MODEL-GUIDED ASSESSMENT

I. 1. Introduction: **Inspiration**

Inspirations from:

=> “ Perhaps the most important influence, operating through changes in the rate of interest, on the readiness to spend out of a given income, depends on the effect of these changes on **the appreciation or depreciation in the price of securities and other assets.**”

John M. Keynes, The General Theory of Employment, Interest, and Money

=> Hyman Minsky: “Watch the **value of capital assets**...”

⇒ **Financial market** as a driver for **wealth inequality** (work with Chappe, Comp. Economics, 2018)

I. 1. INTRODUCTION: HETEROGENEOUS ASSET HOLDERS AND WEALTH DISPARITY

Empirical Fact: The **Gini Coefficient for wealth distribution** is worse than for income

⇒ **Monetary policy: UMP as QE** has the effect of asset price boom through the zero interest rate and **de-risking** of assets

- Asset prices: Equity, bonds, and housing prices; less so wage increase; Asset pricing research asks: **did QE** and decreasing wage share and stimulate an **asset price boom and wealth inequality?**
- CBs response: CBs' QE allowed employment and **wage income to rise**, see Draghi and Furceri, Lounyani et al. (2016). Yet the asset price effects were less investigated

⇒ **Model-guided assessment: Heterogeneous Asset Holders**, finite horizon decision-making

- Kaldor- Pasinetti (Kaldor: Workers are saving, Pasinetti: then they are also holding assets); Pasinetti: Saving rates different, but asset return the same, then wealth proportion constant over time, co-integrated?)
- Piketty (2014): $r-g$ statement, r , and g are time-varying, with two types of asset holders (asset income and labor income)
- Brunnermeier/Sanikov(2014): Households that save (supply of savings) and borrowers (experts, higher return)
- **Our dynamic** model: Heterogeneous households (dominantly asset income, dominantly labor income)

⇒ **Empirical Assessment:** With **Survey of Consumer Finance (SCF)**, cluster analysis not Gini Coeff

- Gordon in his book (2016) : “If the stock market continues to advance, we know that the inequality will increase, for capital gains on equity accrues disproportionately to the top income brackets” (Gordon 2016): 23).

I. 1. WEALTH INEQUALITY: MODEL-GUIDED ASSESSMENT (FINANCIAL MARKET AS DRIVER FOR WEALTH INEQUALITY?)

- ⇒ Asset owners: With a higher proportion of asset income than salary income
- ⇒ Workers: With a higher proportion of labor income than asset income

Financial market drivers for wealth inequality:

- 1. Information advantages and higher expected **returns** for owners of capital (see Arrow 1965), $r_c > r_w$ (r includes capital gains), (see also Chappe and Semmler, Comp. Economics, 2018); differential returns neglected in Pasinetti
- 2. Consumption (saving) rates are income dependent (consumption limit): **higher saving rates** for owners of capital, $s_c > s_w$, differential saving rates, capital formation neglected in Kaldor
- 3. If 1. and 2. holds: then capital share is rising faster (when more assets are owned); neglected in Piketty (2014)

Other drivers:

- Asset size as scale advantages **for leveraging**: Higher income and wealth allow for greater borrowing (collateral constraints for borrowing) and **net worth** rises
- **Labor income**: loss of bargaining power (Kumhof), lagging wage increase, higher corporate earnings discounted generates higher asset prices, (plus discretionary power of salary setting board rooms) (Krugman)
- Decline of **marginal tax rates**, see historical trends in Piketty' (2014) book

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I. 2. HETEROGENEOUS HOUSEHOLDS AND WEALTH ACCUMULATION- A MODEL

DID **QE** FAVOR TOP ASSET HOLDERS? DYNAMIC MODEL WITH CLUSTER ANALYSIS; PARKER/SEMMLER (**SSRN**), **FINITE HORIZON** DECISION-MAKING HOUSEHOLDS, CYNAMON AND FAZZARI (2016); **NMPC** SOLUTION METHOD

Two Households; one a **consumption smoother**, the other **adaptee** (see Veblen's work on conspicuous consumption)... $c_1, t = \text{optimal}$; $c_1, t = \text{adaptive}$,

$$\max_{\{C^*, \alpha\}} \int_0^{\infty} e^{-\delta} U(C_t) dt$$

Evolution of aggregate wealth (Guevenen, 2011): $\Rightarrow$

$$\dot{W}(t) = \alpha_t R_{e,t} W_t + (1 - \alpha_t) R_{f,t} W_t - C_t$$

Evolution of wealth of heterogeneous households; type 1 and 2 (stochastic):

$$V = \max_{\{c_{1,t}\}} \sum_0^T \beta^t U(c_{1,t} - \kappa \sigma_t^2) dt$$

such that

$$\sigma^2 = ((x(1) - \mu) + (x(2) - \mu))^2 / 2$$

$$\dot{x}_{1,t+1} = x_{1,t}(1 + \nu_1 x_{3,t}) - c_{1,t} - \varphi(x_{2,t})$$

$$\dot{x}_{2,t+1} = x_{2,t}(1 + \nu_2 x_{3,t}) - \tilde{c}_{2,t} + \varphi(x_{2,t})$$

$$\tilde{c}_{2,t+1} = \gamma_1 x_{1,t} + \gamma_2 c_1$$

$$x_{3,t+1} = \exp(\rho \ln x_{3,t} + z_k).$$

I. 2. HETEROGENEOUS HOUSEHOLDS AND WEALTH ACCUMULATION;

SEE PARKER/SEMMLER PAPER (SSRN), DID QE FAVOR TOP ASSET HOLDERS?

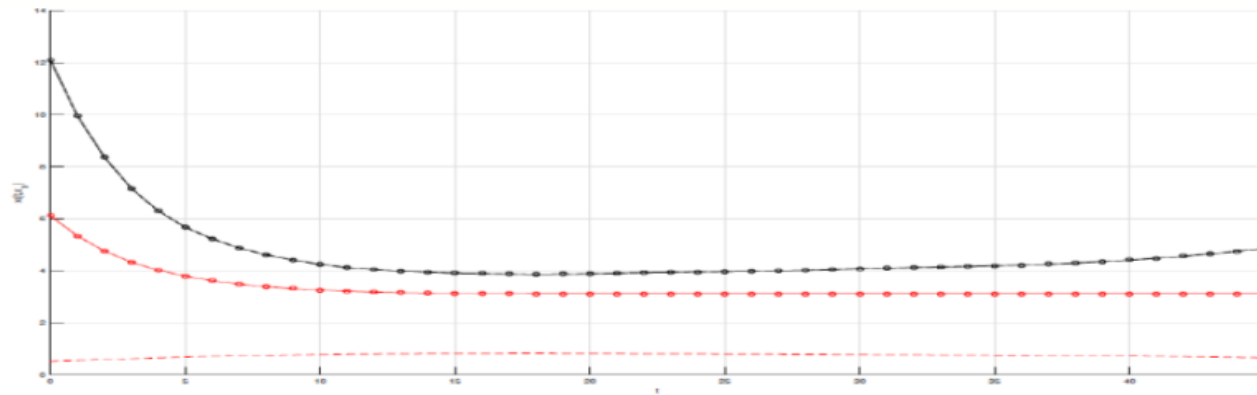


Figure 1: Dynamics of wealth: wealthy households (black), adaptive households (red), ratio of the two (broken line).

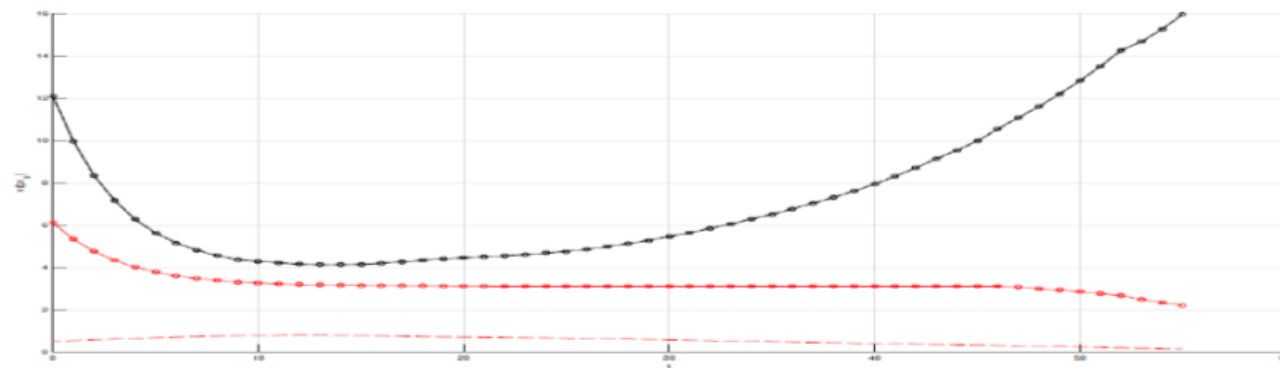


Figure 2: Dynamics of wealth: wealthy households (black), adaptive households (red), ratio of the two (broken line).

I. 3. LITERATURE: HETEROGENEOUS HOUSEHOLDS AND WEALTH ACCUMULATION

- Aggregate wealth dynamics, (Brunnermeier/Sannikov (2014), with **heterogeneous households** preferences (less patient and patient households, with different discounting of future incomes): $q_{kt+1} = x_{1,t} + x_{2,t}$
- We have **stochastic version** of wealth accumulation, See Lichtenberger, Braga and Semmler (Econometrics, 2022), for green and brown assets

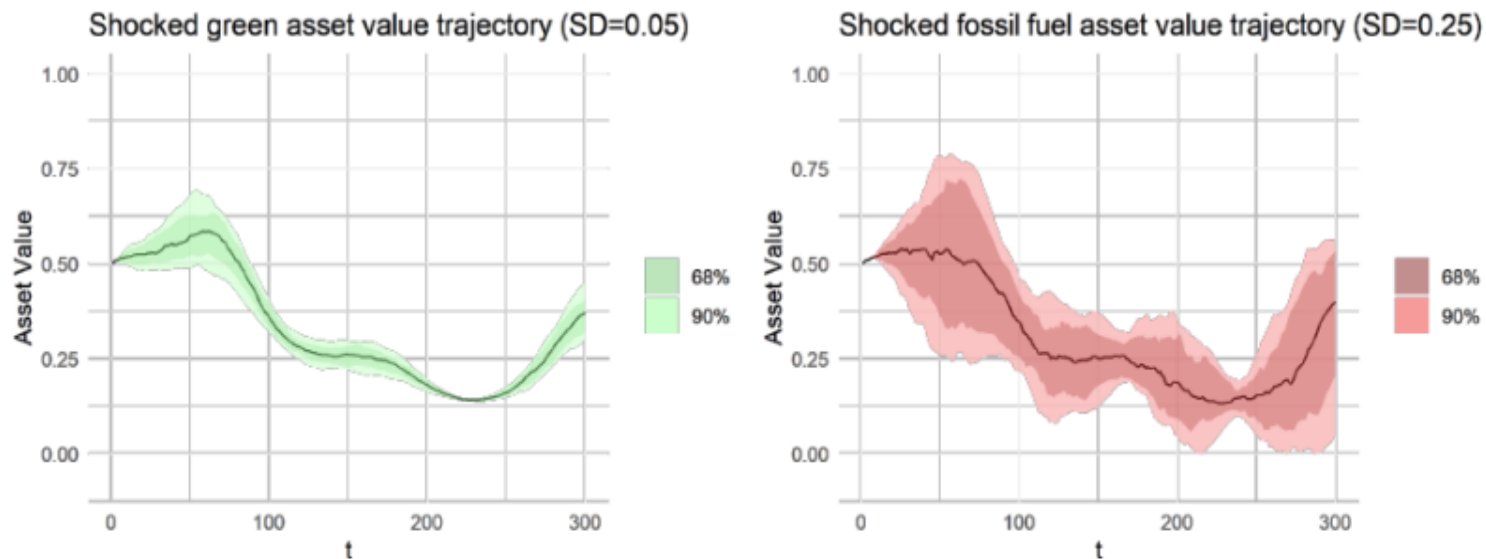


Figure A1. Stochastic version of the model shown in Figure 2 ($\sigma(SD) = 0.05$ for green bonds; $\sigma(SD) = 0.25$ for fossil fuel bonds; $\alpha = 0.034$).

PART II

The Empirical Assessments

⇒ Empirical Assessment: With SCF and cluster analysis

- Data utilized: Survey of Consumer Finances (SCF)... Detailed description following so as to explain the relation of weighted multiple imputation (utilizing five (5) imputates) and weighted Expectations Maximization (WEM) as a choice for empirical analysis.
- Our empirical analysis follows the literature in that the effects of large recent meltdowns (i.e. the financial crisis) have negatively affected the 80 – 90 % of wealth ownership shares, see (Wolff 2012).
- The various direct and indirect channels through which policy intervention can be shown to affect **heterogeneous** households is well documented, as per Coibion, Gorodnichenko, Kueng and Silvia 2017; Cui and Sterk 2021; Parker 1999). Our focus is on the income and capital gains channels through which policy affects distributions.
- However, unlike recent studies, (Andersen, Johannsen, Jørgensen and Peydro 2020; Holm, Paul and Tischbirek 2021), we utilize **statistical learning methods** regarding the effects of recent monetary policy on inequality.
- We further distinguish our analysis from use of liquid assets and/or income percentiles via use of **the Gini coefficient**, but rather utilize the **Malhalanobis Distance** and unsupervised machine learning which possesses benefits over traditional methods as we will highlight shortly.

PART II

Federal Reserve Survey of Consumer Finance (SCF) data

- The data used herein consists of the Federal Reserve's **Survey of Consumer Finance** Data primarily from the years 1995 through 2020.
- This is a multiply imputed data set with five (5) imputates to account for uncertainty and in which the sum of the weights represents the total tax paying population of the Nation.
- The process is similar to Gibbs Sampling implementation in the FRITZ software package. Further details are readily available, see (Kennickell 1998, Kennickell, Mcmanus and Woodburn 1996 and Montalto and Sung 1996).
- Our subset of the data consists of those households with more than a dollar across **five dimensions** (normal income, net worth, realized capital gains, unrealized financial capital gains and unrealized non-financial capital gains), all log transformed.
- Our data represents 10 – 15% of survey data where positive values were reported.
- Due to complexities in the tax code, we currently have omitted analysis of negative values.

PART II

Federal Reserve Survey of Consumer Finance (SCF) data

- Unlike researchers focusing on the Consumer Expenditure Survey (CEX) (Coibion et al. 2017) or administrative micro-data (Andersen et al. 2020), we utilize a data set that provides substantial access to the top 1% of the distribution as well.
- This also allows us to avoid the use of ex ante market values, as our focus is on reported capital gains, as reported in IRS tax filings.
- Due to the weighting schematics, the data is constructed in conjunction with the census in the U.S. so as to reflect household populations
- Additionally, as shown by researchers (see Andersen et al. 2020; Holm et al. 2021), the effects of monetary policy shocks on the income gradient can be seen for periods of approximately two years and persistent effects lasting from three to as long as five years. Hence, the triennial nature of SCF data is well substantiated in the literature.
- **Multiple imputation** (making use of over/under sampling) gives a good understanding of uncertainty due to non-response in survey data of such a complex nature.

PART II

Survey of Consumer Finance (SCF) - Multiply Imputed data

The weighted mean $\bar{x}_{imp}^{(j)}$ for implicate j may be expressed, as per (Kennickell, McManus and Woodburn 1996); (Kennickell and Woodburn 1999):

$$\bar{x}_{imp}^{(j)} = \frac{\sum_{i=1}^{N_j} x_i^{(j)} * \omega_i^{(j)}}{\sum_{i=1}^{N_j} \omega_i^{(j)}}$$

Such that i reflects the sample within implicate j. Therefore, the sample variance σ_x^2 due to the implicates is given by:

$$\sigma_{x,imp}^2 = \frac{\sum_{j=1}^5 (\bar{x}^{(j)} - \bar{x})^2}{4}$$

Such that $\bar{x}$ is the average of the implicate means $\bar{x}^{(j)}$

An additional source of variability is captured by the replicate weights systematically applied to the first implicates values. This represents random sizes of subpopulations associated with each measurement.

Multiply-imputed income and wealth surveys lend themselves to utilization of applied data science models such as weighted expectations maximization (WEM) **cluster modeling** where explicit use of the weights is required for maximization of the likelihood function.

PART II

The **Empirical Validation** of the Theoretical Heterogeneous Models

Unsupervised Machine Learning:

Choice of Model - Weighted Expectations Maximization (WEM) **Cluster Modeling**

WEM is a form of cluster modeling quite similar to the well known method of k-means. However, the use of probabilities allows for observations that may possess attributes of multiple distributions whereby assignment is more likely to be with one clustering than another.

Unlike k-means or other methods, the weighted nature of SCF data lends itself directly to application of a methodology that utilizes the weights in probabilistic assignment of observations to one cluster or another, whereas further research into the assignment probabilities may be well within grasp of analysts.

PART II

Weighted Expectations Maximization (WEM) Cluster Modeling

The WEM Algorithm for Cluster Modeling:

- The algorithm involves first an E-step, creating a function for the expectation of the log-likelihood (based upon the parameter estimate), followed by an M-step, such that the later is calculating the parameters that maximize the expected log-likelihood from the former step.
- This iterative process has been extended to weighted observations as demonstrated in common literature on the topic (Gebru, Alameda- Pineda, Forbes and Horaud 2016).
- This is especially applicable to multiply imputed data such as the Federal Reserve Board Survey of Consumer Finance (SCF) data, whereby no one implicate can be considered independent of the others for a given year.
- The mathematics are rather rigorous and do not appear intuitive from initial research on the topic. However, the algorithm is often found practical in the presence of latent variables and multi-modal data.

PART II

Weighted Expectations Maximization (WEM) Cluster Modeling

- Overall, we maximize the log-likelihood of the data to fit mixture parameters incorporating a number of important elements that may remain of interest for a closer examination of the algorithmic process.
- As such we define the density for observation of variable x over the latent variable z , such that:
- $p(x; \theta) = \sum_z p(x, z; \theta);$
- $\theta = \{(\mu_1, \Sigma_1, \dots, \mu_k, \Sigma_k)\}$, where:
- $\mu_k = \text{the Gaussian distribution mean};$
- $\Sigma_k = \text{the covariance matrix of the } k\text{th mixture component } k$
- $\omega_i = \text{The } i\text{-th weight}$
- $\pi_k = \text{the prior proportion of component } k$

PART II

Weighted Expectations Maximization (WEM) Cluster Modeling

- Thereby the (r+1)-th Expectation Maximization (EM) iteration consists of both the E-step (with posterior weights ω) and the M-step until convergence ($1 * e^{-8}$ in the observed data log likelihood), with the number of reps set to 20 – 500, is reached with η_{ik} as the specific posterior probability of assignment of point of type i to cluster k, conditional upon the weight, ω_i along with all of the parameters of all of the clusters.
- We then utilize likelihood and that:

$$\pi_k^{(r+1)} = \frac{1}{n} \sum_{i=1}^n \eta_{ik}^{(r+1)}$$

- Such that then this iterative process continues allowing us to arrive at the following expression:

$$\Sigma_k^{(r+1)} = \frac{\sum_{i=1}^n \omega_i \eta_{ik}^{(r+1)} \left(x_i - \mu_k^{(r+1)} \right) \left(x_i - \mu_k^{(r+1)} \right)^T}{\sum_{i=1}^n \omega_i \eta_{ik}^{(r+1)}}$$

- We thereby define cluster groupings according to $k = 2$ as per the heterogeneous framework for analysis and compare the BIC against $k=1$.

PART II

Weighted Expectations Maximization (WEM) Cluster Modeling

- A k-cluster, in this case two (2) cluster model better explains the data than a single distribution. The application of this methodology is well suited for economic data designed as multiply-imputed income and wealth surveys.

The benefits of this type of analysis, may be summarized simply:

- Discovering the structures underlying economic systems... if defensible.
- It's a non-linear problem, as such regression is inappropriate.
- It's an unsupervised problem that requires breaking away even from the traditional heterogeneous framework (i.e. there are micro populations).
- Hence, we can even condition the data with clusters to learn more as more data becomes available to help understand the structure of economies.
- It's a latent variable problem, within a class of such problems where we conjecture the existence of subpopulations and infer population parameters to connect actual data points to these subpopulations for our WEM inference based upon probabilities.

PART II

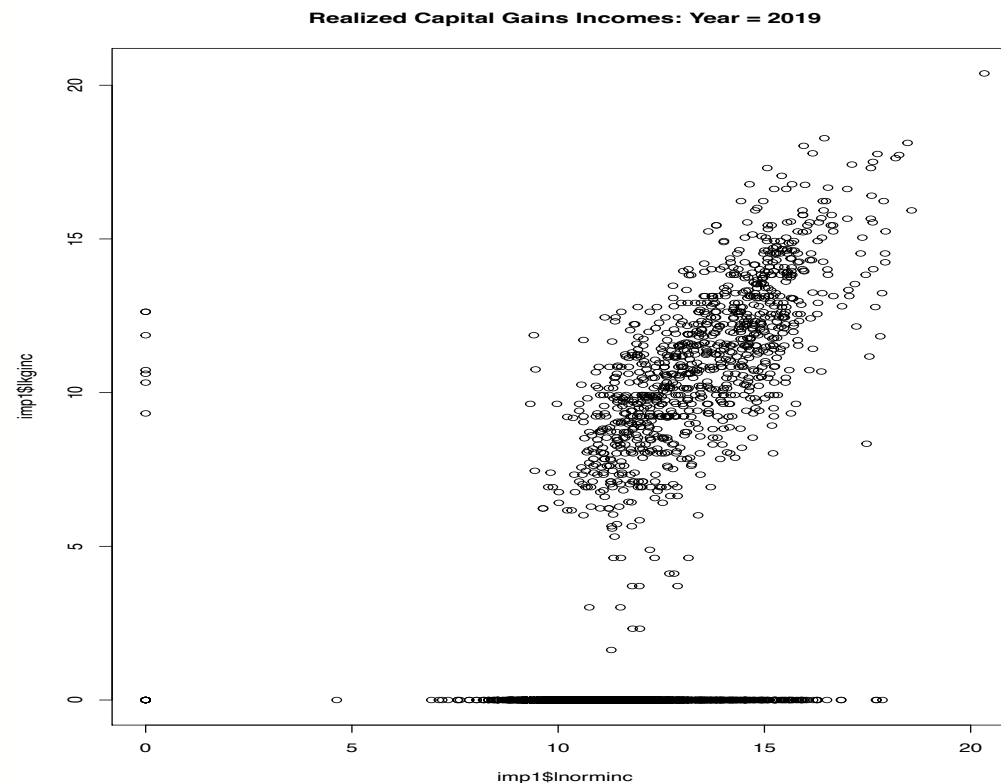
Weighted Expectations Maximization (WEM) Cluster Modeling

- Future research will continue to focus on this **multidimensional** problem where the latent variable is a real unobservable variable.
- After all, **even the notion of a "class" is not clearly well defined** and/or consistent under all economic structures across the entire world and through time.
- Hence, class assignment is both the task while also discovering the classes themselves (i.e. not a classical latent variable problem).
- So to be defensible, **all research must account for data variability** (as necessary) and finally variability within weighted expectations maximization (WEM) reps.
- Repeat Imputation Inference (Rii) techniques may be utilized in some data sets, but as per (Rubin 2004), this may also be viewed as a natural gesture for post-analysis of multiple imputed data sets, but is by no means, the only available choice as it may be a synonym for an averaging of imputation values.
- It should be noted, each implicate of each year is never considered in our analysis in isolation from the remaining implicates even if graphically demonstrated as such for exposition purposes.

PART II

Federal Reserve Survey of Consumer Finance (SCF) data

- Frequently **observed non-linearity** (log of capital gains (y-axis), log income (x)):



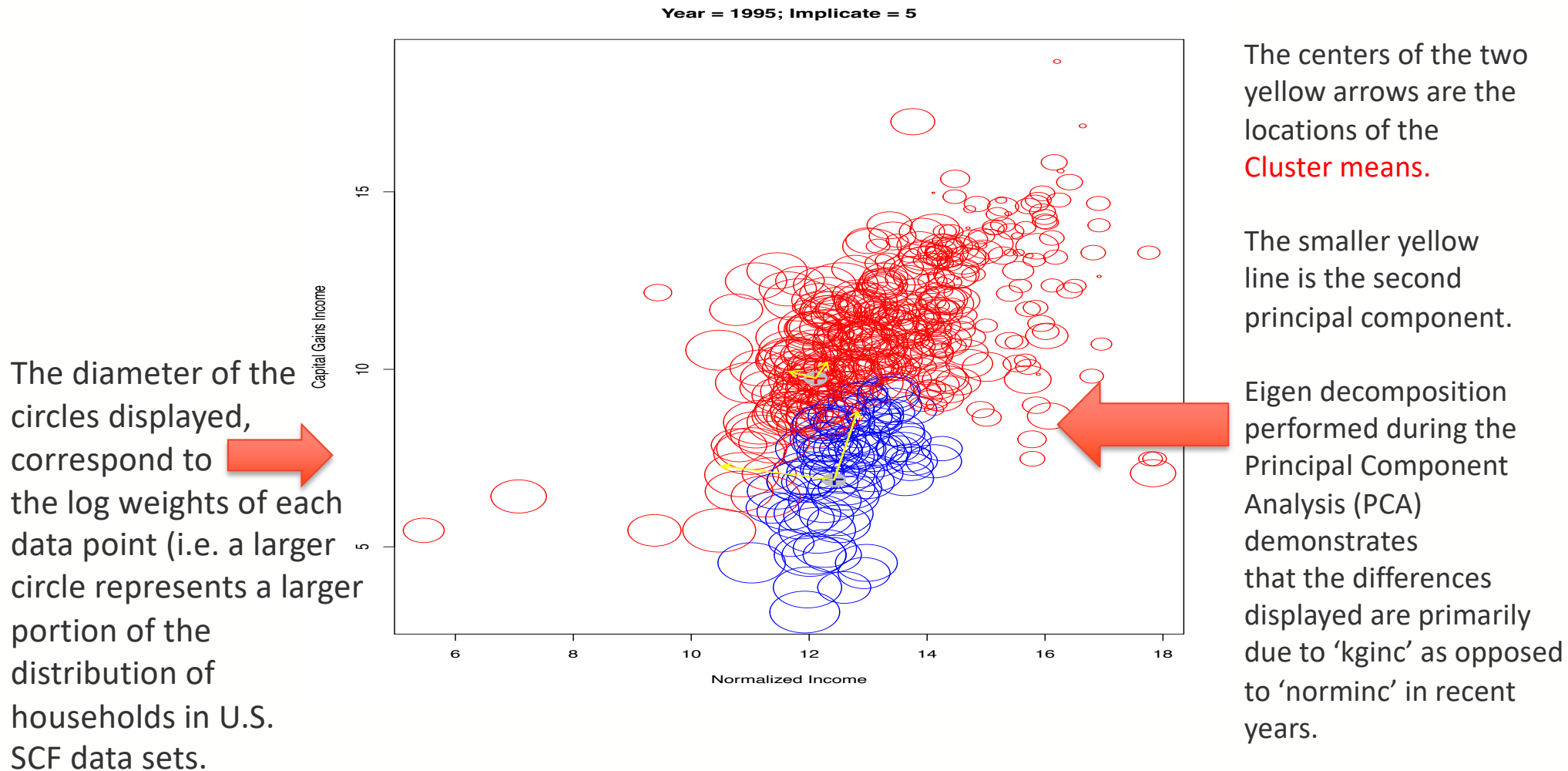
PART II

K = 2, Cluster Model Output for log of Normalized income and log of Capital Gains Income (two of the five dimensions)

- The cluster means, principal components, eigenvalues and eigenvectors were calculated for all five implicates within the SCF data.
- The model was created on the subset across five dimensions of log transformed variables whereas it was run on the first two of the five listed below:
 1. Realized Capital Gains (from tax filings)
 2. Normalized Income
 3. Unrealized Market Estimates of Real Estate Capital Gains
 4. Unrealized Market Estimates of Financial Capital Gains
 5. Net Worth Estimates
- The hypothesis is that the proposed distribution is a Gaussian Mixture model
 - Graphically, the weights from the data were utilized to determine the size of each circular observation in the cluster results that follow, while the yellow arrows reflect the principal components and eigenvalues/vectors.

PART II

K = 2, Cluster Model for log of Normalized income and log of Capital Gains Income (Red denotes the larger population in a given run)



PART II

Principal Component Analysis (PCA) and Dimensionality

Eigenvalues [1] and [2] refer to the first and second axis respectively

- The first principal component of a given cluster is displayed by a larger arrow, such that the line passes through the cluster mean minimizing the sum of squares of the distance from the points to the line.
- The components were calculated based on each cluster of households using the corresponding covariance matrices.
- During eigen decomposition the distance to the line is minimized and displays a trend in recent years whereby the difference is primarily attributable to 'kginc', as opposed to 'norminc' in SCF data.
- As such the greatest source of variance in wealth accumulation among households within each cluster could also then be verified as appearing as an almost vertical yellow line.
- The next step was to study the distance between cluster means through time.

PART II

Empirical Results of the Eigen Decomposition (ex: Implicate 1)

Year (Imp 1)	Cluster 1 Eigenvalue[1]	Cluster 1 Eigenvalue[2]	Cluster 2 Eigenvalue[1]	Cluster 2 Eigenvalue[2]
1995	2.33	0.19	2.5	0.55
1998	2.73	0.12	4.6	0.28
2001	2.36	0.18	3.7	0.38
2004	4.61	0.72	3.28	0.10
2007	8.13	0.31	2.87	0.30
2010	3.1	0.44	3.4	0.16
2013	1.17	0.64	1.92	0.84
2016	2.38	0.75	2.22	0.42
2019	2.75	0.17	6.22	0.34

PART II

Empirical Results of the chosen methodological approach detailed herein

First Step – Calculate the Mahalanobis Distance (MD) from the full-sample covariance matrix, between $k = 2$ cluster means.

$$D_m(\vec{\mu}_i, \vec{\mu}_j) = \sqrt{(\vec{\mu}_i - \vec{\mu}_j)^T \Sigma^{-1} (\vec{\mu}_i - \vec{\mu}_j)}$$

$$\text{s.t. } \mu_i \text{ and } \mu_j \in \mathbb{R}^2$$

In this analytical exercise, the distance measures how many standard deviations one point is from the other point within n-dimensional data.

Our findings suggest the distance between cluster means increased in the years 1998 through 2004, and again in 2007 through 2020 to an MD of approximately 1.64.

PART II

Empirical Results of the Mahalanobis Distance between Cluster Means

Implicate	1995	1998	2001	2004	2007	2010	2013	2016	2019
1	1.62	1.62	1.55	1.16	1.64	1.40	1.61	1.62	1.61
2	0.34	1.60	1.37	1.18	1.64	0.89	1.63	1.60	1.63
3	1.66	1.62	1.46	1.17	1.64	1.58	1.60	1.65	1.62
4	1.64	1.60	1.08	1.59	1.64	1.66	1.63	1.63	1.63
5	1.60	1.60	1.55	1.37	1.66	1.61	1.63	1.63	1.60
Between-imp mean	1.37	1.61	1.40	1.29	1.64	1.43	1.62	1.63	1.62
Between-imp std	0.52	0.01	0.17	0.17	0.01	0.28	0.01	0.02	0.01

PART II

Empirical Results of the chosen methodological approach detailed herein

Second Step – Use MD to study granger causality (two tests for QE & FFR).

$$y_t = \alpha + \omega_1 y_{t-1} + \omega_2 y_{t-2} + \epsilon_t$$
$$y_t = \alpha + \omega_1 y_{t-1} + \omega_2 y_{t-2} + \emptyset_1 x_{t-1} + \emptyset_2 x_{t-2} + \epsilon_t$$

The above restricted and unrestricted models were applied to the mean MD (y), for All five implicates for a given year along with the triennial variables in the form Of a dummy variable for the years of economic shocks in which Quantitative Easing (QE) Was implemented in the US, with the three year average Federal Funds Rate (FFR).

Our findings are that the effects of QE and the Federal Funds rate are rather persistent With statistically significant two period lags at the 8% and 3% levels for alpha, respectively.

Furthermore, the findings are not bi-directional when auto-regressive terms are switched as evidenced by the f-statistics.

PART III — EMPIRICAL CONCLUSIONS

We find there is a **statistically relevant increase** in the Malhalanobis distance (MD) **between cluster means** (populations) in the years following the financial crisis (noted by rows six and seven of the MD table), and notable dips in the mid 1990s and mid 2000s.

We see that there is additionally statistical evidence in the form of one **directional granger causality** for the roles of both unconventional and conventional monetary policy in regards to the distance between wealth generating populations in the US SCF data (in the form of alphas of 8% and 3% with regards to QE and the Federal Funds Rate respectively) .

Due to a loss of degrees of freedom these results **warrant further study** as more data becomes available and as this approach can be applied to alternative yet similar data sets from various regions with a greater number of observations (ideally based upon annual data).

VAR and SVAR models with sufficient observations (df) would help shed light on the persistence of shocks to models involving unsupervised methodological approaches, but due to parameter estimates required, once again a high number of observations is required.

The **evidence** herein supports the **heterogeneous framework** of analysis as presented in the recent literature contained herein (**see Part I**, see also Brunnermeier/Sannikov (2014)).

Additional research could **relax the restrictions of k=2** clusters presented herein and increase the dimensionality (variables) to further study the structure of economic systems beyond a traditional framework.

PART III

POLICY CONCLUSIONS

- ⇒ There are **other model-guided** and **empirical assessments** of wealth disparity (DSGE, ABM..)
- ⇒ There are **policy proposals** for wealth tax, and in particular inheritance tax
- ⇒ We propose in Bastos and Semmler, A **Proposal for a Carbon Wealth Tax: Modelling, Empirics, and Policy** (May 19, 2022), SSRN
 - Carbon-intensive wealth is measured via “**disclosure requirement**”
 - **Tax on public “bads”** is justified by the proportionality principle (Wicksell and Lindahl)
 - Tax on **carbon-intensive wealth**, can be a **tax on the flow** of wealth (not stock of wealth)

APPENDIX. HETEROGENEOUS HOUSEHOLDS AND WEALTH ACCUMULATION, PORTFOLIO APPROACH, WITH LEVERAGING; SEE PARKER AND SEMMLER 2017 (SPRINGER VOLUME), SEE ALSO CYNAMON AND FAZZARI ET AL CJE

- 1) Higher returns, 2) higher saving rates, 3) leveraging, 4) higher income share from capital with more capital=> inequality in net worth

$$\alpha_2 = (1 + f_2), \text{ with } \alpha_2 < 1$$

$$V = \max_{\{c_{2,t}, \alpha_{2,t}\}} \sum_{k=0}^N (1 + \rho)^{-t} U(c_{2,t}, x_{2,t}) \quad (4)$$

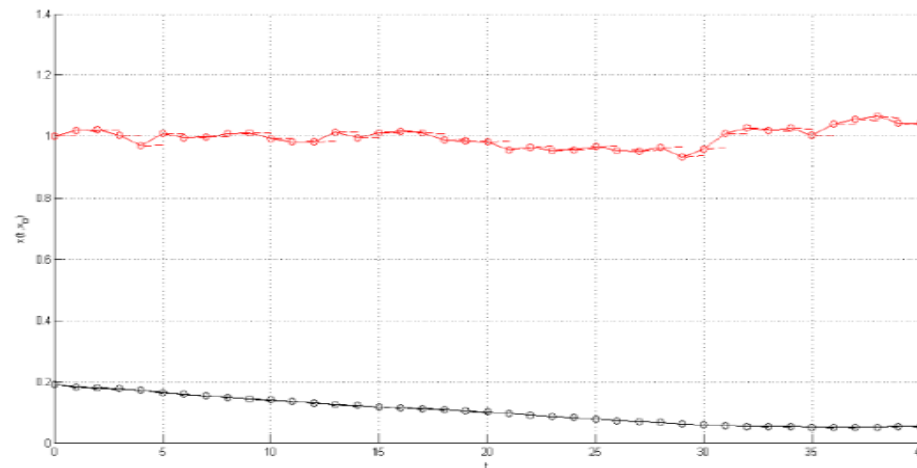
such that

$$x_{2,t} = x_{2,t-1}(1 + \Delta_h)(\alpha_{2,t}(y_2 + v_2 \ln z_t + r_2) + (1 - \alpha_{2,t})(i - v_2 \ln z_t) - \varphi(x_{2,t}) - \tilde{c}_2) \quad (5)$$

$$z_t = \exp(\rho \ln z_{t-1} + \epsilon_t). \quad (6)$$

Here we may normally have $\alpha_2 = (1 + f_2)$, with $\alpha_2 < 1$, which means there is income from the risk free rate as households may deposit savings in a bank with

Figure 2: Borrowing by consumers: Path of net worth, dark line, and stochastic process, red line



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Mathematical Appendix

For a full treatment of the mathematics of WEM Cluster modeling:

Semmler, Willi and Parker, Damien, “Monetary Policy and the Evolution of Wealth Disparity: An Assessment Using US Survey of Consumer Finance Data, Computational Economics, by Springer Nature, 2024

<https://doi.org/10.1007/s10614-024-10560-1>

Mathematical Appendix

Some key equations for the E-step and M-step, modified for weighted data, as per (Gebru, Alameda-Pineda, Forbes and Horaud, 2016) are as follows:

Density at x_i due to mixture component k and multiplicity weights given by ω_i :

$$\hat{p}(x_i; \Theta, \omega_i k) = [N(x_i, \mu_k, \Sigma_k)]^{\omega_i} = A_i N(x_i, \mu_k, \frac{1}{\omega_k} \Sigma_k)$$

Where the proportionality constant $A_i = (2\pi)^{-\frac{d}{2}} |\Sigma|^{1\omega_i/2}$ depends upon dimension d and weight ω_i , but is not a normalization constant.

This allows us to express all mixture components in the full density representation:

$$\tilde{p}(x_i; \Theta, \omega_i) = \sum_{k=1}^k \pi_k A_i N(x_i, \mu_k, \frac{1}{\omega_i} \Sigma_k)$$

Where $\sum \pi_k = 1$, for π_k prior proportion of component k .

Mathematical Appendix

Therefore, adjusting covariances accordingly and even increasing variances may even be necessary so as to maximize the observed data likelihood. This is of particular relevance in calculation of the moments, means and covariances.

Maximizing the log-likelihood of the data to fit the mixture parameters:

$\Theta = (\pi_1, \dots, \pi_k, \theta_1, \dots, \theta_k)$ and $X = (x_1, \dots, x_n)$ and $W = (\omega_1, \dots, \omega_n)$:

$$\ell(\widetilde{p}(X; \Omega, W)) = \sum_{i=1}^n \log \left(\sum_{k=1}^k \pi_k A_k N(x_{(i)}, \mu_{(k)}, \frac{1}{\omega} \Sigma_k) \right)$$

Provided Q is a distribution over potential z values and so long as $\sum_z Q(z) = 1$; $Q(z) \geq 0$, we then find the lower bound.

Writing the complete expected log-likelihood:

$$Q_c(\Theta, \Theta^r) = E_{p(z|X; W, \Theta^r)}(\log P(X, Z, ; W, \Theta))$$

Mathematical Appendix

Therefore, (r+1)-th expectation maximization iteration consists of both the E-step (evaluating posterior with weights w) and the M-step such that, the Q_i attaining equality satisfies:

$$\Theta^{(r+1)} = \underset{\Theta}{\operatorname{argmax}} Q_c(\Theta, \Theta^{(r)})$$

The E-step is then written with the posterior $\eta_{ik}^{(r+1)}$ iterating such that we can cancel and utilize Q_c :

$$Q_c(\Theta, \Theta^{(r)}) \underset{\Theta}{=} \sum_i^n \sum_{k=1}^k \eta_{ik}^{(r+1)} \left(\log \pi_k - \frac{\omega_i d}{2} \log(2\pi) - \frac{\omega_i}{2} \log |\Sigma_k| - \frac{\omega_i}{2} (x_i - \mu_k)^T \Sigma_k^{-1} (x_i - \mu_k) \right)$$

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<https://www.federalreserve.gov/econresdata/scf/files/impute98.pdf>

SCF data and variable descriptions may be found at the following Federal Reserve URL:
<https://www.federalreserve.gov/econres/scfindex.htm>

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