Economic Transition following an Emission Tax in a RBC Model with Endogenous Growth

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Main findings

- This paper addresses the inter-temporal costs and benefits of an environmental tax reform in a dynamic general equilibrium model featuring endogenous growth. It highlights economic features that impact the time needed for an environmental tax reform to produce a superior performance in relation to a business as usual scenario.

- The business as usual scenario assumes that as emissions remain high and pollution continues to accumulate, environmental damages become so large that they negatively impact total factor productivity and thus economic performance and growth. An environmental tax reform charges firms a tax on their emissions produced, which causes them to engage in abatement spending, thus reducing emissions and pollution. We assume that the environmental tax is sufficiently large to prevent further damages to the environment.

- There are five main results. First, the pure introduction of an emission tax will have a negative impact on employment. Since the tax alters the behavior of firms to cut their emissions, it distorts the market outcome. In the long run this distortion through the tax will be smaller in most calibrations than the distortion generated by environmental damage, thus allowing output and growth to be higher under green policy eventually.

- Second, using the revenue from the emission tax to lower wage taxes has positive effects on employment. Additionally, net remuneration as a percentage of GDP for workers increases. Furthermore, cutting wage taxes also raises long run output and growth.

- Third, the introduction of the emission tax can have a positive short run effect on output and growth during the initial transition phase. The emission tax changes both the optimal use of factors of production and also acts as a demand shock due to higher abatement spending it induces. During the adjustment period of capital the transition dynamics could actually increase output.

- Fourth, the medium run effect of the emission tax is negative in most cases. The distortions due to the tax, despite cut in wage taxes, have a net negative effect on capital use and output compared to the initial situation. In the business as usual scenario, it takes many years until the environmental damages become strong enough to lower growth and output below the green scenario. The time length of this process depends mostly on the pollution decay parameter, where a smaller decay implies a slower transition of the business as usual scenario to its steady state.

- Fifth, taking into account endogenous growth effects can have a large impact on the analysis concerning the costs and benefits of a green policy. If an economy operates at a lower growth rate for a number of years, then this difference first has to be caught up before an actual positive output effect is achieved.

1 Introduction

In recent years a large consensus has emerged that economic activity has a strong impact on the environment. It has become clear that a continuation of economic activity as we have seen in the past will lead to unsustainable increases in environmental pollution. Furthermore, there is also consensus that environmental degradation will eventually impact on economic performance and development, even though the precise size of this impact is very hard to estimate.

On the other hand, there are voices calling that stringent environmental regulations will cost too much in terms of output, jobs and growth, and that such policy should be avoided due to the high uncertainty of eventual negative effects of environmental damage.

This paper develops a modeling approach that takes both the eventual environmental damage on the economy as well as the distortion of environmental regulation into account. Economic activity causes emissions, which accumulate as pollution and thereby can have a negative impact on output. The fact that pollution takes time to accumulate means that damaging effects from economic activity are not seen immediately.

The starting point of this paper is to better understand the economics dimension of green policies. In UNEP (2011) a detailed discussion of what constitutes a green economy is made. Specifically, green policy requires fixing market failures and often does not even require actual resource investments. In this paper, based on a modeling framework, we focus on environmental investments that require resources.

We assume a simple framework where the government imposes a tax on emissions produced by firms to fix the market failure. Firms can reduce their emissions and tax payments by spending on abatement. The tax on emissions alters the market outcome in two ways. On the one hand it lowers emissions, pollution and thus the output reducing impact of environmental damage, on the other it changes demand for factors of production and thereby can causes an immediate negative effect on output.

Furthermore, this paper combines both real business cycle fluctuations and endogenous growth as in Comin & Gertler (2006). The economic literature on climate change traditionally adopt a long term (exogenous or endogenous) growth framework. The main reason is that given the slow decay of CO2 the reduction in the stock of pollution takes place over long time periods. In Nordhaus & Boyer (2000) for instance, simulations of green policies take place over 60 decades. The few real business cycle with pollution on the contrary shows that damages have no effects on the dynamic of the main macroeconomic variables over the business cycle (see Heutel (2011)). Real business cycle model are however well suited to discuss the short run effects of government policies such a carbon tax for instance.

These methodological differences point to a time inconsistency related to green policies. Green policies have short term economic costs while their benefits are felt in the long run. In order to tackle both dimensions, we develop a model with both a real business cycle dimension and an endogenous growth dimensions. Both environmental damage as well as market distortions have detrimental effects on growth. Therefore, this paper allows a quantitative analysis of when, if ever, the output effect as well as the accumulated growth effects of environmental policy pay off compared to a business-as-usual (BAU) scenario.

We present the BAU scenario as a scenario where no environmental policy action is taken and the slowly accumulating pollutants cause a gradual decay of total factor productivity and economic growth. As an alternative, we investigate a scenario where a sufficiently large emission tax is introduced to cut emissions so much that further accumulation of pollutants is stopped. The effect of environmental policy is thus presented as a net effect between the two scenarios.

The paper draws a number of important conclusions. First, the pure introduction of an emission tax will have a negative impact on employment, and might have a negative impact on long run growth and output as well. The latter effect depends on the marginal efficiency of abatement spending to reduce emissions, as a low efficiency requires a high tax rate to achieve the emission target. This in turn raises distortions, which lowers output and growth.

Second, using the revenue from the emission tax to lower wage taxes has positive effects on employment. The higher the tax revenue (which means the lower the marginal efficiency of abatement spending), the larger can be the cut in wage taxes, so that for most parameterizations the introduction of an emission tax actually has a positive impact on employment due to the double dividend. Furthermore, cutting wage taxes also raises long run output and growth.

Third, the introduction of the emission tax can have a positive short run effect on output and growth during the initial transition phase. The emission tax changes both the optimal use of factors of production and also acts as a demand shock due to higher abatement spending it induces. During the adjustment period of capital the transition dynamics could actually increase output.

Fourth, the medium run effect of the emission tax is negative in most cases. The distortions due to the tax, despite cut in wage taxes, have a net negative effect on capital use and output compared to the initial situation. In the BAU scenario, it takes many years until the environmental damages become strong enough to lower growth and output below the green scenario. The time length of this process depends mostly on the pollution decay parameter, where a smaller decay implies a slower transition of the business as usual scenario to its steady state.

Fifth, taking into account endogenous growth effects can have a large impact on the analysis concerning the costs and benefits of a green policy. If an economy operates at a lower growth rate for a number of years, then this difference first has to be caught up before an actual positive output effect is achieved.

The rest of the paper is organized as follow. Section 2 discusses the existing literature and puts into perspectives our methodological choices. The model is presented in section 3. The steady states and calibration of the main parameters are discussed in section 4. The results and the numerical simulations are displayed in section 5. Section 6 discusses optimal policies, while section 7 concludes.

2 Literature Review

This section provides a short overview of recent studies analyzing the environmental issues in macroeconomics. Most of the existing literature relies on growth model to discuss the effects of pollution on economic activity in the long-run. These models are either exogenous growth model à la Ramsey or endogenous growth model (see for instance Greiner & Semmler (2008)). Koesler (2010) uses a standard Schumpeterian growth model in order to analyze the effect of pollution on economic growth. In this model, pollution is defined as an externality of the production of differentiated intermediate goods. Besides, the quantity of pollution emitted in the economy depends on the level of technological progress. The author shows that pollution slows down the economic growth if the pollution intensity does not decline following an improvement in technological progress.

Rezai *et al.* (2011) use a standard neoclassical growth model to study the effects of global warming on economic activity. They argue that business as usual scenarios are miss-specified in the literature and that they under-estimate the costs of climate externalities. They then shows that correcting for this shortcoming implies that green scenario yields a superior outcome than business as usual scenario. In this model, pollution emissions are caused by the production of the final goods. Besides, pollution is assumed to affect negatively the output through a damage function. Rezai *et al.* (2011) show that pollution leads an over-accumulation of capital and under-investment in mitigation, which in turn imply environmental damages. After several periods, no more capital accumulation is possible because of the damages. It follows that consumption and output decline.

Another strand of the literature focuses on the effects of environmental policy in the perspective of the business cycle. Heutel (2011) points out that environmental policies, designed to fight against climate change, are usually implemented without taking into account the macroeconomic fluctuations. In order to assess whether environmental policy should react to macroeconomic economic fluctuations or not, the author used a real business cycle model in which pollution appears as a negative externality. In this model, the damage of pollution on the efficiency of the production is explicitly modeled. Hence, using the model, the author evaluates how an emission tax rate should respond to a productivity shock. First, he finds that emissions are pro-cyclical. Second, he shows that the emission tax rate should increase during expansions and decline during recessions. Therefore, the emission tax rate comes to dampen the pro-cyclicality of pollution emissions.

Fischer & Springborn (2011) consider a real business cycle model in which the production process uses three inputs, being labor, capital, and a polluting intermediate good. The damage caused by pollution is not explicitly modeled. Those authors evaluate three environmental policy instruments that aim at limiting pollution emissions from the intermediate good. These policy instruments are an emission cap (fixed quantity of intermediate good), an emission tax, and an intensity target (a maximum emissions to output ratio). Fischer & Springborn (2011) find that output is higher when the intensity target policy is considered than when the emission tax or emission cap policy are.

The model presented in the following section follows Heutel (2011) and can thus be used to study business cycles. The long-term damaging effects of pollution are similar to both Koesler (2010) and Rezai *et al.* (2011). A novelty is that the model is path dependant as in Comin & Gertler (2006). The short run economic dynamic affects potential output such that hysteresis effects may take place. The model therefore takes the growth dimension into account, as in Koesler (2010).

This framework allow us to study of the transition costs towards a green economy, starting from a business as usual economy. The method employed thus focuses on the relative performance of an economy continuing with business as usual versus an economy where green policy is implemented. A large number of factors affecting the relative performance of these two economies is analyzed.

3 The Model

The economy is populated by the government and many atomistic households as well as firms. Households are standard, maximizing inter-temporal utility by choosing consumption, hours worked and capital accumulation.

Firms hire capital and labour to produce output, which they sell under monopolistic competition on the goods market. However, firm's productive activity produces emissions, which accumulate to increase the stock of pollutants in the air. This stock of pollutants in turn causes environmental damage, which in turn also impacts negatively total factor productivity.

Due to the fact that firms are atomistic, their marginal impact on total emissions, and thus on the stock of pollutants, is zero. Hence, they do not consider the effect of their production activity on pollution and damages, thus emission is a true externality.

Environmental policy aims to force firms to somehow internalize the externalities they produce. For this reason, we introduce a tax on emissions made by firms. Furthermore, we assume that firms can reduce their emissions by performing abatement activities, which cost resources. Given this setup, firms will find it optimal to spend as much on abatement so that their marginal savings on emission tax paid equals the marginal cost of abatement spending. The presence of abatement spending reduces emissions and pollution accumulation.

Technology and Pollution

Technology

A common claim by papers advocating environmental policy is the existence of a beneficial effect of such policy on the rate of technological progress. Specifically, it is claimed that more stringent regulation or higher costs of emissions leads to more innovation to sustain economic profits, thus accelerating the rate of technology growth.

In this paper we apply a standard learning-by-doing technology accumulation process without taking into account the potential innovation-accelerating aspects of environmental regulation or taxes. We motivate this choice with the unknown effect of environmental taxes on technology accumulation, which would introduce another dimension of parameter uncertainty. By relying on a standard mechanism, we can show the basic effects of green policy on growth. However, we acknowledge that a more sophisticated specification of the technology accumulation process, albeit being very difficult, could allow taking such mechanisms into account in an extended version of this model.

The aggregate technology is accumulated through aggregate output as follows

$$T_t = B T_{t-1}^{1-\eta} Y_t^{\eta} \tag{1}$$

where B is scaling parameter and η is the strength of the endogenous growth effect. Note that $\eta = 0$ implies exogenous growth. Let's define the gross growth rate of technology as follows: $g_{T,t} \equiv \frac{T_t}{T_{t-1}}$. Thus, using equation (1), we can show that the technology gross growth rate is a positive function of aggregate output

$$g_{T,t} = B\left(\frac{Y_t}{T_{t-1}}\right)^{\eta} \tag{2}$$

Technology growth is thus determined by the exogenous growth parameter B as well as by the endogenous growth parameter η.

An inspection of equation (2) reveals that in a steady state with a constant technology growth rate, the growth rate of output and technology are equal, thus $g_Y = g_T$. This also means that the term $\frac{Y}{T}$ will be stationary.

Pollution

In order to introduce pollution into the model we need to consider three aspects of pollution: the nature of pollution, the source of pollution, and the effects of pollution on the agents of the economy (see Koesler, 2010). With regard to the nature of pollution, we assume that pollution corresponds to greenhouse gases. As a consequence, pollution must be considered as a stock variable (see Stokey, 1998). Concerning the source of pollution, we assume that the emissions of pollution arise as an externality of the production of intermediate goods (see Koesler, 2010). Finally, aggregate pollution is assumed to affect negatively the production of intermediate goods (see Heutel, 2011). These three aspects of pollution are formalized as follows.

Emissions occur at the firm level. For each firm i we specify relationship between emissions, $E_{i,t}$, output, $Y_{i,t}$, and abatement, $S_{i,t}$, as

$$E_{i,t} = A_E\left(1-\theta_1\left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2}\right)Y_{i,t}^{\chi} \tag{3}$$

This equation shows that a firm can reduce its emissions by spending $S_{i,t}$ on abatement. This specification follows Heutel (2011). Aggregate emissions are simply the sum over all $E_{i,t}$.

Equation (3) shows that in a steady state with a constant share of abatement spending in output, the relationship between the growth rates of emissions and output is given by $g_E = g_Y^{\chi} = g_T^{\chi}$. Since $\frac{Y}{T}$ is stationary, we can also derive that $\frac{E}{T^{\chi}}$ is stationary.

The stock of pollution, denoted by X_t, is accumulated through aggregate emissions, denoted by E_t. We consider a linear accumulation process (see Heutel, 2011), adjusted for the stationarity requirements of a growing economy.

$$X_t = g_{T,t}^{\chi} \left(\rho_x X_{t-1} + E_t \right), \quad 0 < \rho_x < 1 \tag{4}$$

where ρ_x is the depreciation rate of pollution. Since $\frac{E}{T^{\chi}}$ is stationary, so will be $\frac{X}{T^{\chi}}$. We adjust (4) by the technology technology growth rate since we require the use of stationary pollution in a damage function.

We specify the damage of pollution on the production of intermediate goods, D_t, by the following damage function (see Rezai *et al.*, 2011)

$$D_t = D\left(\frac{X_{t-1}}{T_{t-1}^{\chi}}\right) = \left(1 - \left(\frac{X_{t-1}/T_{t-1}^{\chi} - \underline{x}}{\bar{x} - \underline{x}}\right)^{\frac{1}{\gamma}}\right)^{\gamma} \tag{5}$$

It is worth noting that this specification of the damage function implies that the stock of pollution is bounded by $\underline{x}$ (lower bound) and $\bar{x}$ (upper bound), and the damage variable takes value between 0 and 1: $D_t : [\underline{x}, \bar{x}] \to (0,1), \frac{X_{t-1}}{T_{t-1}^{\chi}} \mapsto D\left(\frac{X_{t-1}}{T_{t-1}^{\chi}}\right)$.

Emission Tax

Given a constant share of abatement spending in output, a unit percent increase in output increases emissions by $\chi\%$. Emission taxes need to grow in line with the size of the economy to have a constraining effect. Indeed, if emissions were to be taxed at a fixed rate, then the tax payable from a unit percent increase in output would also increase by $\chi\%$, while the cost of abatement spending increases by 1% , to keep its share in output constant. Thus, with a fixed tax rate on emissions it will not be optimal for firms to spend a stationary share of output on abatement in a growing economy. Rather, this share will eventually be driven to zero.

To counter this effect, we assume the following firm emission tax spending $M_{i,t}$:

$$M_{i,t} = \tau_E Y_t^{1-\chi} E_{i,t} \tag{6}$$

The tax rate on emission is time variable, but not progressive. More specifically, it grows over time with aggregate output, which the firm takes as given, as the tax is set by the government. This is a sensible assumption, since the government cares about the emission constraining effects from its emission tax and thus will adjust the tax to grow in line with the economic output.

Households

The preferences of the representative household over consumption, C_t, and leisure, $1-h_t$ where h_t denotes the hours worked, are given by the following intertemporal utility function

$$U\left(C_t, 1-h_t\right) = \mathscr{E}_0 \Sigma_{t=0}^{\infty} \beta^t \frac{\left[C_t\left(1-h_t\right)^{\psi}\right]^{1-\sigma}}{1-\sigma} \tag{7}$$

where β denotes the discount factor, ψ the preference for leisure parameter and $\mathscr{E}$ the expectation operator.

In the literature, pollution sometimes affects the utility of households negatively. When discussing pollution in an economic model, it is a straightforward idea to also have pollution affect utility. We refrain from doing this here for two reasons. If one were to introduce pollution additively into the utility function, there would be no effect of that on household utility maximization in a decentralized economy as we consider it here since households cannot chose emissions or pollution. In case of a multiplicative introduction of pollution in the household utility function accumulating pollution should negatively affect the marginal utility of consumption and leisure. However, depending on the specification, the impact on the first order conditions could still be minimal. In Section we discuss the impact on optimal policy of allowing for pollution to affect household utility.

The intra-temporal budget constraint of the household is given by

$$C_t + I_t + \tau_t + \Pi_t \leq (1 - \tau_w) W_t h_t + r_t K_{t-1} \tag{8}$$

where I_t stands for investment, W_t the real wage, τ_w the labour tax rate, K_t capital stock, and r_t the real rental rate of capital. τ_t stands for government lump sum transfers and Π_t for payouts of firms' profits.

The household accumulates capital

$$K_t = (1 - \delta) K_{t-1} + I_t \tag{9}$$

where δ is the capital depreciation rate.

The households' constrained optimization problem is described in Appendix .

Households maximize their lifetime utility (7) subject to series of budget constraints (8) and the capital accumulation identity 9 by choosing a path for consumption, hours worked and investment.

The familiar first order conditions are:

$$(1-\tau_w)W_t = \psi \frac{C_t}{1-h_t} \tag{10}$$

$$C_t^{-\sigma}(1-h_t)^{\psi(1-\sigma)} = \beta E_t C_{t+1}^{-\sigma}(1-h_{t+1})^{\psi(1-\sigma)}(r_{t+1}+1-\delta) \tag{11}$$

Final goods producers

The final good, Y_t, is produced in a competitive market according to the following CES technology:

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{1}{\mu}} di\right)^{\mu}, \quad \mu \geq 1 \tag{12}$$

where each input $Y_{i,t}$ is a differentiated intermediate good. The term $\frac{\mu}{1-\mu}$ indicates the price elasticity of the demand for any intermediate good i.

The fact that demand for intermediate goods is price elastic means that each intermediate good i has a downward sloping demand curve given by

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{\frac{\mu}{1-\mu}} Y_t \tag{13}$$

The derivation of (13) is shown in Appendix .

Intermediate goods producers

Intermediate goods are produced by a continuum of firms, indexed by the letter i, operating in a monopolistically competitive market. Each variety of intermediate goods is produced by the following technology,

$$Y_{i,t} = D_t F\left(K_{i,t-1}, T_{t-1} h_{i,t}\right) = D_t A K_{i,t-1}^{\alpha} \left(T_{t-1} h_{i,t}\right)^{1-\alpha} \quad 0 < \alpha < 1 \tag{14}$$

where $K_{i,t-1}$ denotes firm's i physical capital stock, $h_{i,t}$ firm's i labor (hours worked), and T_{t-1} the aggregate level of technology, A is a scaling parameter, D_t is the pollution damage function (5), and α is the capital share. Here we assume that intermediate goods producers are not aware that their production activities affect the aggregate levels of pollution and technology. In other words, firms take D_t and T_t as given.

Intermediate goods producer i sells the proceeds of its output at (relative) price $\frac{P_{i,t}}{P_t}$ out of which it pays its wage bill, $W_t h_{i,t}$, rental costs of capital, $r_t K_{i,t-1}$, the emission tax, $M_{i,t}$, and abatement spending, $S_{i,t}$. It selects optimally its production, price, capital outlays, working hours and abatement according to the following dynamic optimization problem:

$$\max_{P_{i,t}, K_{i,t-1}, h_{i,t}, S_{i,t}} E_0 \Sigma_{t=0}^{\infty} \zeta_t \left[\frac{P_{i,t}}{P_t} Y_{i,t} - W_t h_{i,t} - r_t K_{i,t-1} - \tau_E Y_t^{1-\chi} A_E \left(1 - \theta_1 \left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2}\right) Y_{i,t}^{\chi} - S_{i,t} \right]$$

subject to (3), (13) and (14). ζ_t is the endogenous discount factor.

We can derive the following first order conditions (see Appendix).

$$r_t = \lambda_{i,t} \alpha \frac{Y_{i,t}}{K_{i,t-1}} \tag{15}$$

$$W_t = \lambda_{i,t} (1-\alpha) \frac{Y_{i,t}}{h_{i,t}} \tag{16}$$

$$\lambda_{i,t} = \frac{P_{i,t}}{P_t} - \mu_{i,t} - \tau_E A_E \left(\frac{Y_t}{Y_{i,t}}\right)^{1-\chi} \left(\theta_1 \theta_2 \left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2} + \chi \left[1 - \theta_1 \left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2}\right]\right) \tag{17}$$

$$\left(\frac{S_{i,t}}{Y_{i,t}}\right)^{1-\theta_2} = \tau_E A_E \theta_1 \theta_2 \left(\frac{Y_t}{Y_{i,t}}\right)^{1-\chi} \tag{18}$$

$\lambda_{i,t}$ is the Lagrange multiplier on the production function.

Distortions

We see that when $\lambda_{i,t} < 1$, there is a distortion which causes the factors of production to be paid less than their marginal product, and which consequently tends to reduce the employment of these factors. In a flexible price equilibrium, we can solve for $\mu_{i,t} = -\frac{1-\mu}{\mu}$ by optimizing with respect to $P_{i,t}$. Without an emission tax, the distortion will then be the familiar distortion from monopolistic competition of size $\lambda_{i,t}\frac{1}{\mu}$. From now on, we will define the aggregate distortion as $\kappa_t = \sum \lambda_{i,t}$.

Equation (17) shows that with an emission tax, there will be an additional distortion. This counters the claims of other literature on environmental policy [CITATIONS NEEDED] that an emission tax is a pure source of revenues and does not produce dostortions. However, the emission tax as introduced in this paper serves to provide incentives to change the behavior of economic agents, and thus will alter the use of factors of production.

Specifically, there are two dimensions along which the existence of tax payments, given by $\tau_E A_E$, affects the firm's decision. The first, given by the first term in the bracket in (17), relates to the fact that firms can engage in abatement spending to avoid spending on the tax. This term disappears when abatement spending is inefficient, thus when $\theta_1 = 0$ of $\theta_2 = 0$. The second term relates to the fact that the firm can reduce its output to avoid emission tax spending.

Marginal costs

In Appendix , we also derive real marginal costs of a firm. These will be given by

$$mc_{i,t} = \frac{1}{D_t} \frac{\left(\frac{W_t}{T_{t-1}}\right)^{1-\alpha} r_t^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} + \tau_E A_E \left(\frac{Y_t}{Y_{i,t}}\right)^{1-\chi} \left(\theta_1 \theta_2 \left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2} + \chi \left[1 - \theta_1 \left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2}\right]\right) \tag{19}$$

$$= \frac{P_{i,t}}{P_t} - \mu_{i,t} \tag{20}$$

Equation (20) is in fact the same as in a standard model of monopolistic competition. Firms charge a price as a mark-up on their marginal costs. However, these marginal costs are higher when there is an emission tax, which produces a larger wedge in the economy. The flexible price equilibrium is given by symmetric firms, thus by $mc_{i,t} = mc_t$ and $P_{i,t} = P_t$. In that case, we obtain the familiar $mc_t = \frac{1}{\mu}$.

Government budget

Since we study a long-run model, we assume that the government cannot rely on debt financing. To keep notation as simple as possible, we only specify the aspects of the government budget we are interested in. Specifically, the government collects emission taxes and can redistribute these either as lump-sum payments to households or can use them to cut labour taxes. We therefore specify

$$\tau_E Y_t^{1-\chi} E_t = -\tau_t - \tau_w h_t W_t \tag{21}$$

Market clearing

The economy's aggregate resource constraint is given by

$$Y_t = C_t + I_t + S_t \tag{22}$$

This also coincides with the bond-market equilibrium as well as with the households aggregated budget constraint.

Equilibrium

Due to the fact that the model has a positive technology growth rate, it has a non-linear non-stationary equilibrium described in Appendix . In order to solve the model with available mathematical tools, we need to make it stationary. This is done by dividing non-stationary variables by a common co-integrating factor, in our case by technology T.

By doing this, we can derive the non-linear stationary equilibrium, which is described in Appendix .

4 Quantitative Analysis

Calibration and steady states

In order to solve the model, we need to assign values to the parameters and compute the steady state. The model has 17 structural parameters, namely β, α, δ, μ, σ, η, B, ρ_x, γ, θ_1, θ_2, A, A_E, $\bar{x}$, $\underline{x}$ and ψ. There are also the tax parameters τ_E, τ_w and τ, where the lump sum tax τ remains endogenous to fulfill the budget constraint. Furthermore, there are 13 non time-varying variables y, c, i, h, k, w, r, d, x, e, g_T, κ and $\frac{y}{s}$.

The steady state values of the variables will be uniquely determined by the structural as well as the tax parameters. However, we proceed in the calibration process by calibrating target values for h, g, d, $\frac{s}{y}$ and the half-life time of pollution decay ν. This way, we determine 5 parameters, A, ψ A_E, θ_1, and ρ_x endogenously. Table 1 shows the parameter restrictions imposed.

Most of the parameters are standard. The motivation for the parameter choice related to pollution is taken from Heutel (2011). The target abatement spending in the green scenario of 2% is taken from UNEP (2011). The technology scaling parameter B is chosen to allow 2% annual steady state growth if growth were exogenous. We perform sensitivity analysis for the choice of η.

The steady state equations are the following

$$g_T k = (1-\delta)k + i \tag{23a}$$

$$(1-\tau_w)w = \psi \frac{c}{1-h} \tag{23b}$$

$$c^{-\sigma}(1-h)^{\psi(1-\sigma)} = \beta (g_T c)^{-\sigma}(1-h)^{\psi(1-\sigma)}(r+1-\delta) \tag{23c}$$

Table 1: Restrictions.

Structural parameters	
Discount factor	$\beta = 1.03^{-\frac{1}{4}}$
Technology scale	$B = 1.005$
Capital share	$\alpha = 0.3$
Capital depreciation rate	$\delta = 0.025$
Relative risk aversion parameter	$\sigma = 1$
Technology parameter	$\eta = 0.1$
Emission parameter	$\chi = 0.696$
Damage parameters	$\gamma = 0.5$
Damage parameter	$\bar{x} = 800$
Damage parameter	$\underline{x} = 300$
Efficiency of abatement spending	$\theta_2 = 0.5$
Target Values	
Steady state growth rate target	$g_T = 1.005$
Steady state labour supply target	$h = 0.6$
Abatement spending share in green scenario	$\frac{s}{y}(G) = 0.02$
Damage in green scenario	$d(G) = 1$
Abatement spending share in BAU scenario	$\frac{s}{y}(B) = 0.002$
Damage in BAU scenario	$d(B) = 0.95$
Half life time of pollution in years	$\nu = 60$

$$y = zdAk^{\alpha}h^{1-\alpha} \tag{23d}$$

$$r = \kappa\alpha\frac{y}{k} \tag{23e}$$

$$w = \kappa(1-\alpha)\frac{y}{h} \tag{23f}$$

$$g_T = By^{\eta} \tag{23g}$$

$$x = \rho_x x + e \tag{23h}$$

$$e = A_E\left(1-\theta_1\left(\frac{s}{y}\right)^{\theta_2}\right)y^{\chi} \tag{23i}$$

$$d = \left(1-\left(\frac{x-\underline{x}}{\bar{x}-\underline{x}}\right)^{\frac{1}{\gamma}}\right)^{\gamma} \tag{23j}$$

$$y = c + i + \frac{S}{Y}y \tag{23k}$$

$$\left(\frac{s}{y}\right)^{1-\theta_2} = \tau_E A_E \theta_1 \theta_2 \tag{23l}$$

$$\kappa = \frac{1}{\mu} - \tau_E A_E\left(\theta_1\theta_2\left(\frac{s}{y}\right)^{\theta_2} + \chi\left[1-\theta_1\left(\frac{s}{y}\right)^{\theta_2}\right]\right) \tag{23m}$$

The above system of equations is not block-recursive and hence cannot be solved in a step-wise analytical way due to many interdependencies. Moreover, we want to calibrate the model in a way that we can obtain certain target values for g , h , $\frac{s}{y}$, d and ν. We use the following procedure to solve for the structural parameters A, ψ, θ_1, AE and ρ_x.

Calibrating the structural parameters using targets

The first parameter we calibrate is pollution depreciation ρ_x. By specifying a targeted half life time v, the computation is straightforward:

$$\rho_x = 0.5^{\frac{1}{v}} \tag{24}$$

When calibrating an endogenous growth model, it is useful to specify the targeted steady state growth rate, and set $B = g_T$. From this, it follows immediately that $y = 1$. Then using equation (23c) we get the steady state value of real rental rate of capital

$$r = \frac{g_T^{\sigma}}{\beta} - 1 + \delta \tag{25}$$

Using equation (23j), we get the steady state value of capital

$$k = \kappa \frac{\alpha}{r} y \tag{26}$$

To solve this we need to make an assumption on κ belonging to our targeted g_T. For the calibration of our parameters belonging to the scenario $g_T = B$, we assume a case where there are no distortions from emission taxation, and thus $\kappa = \frac{1}{\mu}$. Furthermore, we assume in this calibration scenario that there is non-distortionary abatement spending such that a green economy is reached (meaning $\frac{s}{y} = 0.2$), and thus that there is no damage to the economy, so that $d = 1$. Using this information, we can calibrate A

$$A = \frac{y}{dk^{\alpha}h^{1-\alpha}} \tag{27}$$

Using equation (23a) we get the steady state value of investment

$$i = (g_T - 1 + \delta)\, k \tag{28}$$

Using equation (23k) we get the steady state value of consumption

$$c = y - i - \frac{s}{y} y \tag{29}$$

Combining (23b) and (23f), we can solve for the next structural parameter

$$\psi = \frac{\kappa(1-\alpha)\; y}{c} \frac{1-h}{h} \tag{30}$$

Investigating equation (23i), we see that to specify the parameters θ_1 and A_E we require a set of target variables $\left[y, e, \frac{s}{y}\right]$ for two different scenarios. The first scenario is the one described above, while the second scenario is the business as usual scenario. Given the structural parameters as well as our assumed target damage, we can solve for y in that scenario. Furthermore, using the implied damages in equations (23j) and (23h), we can solve for targeted emissions e. Using the targeted abatement spending shares, we can then solve to θ_1 and A_E.

Simulation Setup

We will compare different scenarios by targeting a certain abatement spending and determining the corresponding emission tax using equation (23l).

When the government imposes a tax on emissions, it not only induces more abatement spending by firms, it also obtains revenues. These can either be redistributed in a lump-sum fashion to households, or they can be used to cut other taxes.

Here, we investigate a cut in the wage tax.

A balanced budget approach would suggest the following relationship

$$\tau_E Y_t^{1-\chi} E_t = -\tau_w h_t W_t \tag{31}$$

Using (23f) and (23i), we obtain

$$\tau_E A_E \left(1 - \theta_1 \left(\frac{S}{Y}\right)^{\theta_2}\right) Y = -\tau_w \kappa (1-\alpha) Y \tag{32}$$

This directly allows the calculation of the wage tax given the emission tax.

Given the full specification of all structural parameters as well as an emission and wage tax rate, we can simulate scenarios in which all variables will be determined endogenously, including these variables that are used as target values in the calibration section.

Simulation exercises

Our main interest lies in comparing the effect of a business-as-usual (BAU) scenario to a green scenario. For that aim, we define the BAU scenario as a scenario where there is only a very small tax rate on emissions, so that the abatement spending share specified in Table 1 is obtained. The alternative scenario is a green scenario, where the emission tax rate is set to such a level that the green abatement spending share specified in Table 1 is obtained.

There are two basic simulation exercises we can perform. The first is a standard impulse response analysis to a productivity shock, comparing the response of a green to that of a BAU economy. However, there is only a minimal difference in terms of economic volatility. A positive productivity shock increases output, which increases emissions. In the BAU economy the increase in emissions and pollution will be somewhat larger since there is less abatement spending. This means that there will be a slightly larger negative medium run effect as damage is somewhat higher. However, the effect on damage is very small, being 0.025% of its steady state value in the green scenario and 0.027% of its steady state value in the BAU scenario. The similarity of these two scenarios stems from the fact that this simulation performs a productivity shock around the steady state, at which point the transition equations of both scenarios are almost identical.

The second simulation exercise is a transition analysis: We assume that the BAU scenario has not reached its final steady state in pollution levels and damages. We can therefore compare the development of the economy over time when we impose an emission tax and compare it to the development of BAU. In these scenarios, we run the simulation starting from initial values representing the current status of the economy, meaning a low value of emission spending, but also a (still) low level of pollution and damages. For simplicity, we will assume that in the green scenario the emission tax will be high enough to keep pollution in check so that the current level is also the steady state level.

We set up the initial situation by setting damages to zero, and solving for the corresponding pollution level. Furthermore, we set the amount of abatement spending to the current BAU scenario. Using this, we can obtain the emission tax rate and the real marginal costs. With these variables, we can solve for the equilibrium values of the other variables, where we assume that they are as if in steady state since the only variables not in steady state – the pollution level and damage – actually will take a very long time to reach steady state and thus intertemporal influences on equilibrium determination should be very small.

5 Results

This section presents the results from introducing an emission tax in the baseline calibration as well as in alternative calibrations. In the baseline calibration, we utilize the parameters specified in Table 1 and also assume that all tax revenue of the emission tax is used to cut wage taxes.

Baseline calibration

Figure 1 presents the development of output in the business as usual scenario as well as in the scenario with a high emission tax under the baseline calibration. Since we perform our analysis in a growing economy, both lines slope upwards. After the introduction of the emission tax in Year zero there is a widening gap between the two scenarios, which then starts to close again after around 50 years, while after around 100 years output in the green scenario surpasses that in the business as usual scenario.

In the following, we will continue by presenting variables as a percentage difference between the green scenario and the BAU scenario. Specifically, we will present graphs showing $(Y_{green} - Y_{BAU})/Y_{BAU}$, with the exception of technology growth rates, where we present their actual values. Figure 2 shows the relative path of output, consumption, investment and labour supply in our baseline calibration.

The introduction of the emission tax has two major effects on the economy.

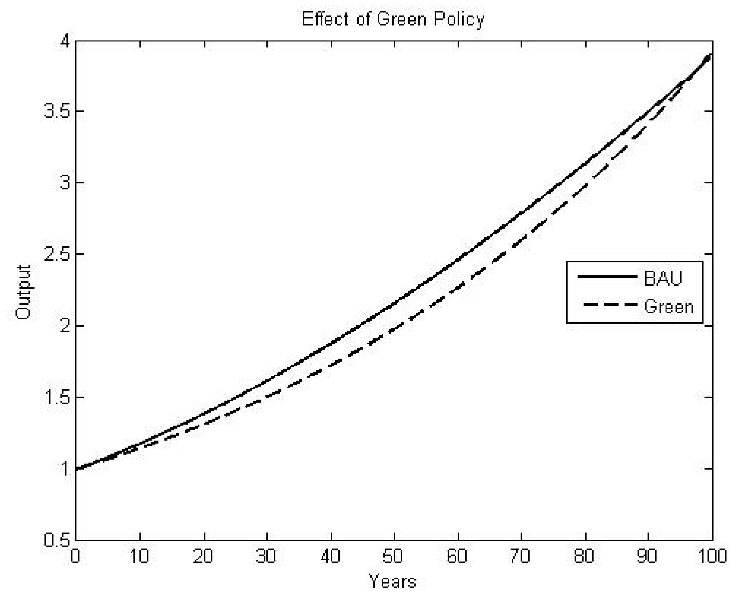

Figure 1: The development of output in two scenarios.

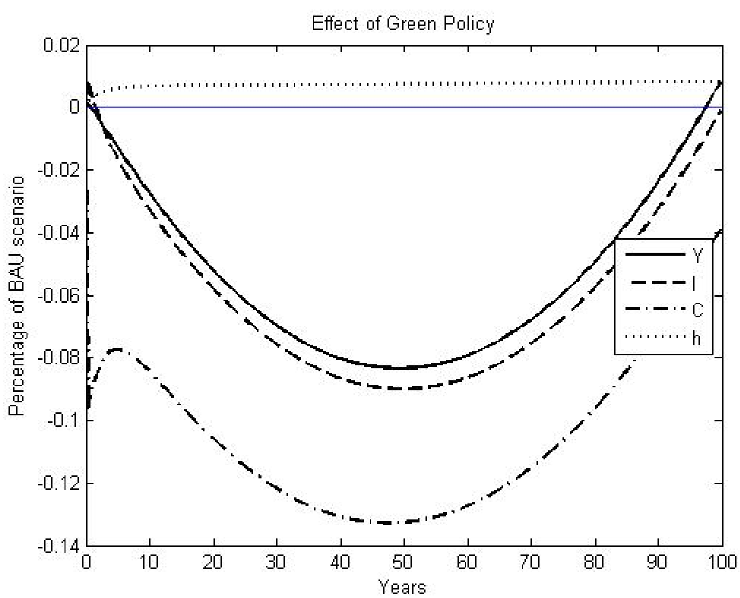

Figure 2: The net effect of higher emission taxes in the baseline calibration.

On the one hand, it creates a distortion in the product market, similar to the price markup, which tends to reduce real payments to the factors of production. On the other hand, the emission tax also serves as a demand shock as now firms have to spend resources on abatement. In the periods immediately following the tax increase, when capital is still "fixed", the demand shock dominates the mark-up shock, and there is a strong fall in consumption, leading to an increase in labour supply, output and growth. Over time (around 10 years in the simulation), the production structure of the economy adjusts to the new situation and the economy grows at its new steady state growth rate (see Figure 3).

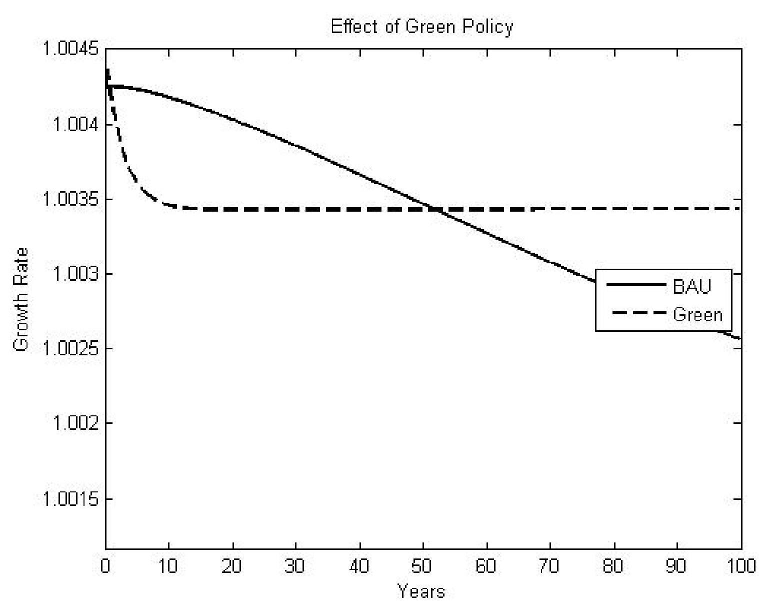

Figure 3: The net effect of higher emission taxes on growth.

The growth rate in the business as usual scenario falls slowly as pollution accumulates further, thus causing further damage. However, the convergence of the growth rate to its final steady state value, represented by the intersection of the y-axis with the x-axis, takes a very long time. In fact, the green economy already has a superior performance in terms of growth, but also in terms of damages in relation to distortions caused by the emission tax, after 50 years. Nevertheless, 50 years of lower growth accumulate to a large relative output gap, which takes a long time of superior growth to catch up.

Discussion on factor shares

The simulation of the baseline calibration shows that there is actually a positive employment effect from the introduction of an emission tax. However, we also note that the emission tax creates an distortion that reduces the payments to factors of production (see equations 15 and 16), or the demand for them. Contrasting this is the rise in labour supply due to the fall in consumption and also the fall in the wage tax, which raises net incomes of households.

We can calculate that payments to labour fall from 52% to 49.4% of output, while the payments to capital fall from 29.2% to 27.8%. The emission tax payments as a share of output are 6%, which actually also implies that firm's profits fall. However, once we consider that the emission tax payments come to the benefit of wage earners, we can see that the effect for wage earners of the emission tax increase is an increase in their net income share of output by 3.5%. As we discuss in Section , the size of tax receipts, and thereby the change in net wage income, depends on the marginal efficiency of abatement spending.

The importance of endogenous growth

In this section we highlight the importance of endogenizing growth. Figure 4 shows a model with exogenous growth, meaning $\eta = 0$. When one ignores endogenous growth, one would draw the conclusion that the green economy delivers a superior performance much earlier than when one takes growth into account. The temporarily worse performance of the economy has no lasting effects in terms of path dependence.

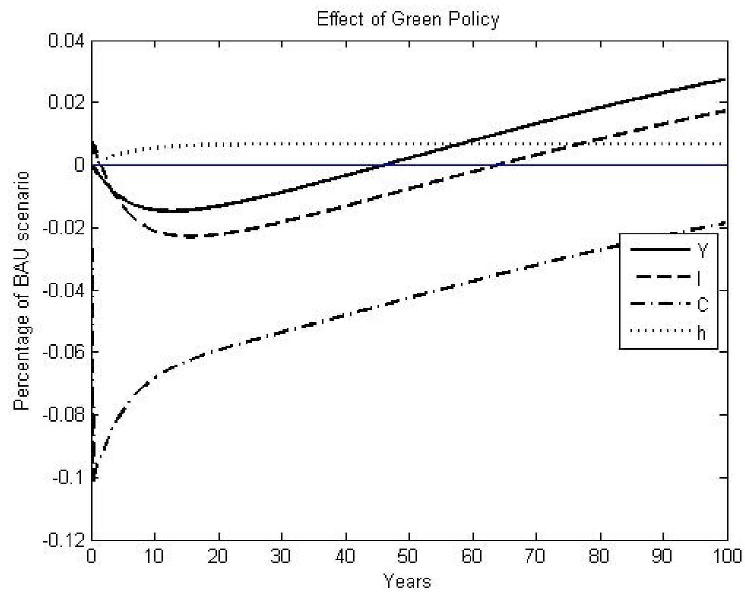

Figure 4: The net effect of higher emission taxes under exogenous growth.

On the other hand, changing the strength of endogenous growth to $\eta = 0.5$ does not alter the conclusions of the model too much (see Figure 5). The timespan until the green economy surpasses the BAU economy is somewhat longer, but the sensitivity to a parameter change in η is quite low.

We stress again that the endogenous growth framework in this model uses a simple learning-by-doing setup depending on output. If one were to specify a framework whereby environmental policy changed the incentive structure in the R&D sector, then there could be further forces changing the endogenous rate of technology growth. The outcomes of such changes can easily be derived from the results in

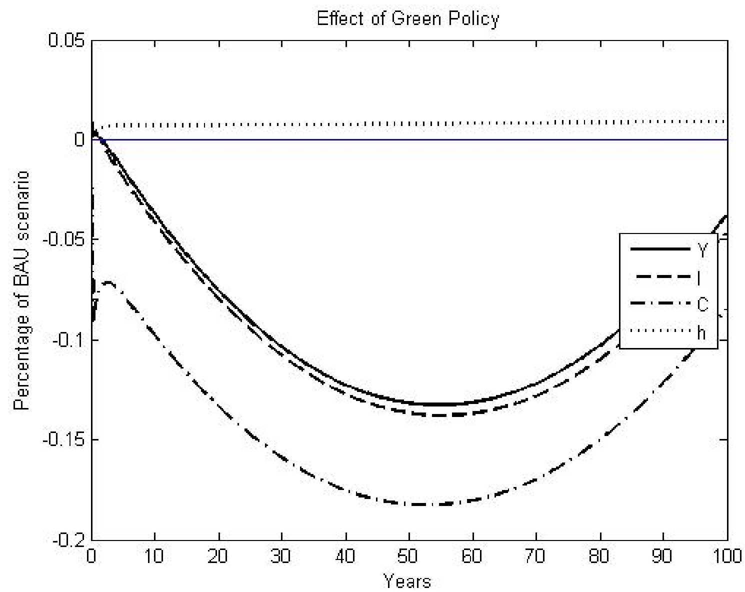

Figure 5: The net effect of higher emission taxes with $\eta = 0.5$.

this section.

No Double Dividend

When we simulate the model for the case where the tax revenues from the emission tax are not used for wage tax cuts but are distributed to households in a lump-sum fashion, then the growth rate in the green economy falls further, and thus the catch-up point is moved back (see Figure 6). Furthermore, employment falls when emission tax receipts are not used to cut labour taxes. The reason is that there is a distortion introduced, while no other distortion is reduced.

The long period of lower growth in the green scenario implies that relatively large output losses are being accumulated compared to the BAU scenario (see Figure 7). Nevertheless, output will surpass the BAU scenario eventually as the growth rate in the green scenario is higher.

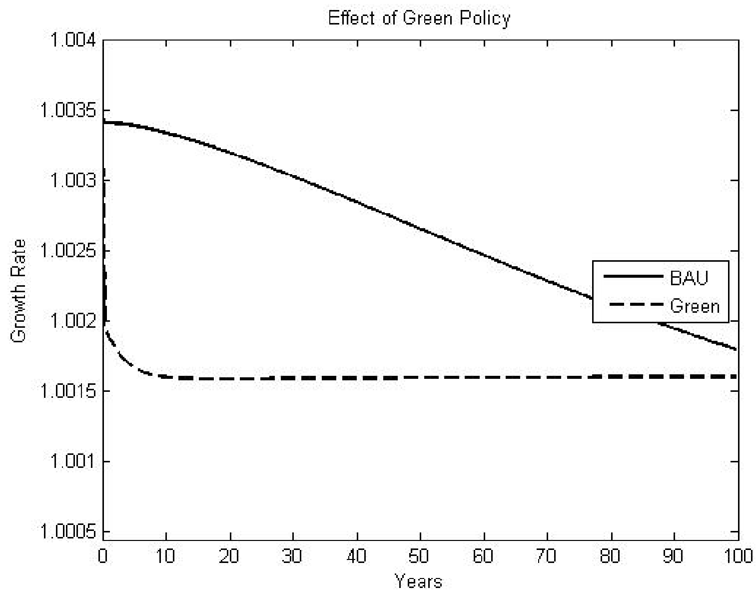

Figure 6: The net effect of higher emission tax without cutting wage taxes.

Total labour compensation as a percentage of GDP hardly changes compared to the baseline scenario where wage taxes are cut. The reason is that both labour supply and employment fall in this scenario, but the real wage is higher. Incidentally, these two effects cancel each other out. Nevertheless, lower employment causes lower output and growth, and thus clearly has detrimental effects. Furthermore, net compensation for employees is lower. One should keep in mind that the government collects a large share of GDP in tax revenues, which it simply distributes in form of lump sum transfers to households in this scenario, while there might be other uses for it as well.

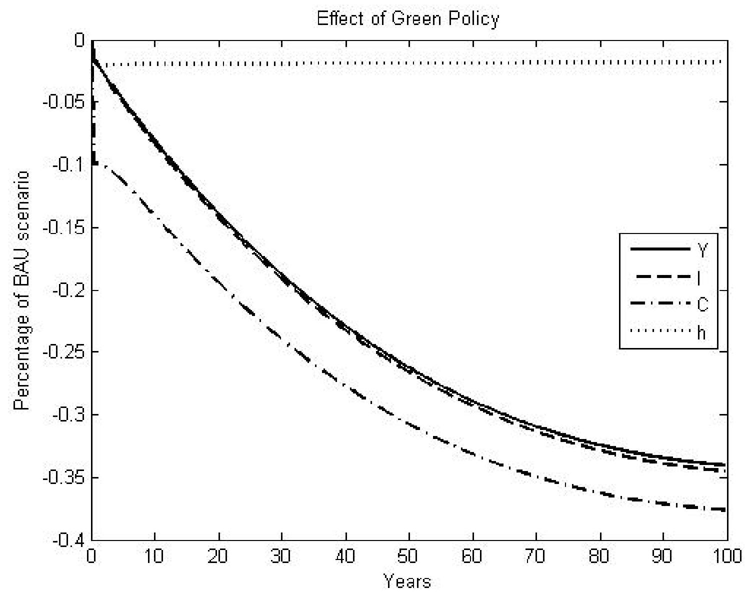

Figure 7: The net effect of higher emission tax without cutting wage taxes.

Changing the half-life time of pollution

Given all other structural parameters, a reduction in the half-life time of pollution implies a faster decay of pollutants and thus a lower steady state level of them. Hence, damages in the BAU scenario would also be lower. However, we calibrate the model assuming a certain steady state damage in the BAU scenario, no matter what the half-life time of pollutants is. Thus, testing different calibrations of the half-life time of pollutants is equivalent to testing different time horizons for the transition of the BAU scenario to its final steady state. A low half-life time of pollution implies a relatively fast transition to the steady state, and thus a relatively fast transition to high damages. Obviously, it is possible to fix all structural parameters and only investigate the effect of a varying half-life time.

There is no proper estimate for the half-life time of pollution. Heutel (2011) cites several sources with estimates ranging from $v = 30$ years to $v = 120$ years. We will reproduce both "extreme" values here. In Figure 8 we present a simulation where we calibrate the half life time of pollution to 30 years. This causes the BAU scenario to reach its steady state much more quickly, which also means that the growth rate in the BAU scenario falls quicker below the growth rate in the green scenario.

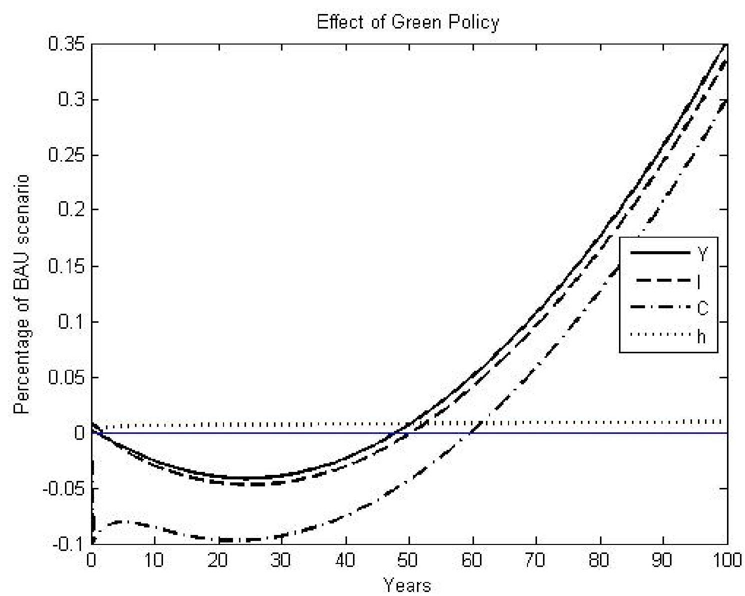

Figure 8: The net effect of a higher emission tax with a low half life of pollution ($v = 30$ Years).

To contrast this, we present Figure 9, where we assume a high half-life time of pollution of 120 years. As the business as usual scenario reaches its steady state more slowly, there will be a larger relative negative effect in the green scenario. Nevertheless, the fundamental conclusions do not change. Introducing an emission tax will eventually increase economic output.

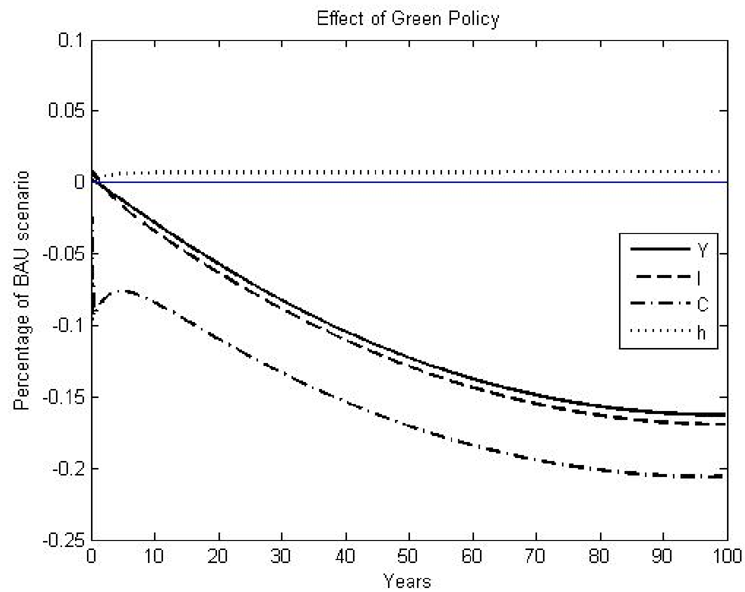

Figure 9: The net effect of a higher emission tax with a high half life of pollution ($v = 120$ Years).

Lower marginal efficiency of abatement spending

A lower efficiency of abatement spending ($\theta_2 = 0.35$) causes a certain share of abatement spending in output, $\frac{s}{y}$, to cut emissions by less. However, the way we calibrate the model's parameters means that the 2% share of output spent on abatement in the green senario is always sufficient to cut emissions so far to ensure a green economy; we adjust other parameters in the emission function accordingly.

Nevertheless, there is an important indirect effect of a lower marginal efficiency of abatement spending. Firms will rather spend money on the tax than spending it on abatement. This means that a higher tax rate has to be introduced to reach the target spending rate of 2%. This in turn means that the distortions introduced by the emission tax are larger, and factor payments to labour fall by a further 0.4% of GDP.

There is a countering effect though. Due to the fact that firms will rather spend money on the tax than on abatement, tax revenues are larger. Tax payments as a share of output increase from 6% to 6.8%. This also means that wage taxes can be cut by more, which in turn counters the increased distortions from the emission tax. Figure 10 shows that, compared to the baseline case in Figure 3, the growth rate falls slightly further and it takes a few years longer for the benefits of the green transition to materialize. While net labour compensation and employment is slightly higher than in the baseline scenario, the lower use of capital due to its lower compensation leads to a slightly negative net effect on output and growth.

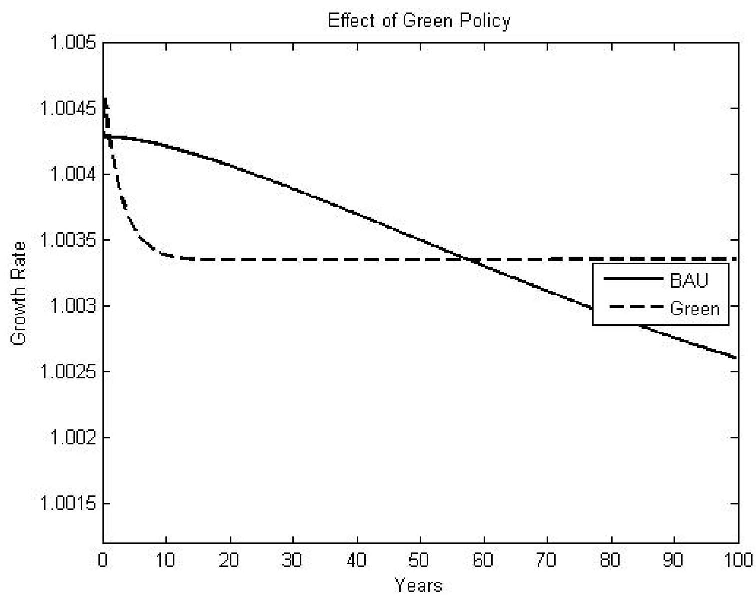

Figure 10: The net effect of a higher emission tax with a lower marginal efficiency of abatement spending ($\theta_2 = 0.35$).

Figure 11 shows that a green transition without a cut in wage taxes when the marginal efficiency of abatement spending is low will take an extremely long time to show benefits. In fact, the growth rate in the green transition is only marginally above the growth rate of the BAU scenario in its final steady state. Naturally, there will also be a strong negative employment effect in this scenario.

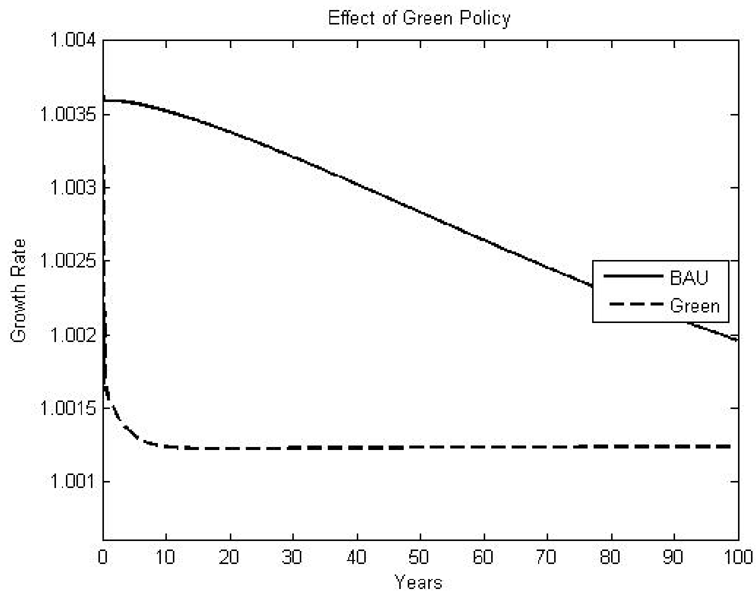

Figure 11: The net effect of a higher emission tax with a lower marginal efficiency of abatement spending ($\theta_2 = 0.35$) and no cut in wage taxes.

The policy conclusion from this experiment is that it is even more important to use revenues from the emission tax to cut labour taxes when the distortions induced by the emission tax are larger. Furthermore, the government should try to impose a tax that causes as little distortions as possible while reaching its emission goal. While we show that the net effect for labour compensation from higher distortions can be compensated, overall distortions increase and lead to slightly worse economic outcome.

Phasing-in the emission tax

In this section we investigate the effect of phasing-in the emission tax, meaning that not the whole amount is increased as a one-time shock but rather that it is slowly increased to its maximum amount. The way we introduce it is by using a simple autoregressive process of the form $\tau_{E,t} = (1-\rho_\tau)\tau_E + \rho_\tau \tau_{E,t-1}$, where τ_E is the final tax rate to be achieved and ρ_τ is the autoregressive parameter. We choose $\rho_\tau = 0.95$, which implies a half-life of 13.5 years, meaning that every 13.5 years half of the remaining tax amount has been levied.

A more realistic introduction of the tax would probably see an introduction in several discrete steps. Due to the simplicity of our model without many rigidities or backward looking variables, this will produce a number of jumps in the variables. Introducing such autoregressive process is a common way to represent policy shocks in DSGE literature.

Figure 12 shows two things, which are directly related. First, consumption falls immediately, despite the fact that the tax is introduced only slowly. Forward looking households smooth their fall in future consumption, due to the resource use of abatement spending, and increase their savings. This crowds in private investment, but leads to fall in demand as well. Second, the maximum negative net effect is smaller in this scenario than in Figure 2.

Figure 13 shows that the growth rate falls more slowly due to the gradual introduction of the distortion in the economy. For this reason, the accumulated gap in output is smaller, and thus the maximum negative effect in Figure 12 is smaller as well.

Finally, a gradual introduction of the emission tax means that emissions are not cut immediately by the required amount. Thus, pollution will initially continue to accumulate. Figure 14 shows the amount of pollution accumulated in this scenario, relative to the maximum pollution reached in the BAU scenario. If the emission tax

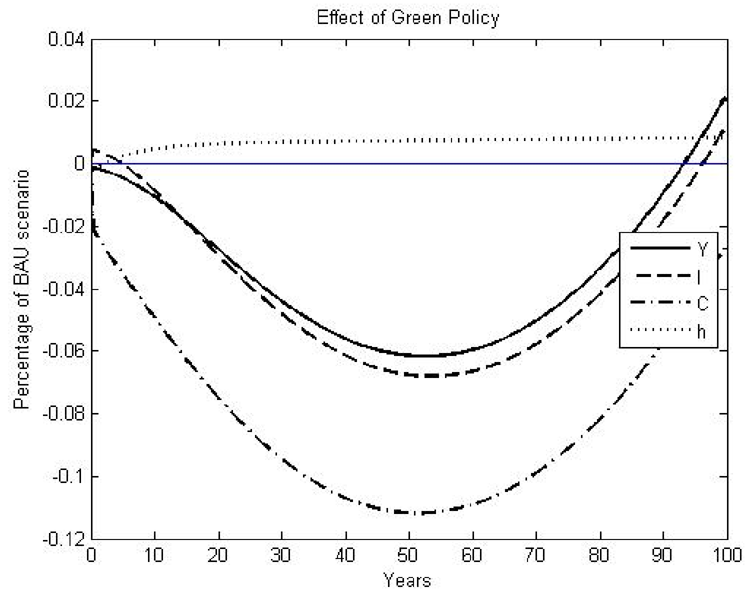

Figure 12: The net effect of a higher emission tax with a phased-in transition of the tax.

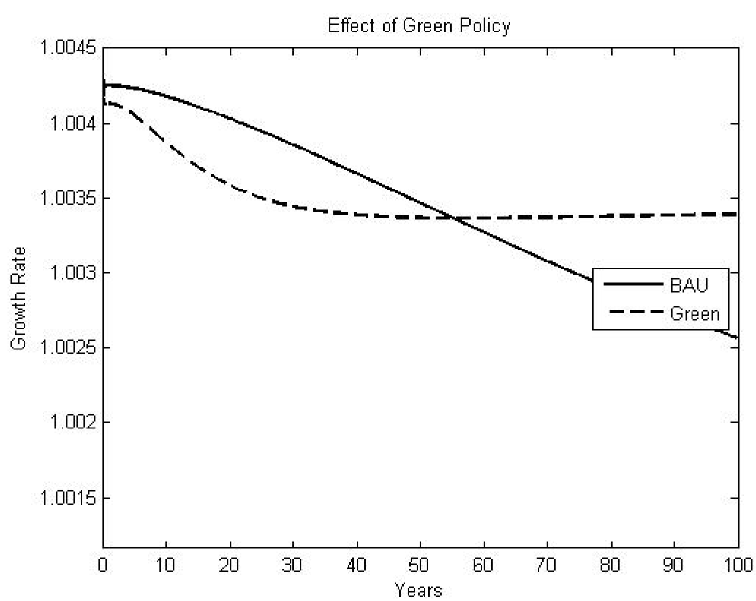

Figure 13: The net effect of a higher emission tax with a phased-in transition of the tax.

were to be introduced fully at once, the line would remain at zero. We see that pollution increases strongly for another 15 years, at which point the turn-around starts.

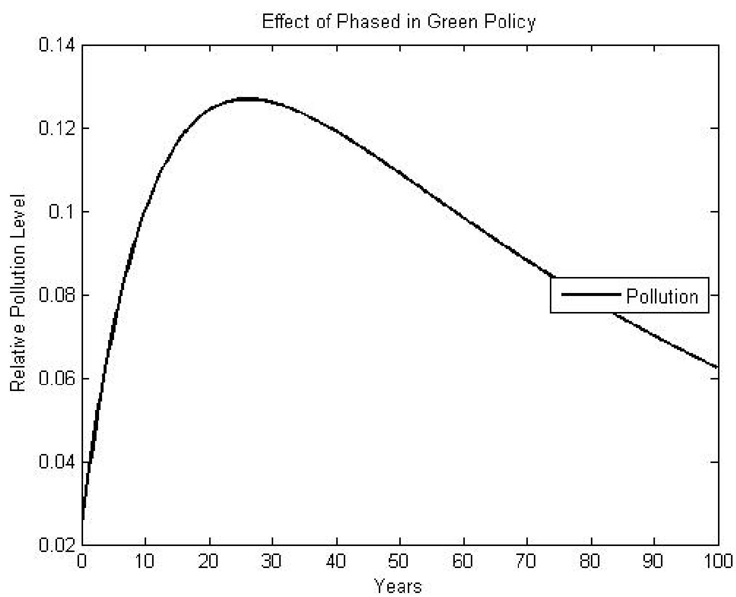

Figure 14: The amount of pollution accumulated in the green scenario with a gradual emission tax introduction relative to the maximum amount of pollution in the BAU scenario. Zero represents an immediate introduction of the emission tax.

6 Discussion on optimal policy

We define optimal policy as the choice of government policy variables, thus the emission tax rate, that maximizes household utility. A full discussion of optimal policy requires that the economy starts in a situation of optimal tax and spending policy, although without an emission tax. From that situation, the changes to the policy instruments given the consideration of an emission tax have to be calculated. This analysis is beyond the scope of this paper. A more realistic option is to focus purely on the emission tax and to follow a balanced budget approach concerning the revenues of that tax in the policy optimization.

Steady state analysis

As a first step, we derive household utility in terms of stationary steady state variables in Appendix .

$$U(c,1-h) = T_0 \frac{\left[c\,(1-h)^{\psi}\right]^{1-\sigma}}{1-\sigma} \frac{1}{1-\beta g^{1-\sigma}} \tag{33}$$

Equation (33) shows that when $\sigma = 1$ (i.e. log-utility), the economy's growth rate does not enter the household's discounted stationary utility. Thus, utility maximization does not have to equal growth maximization. Important to utility maximization are stationary consumption and leisure.

In Table 2, we show the stationary steady state values of consumption, hours worked and period utility (thus not multiplying with the discounted infinite future values) for the baseline calibration. The figure shows that in steady state, households actually have a higher utility when their wage taxes are not cut. The intuition for this result is simple: emission tax revenues are distributed lump-sum to households, so that they receive that amount to consume. The cut in wage taxes raises output, growth and consumption, but also lowers leisure, so that the net effect on utility turns out to be negative.

We can already conclude from the above results that the optimal policy will involve a corner solution in our baseline calibration. This implies that the tax rate is set such that the minimum emissions, pollution and damage will be achieved. A

Table 2: Steady State Values

Scenario	c	h	$\log\left(c(1-h)^{\Psi}\right)$
BAU	0.737	0.592	-0.736
Green with wage tax cut	0.748	0.603	-0.734
Green without wage tax cut	0.737	0.574	-0.715

Steady state stationary values of consumption, hours worked and period utility under different scenarios using the baseline calibration.

formal optimization has to take into account that under many calibrations corner solutions will be achieved. Thus, the specification of constraints is important in the optimization.

Outlining optimization under the ideal price instrument

The paper thus far assumed that firms do not take their impact on pollution and damage into account, as they are too small. This produces a negative externality, which the government tries to correct using the tax on emissions as an instrument. An interesting case to study is based on the idea that there is an ideal instrument available, which causes firms to take their impact on damages into account.

The way to specify this ideal instrument is to conduct the intermediate firm optimzation in Section without an emission tax, but by having firms optimizing also with respect to damages, subject to the pollution accumulation equation. In a sense, the firm assumes it is large. The firm will have to make a trade-off between investment in capital and investment in abatement.

Pollution in utility

So far, the beneficial effect of green policy occurred due to the damages pollution have on the economy. We calibrated the model in a way that green policy leading to the lowest possible pollution outcome is beneficial. However, it is also conceivable that pollution is much less harmful economically, or much more expensive to reduce. In such a case, the presence of pollution in the utility function could increase the optimal tax rate beyond what one would obtain from pure maximization of the utility function.

7 Conclusion

This paper analysis the costs and benefits of a transition to a green economy using a dynamic general equilibrium model featuring endogenous growth. As such, the model does not only allow a static comparison of steady states in a green and a business as usual economy, but allows a quantification of the transition costs to a green economy. The paper draws several important conclusions.

First, the introduction of an emission tax creates a distortion in the economy, which immediately lowers consumption and growth. As the business-as-usual scenario remains on a high growth path until environmental damages to the economy are eventually large enough to lower growth, there will be an initial period of lower growth in the green economy, which accumulates to a large relative output gap compared to the business as usual case. The closing of this output gap can take a very long time, more than 100 years.

Second, using the revenues from the emission tax to cut wage taxes has several benefits. It reduces the distortions on the labour market, thereby increasing employment, output and growth. This significantly lowers the size and duration of the negative relative output gap vis-a-vis the business-as-usual scenario. Furthermore, the share of income in the economy attributed to wage earners rises.

Third, the time for the business-as-usual scenario to reach its steady state has important consequences for when the positive effects of a green transition materialize. If this time is long, then environmental damages depressing the economy appear only very late, and thus investments made in the green scenario take a long time to pay off. Beneficial effects of green policy appear soon if the business-as-usual scenario converges quickly to its steady state.

Fourth, a gradual introduction of the emission tax into the economy has slight benefits in terms of the size and the time length of the relative output gap vis-a-vis the business-as-usual scenario. However, it comes at the cost of having a temporary increase in the pollution stock, as environmental policy is not fully effective from the outset.

Future avenues of research for this model are extensions using standard New Keynesian features such as price stickiness, consumption habits and capital adjustment costs. Furthermore, a more detailed labour market model could be introduced to study employment effects. These aspects will allow a richer analysis of the transition dynamics and will also allow to study the role of monetary policy in such a transition.

A Mathematical Appendix

Household optimization problem

The household solves the following optimization problem

$$\max_{C_t, h_t, K_t, I_t, B_t} U\left(C_t, 1-h_t\right)$$

subject to (8) and (9).

The Lagrangian of the household's problem is

$$\mathscr{L}^{h1} = E_0 \Sigma_{t=0}^{\infty} \beta^t \left\{ \frac{\left[C_t \left(1-h_t\right)^{\psi}\right]^{1-\sigma}}{1-\sigma} + \Lambda_t \left[-C_t - K_t + \left(r_t + 1 - \delta\right) K_{t-1} + (1-\tau_w) W_t h_t\right] \right\}$$

where Λ_t is the Lagrange multiplier associated to the budget constraint.

The first order conditions with respect to C_t, h_t, and K_t are , in that order, given by

$$\Lambda_t = C_t^{-\sigma} \left(1-h_t\right)^{\psi(1-\sigma)} \tag{34}$$

$$\Lambda_t (1-\tau_w) W_t = \psi C_t^{1-\sigma} \left(1-h_t\right)^{\psi(1-\sigma)-1} \tag{35}$$

$$\Lambda_t = \beta E_t \Lambda_{t+1} \left(r_{t+1} + 1 - \delta\right) \tag{36}$$

Plug (34) in (35) and (36)

$$(1-\tau_w) W_t = \psi \frac{C_t}{1-h_t} \tag{37}$$

$$C_t^{-\sigma} \left(1-h_t\right)^{\psi(1-\sigma)} = \beta E_t C_{t+1}^{-\sigma} \left(1-h_{t+1}\right)^{\psi(1-\sigma)} \left(r_{t+1} + 1 - \delta\right) \tag{38}$$

Optimization of final goods producers

The aggregate demand is the result of a cost minimization problem:

$$\min_{Y_{i,t}} \int_0^1 P_{i,t} Y_{i,t} di$$
$$\text{s.t. } Y_t \leq \left(\int_0^1 Y_{i,t}^{\frac{1}{\mu}} di \right)^{\mu}$$

The Lagrangian of that problem is

$$\mathscr{L} = \int_0^1 P_{i,t} Y_{i,t} di + \omega_t \left(Y_t - \left(\int_0^1 Y_{i,t}^{\frac{1}{\mu}} di \right)^{\mu} \right)$$

where ω_t is Lagrange multiplier associated to the final goods technology constraint.

The first order conditions with respect to two varieties of intermediate goods, $Y_{i,t}$ and $Y_{j,t}$, in that order, are

$$\begin{aligned} & P_{i,t} + \omega_t (-\mu) \frac{1}{\mu} Y_{i,t}^{\frac{1}{\mu}-1} \left(\int_0^1 Y_{i,t}^{\frac{1}{\mu}} di \right)^{\mu-1} = 0 \\ \Leftrightarrow \quad & P_{i,t} = \omega_t Y_{i,t}^{\frac{1-\mu}{\mu}} Y_t \left(\int_0^1 Y_{i,t}^{\frac{1}{\mu}} di \right)^{-1} \\ \Leftrightarrow \quad & P_{i,t} = \omega_t Y_{i,t}^{\frac{1-\mu}{\mu}} Y_t Y_t^{-\frac{1}{\mu}} \\ \Leftrightarrow \quad & P_{i,t} = \omega_t \left(\frac{Y_{i,t}}{Y_t} \right)^{\frac{1-\mu}{\mu}} \end{aligned} \tag{39}$$

$$
\begin{aligned}
& P_{j,t} + \omega_t (-\mu) \frac{1}{\mu} Y_{j,t}^{\frac{1}{\mu}-1} \left(\int_0^1 Y_{i,t}^{\frac{1}{\mu}} di \right)^{\mu-1} = 0 \\
\Leftrightarrow \quad & P_{j,t} = \omega_t Y_{j,t}^{\frac{1-\mu}{\mu}} Y_t \left(\int_0^1 Y_{i,t}^{\frac{1}{\mu}} di \right)^{-1} \\
\Leftrightarrow \quad & P_{j,t} = \omega_t Y_{j,t}^{\frac{1-\mu}{\mu}} Y_t Y_t^{-\frac{1}{\mu}} \\
\Leftrightarrow \quad & P_{j,t} = \omega_t \left(\frac{Y_{j,t}}{Y_t} \right)^{\frac{1-\mu}{\mu}}
\end{aligned} \tag{40}
$$

Combining (39) and (40) we get

$$
\begin{aligned}
\frac{P_{i,t}}{P_{j,t}} &= \left(\frac{Y_{i,t}}{Y_{j,t}} \right)^{\frac{1-\mu}{\mu}} \\
\Leftrightarrow Y_{i,t} &= Y_{j,t} \left(\frac{P_{i,t}}{P_{j,t}} \right)^{\frac{\mu}{1-\mu}}
\end{aligned}
$$

We plug the previous expression into (12) (binded) and get

$$
\begin{aligned}
Y_t &= \left(\int_0^1 \left(Y_{j,t} \left(\frac{P_{i,t}}{P_{j,t}} \right)^{\frac{\mu}{1-\mu}} \right)^{\frac{1}{\mu}} di \right)^{\mu} \\
\Leftrightarrow Y_t &= \left(\int_0^1 Y_{j,t}^{\frac{1}{\mu}} \left(\frac{P_{i,t}}{P_{j,t}} \right)^{\frac{1}{1-\mu}} di \right)^{\mu} \\
\Leftrightarrow Y_t &= Y_{j,t}^{\frac{\mu}{\mu}} P_{j,t}^{-\frac{\mu}{1-\mu}} \left(\int_0^1 P_{i,t}^{\frac{1}{1-\mu}} di \right)^{\mu} \\
\Leftrightarrow Y_t &= Y_{j,t} P_{j,t}^{-\frac{\mu}{1-\mu}} \left(\int_0^1 P_{i,t}^{\frac{1}{1-\mu}} di \right)^{\mu} \\
\Leftrightarrow Y_t &= Y_{j,t} P_{j,t}^{-\frac{\mu}{1-\mu}} P_t^{\frac{\mu}{1-\mu}}
\end{aligned}
$$

where

$$
P_t = \left(\int_0^1 P_{i,t}^{\frac{1}{1-\mu}} di \right)^{1-\mu} \tag{41}
$$

Hence the demand function for variety i of intermediate goods is

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{\frac{\mu}{1-\mu}} Y_t \tag{42}$$

Firm Optimization

The Lagrangian of that problem is given by

$$\begin{aligned}\mathscr{L} = E_0 \Sigma_{t=0}^{\infty} \zeta_t \Bigg\{ \left(\frac{P_{i,t}}{P_t}\right) Y_{i,t} - W_t h_{i,t} - r_t K_{i,t-1} - \tau_E Y_t^{1-\chi} A_E \left(1 - \theta_1 \left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2}\right) Y_{i,t}^{\chi} - S_{i,t} \\ + \lambda_{i,t} \left[D_t K_{i,t-1}^{\alpha} \left(T_{t-1} h_{i,t}\right)^{1-\alpha} - Y_{i,t}\right] + \mu_{i,t} \left[\left(\frac{P_{i,t}}{P_t}\right)^{\frac{\mu}{1-\mu}} Y_t - Y_{i,t}\right] \Bigg\}\end{aligned} \tag{43}$$

The first order conditions with respect to $K_{i,t-1}$ and $h_{i,t}$, in that order, are

$$\begin{aligned} & -r_t + \lambda_{i,t} \alpha D_t K_{i,t-1}^{\alpha-1} \left(T_{t-1} h_{i,t}\right)^{1-\alpha} = 0 \\ \Leftrightarrow \quad & r_t = \lambda_{i,t} \alpha \frac{Y_{i,t}}{K_{i,t-1}} \end{aligned} \tag{44}$$

$$\begin{aligned} & -W_t + \lambda_{i,t} \left(1-\alpha\right) D_t K_{i,t-1}^{\alpha} \left(T_{t-1}\right)^{1-\alpha} h_{i,t}^{-\alpha} = 0 \\ \Leftrightarrow \quad & W_t = \lambda_{i,t} \left(1-\alpha\right) \frac{Y_{i,t}}{h_{i,t}} \end{aligned} \tag{45}$$

The first order conditions with respect to $Y_{i,t}$ and $S_{i,t}$ can be solved as

$$\lambda_{i,t} = \frac{P_{i,t}}{P_t} - \mu_{i,t} - \tau_E A_E \left(\frac{Y_t}{Y_{i,t}}\right)^{1-\chi} \left(\theta_1 \theta_2 \left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2} + \chi \left[1 - \theta_1 \left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2}\right]\right) \tag{46}$$

$$\left(\frac{S_{i,t}}{Y_{i,t}}\right)^{1-\theta_2} = \tau_E A_E \theta_1 \theta_2 \left(\frac{Y_t}{Y_{i,t}}\right)^{1-\chi} \tag{47}$$

To derive real marginal costs, we combine (15) and (16) and get

$$h_{i,t} = \frac{r_t\,(1-\alpha)}{W_t\alpha} K_{i,t-1}$$

We plug the previous equation into (14) and derive the capital demand function

$$\begin{aligned} Y_{i,t} &= D_t K_{i,t-1}^{\alpha} \left(T_{t-1} \frac{r_t\,(1-\alpha)}{W_t \alpha} K_{i,t-1} \right)^{1-\alpha} \\ \Leftrightarrow Y_{i,t} &= D_t K_{i,t-1} \left(\frac{1-\alpha}{\alpha} \frac{r_t}{W_t/T_{t-1}} \right)^{1-\alpha} \\ \Leftrightarrow K_{i,t-1} &= \frac{Y_{i,t}}{D_t} \left(\frac{\alpha}{1-\alpha} \frac{W_t/T_{t-1}}{r_t} \right)^{1-\alpha} \qquad (48) \end{aligned}$$

The demand function for labor is thus

$$\begin{aligned} h_{i,t} &= \frac{Y_{i,t}}{D_t} \frac{1-\alpha}{\alpha} \frac{r_t}{W_t} \left(\frac{\alpha}{1-\alpha} \frac{W_t/T_{t-1}}{r_t} \right)^{1-\alpha} \\ \Leftrightarrow h_{i,t} &= \frac{Y_{i,t}/T_{t-1}}{D_t} \left(\frac{\alpha}{1-\alpha} \frac{W_t/T_{t-1}}{r_t} \right)^{-1} \left(\frac{\alpha}{1-\alpha} \frac{W_t/T_{t-1}}{r_t} \right)^{1-\alpha} \\ \Leftrightarrow h_{i,t} &= \frac{Y_{i,t}/T_{t-1}}{D_t} \left(\frac{\alpha}{1-\alpha} \frac{W_t/T_{t-1}}{r_t} \right)^{-\alpha} \qquad (49) \end{aligned}$$

With the demand functions for capital and labor we can derive the expression of the total cost function

$$\begin{aligned}
TC_{i,t} &= W_t h_{i,t} + r_t K_{i,t-1} + \tau_E Y_t^{1-\chi} E_{i,t} \\
\Leftrightarrow TC_{i,t} &= W_t \frac{Y_{i,t}/T_{t-1}}{D_t}\left(\frac{\alpha}{1-\alpha}\frac{W_t/T_{t-1}}{r_t}\right)^{-\alpha} + r_t \frac{Y_{i,t}}{D_t}\left(\frac{\alpha}{1-\alpha}\frac{W_t/T_{t-1}}{r_t}\right)^{1-\alpha} + \tau_E Y_t^{1-\chi} E_{i,t} \\
\Leftrightarrow TC_{i,t} &= \left(\frac{W_t}{T_{t-1}}\right)^{1-\alpha} r_t^{\alpha} \frac{Y_{i,t}}{D_t}\frac{\alpha^{-\alpha}}{(1-\alpha)^{-\alpha}} + r_t^{\alpha}\frac{Y_{i,t}}{D_t}\frac{\alpha^{1-\alpha}}{(1-\alpha)^{1-\alpha}}\left(\frac{W_t}{T_{t-1}}\right)^{1-\alpha} + \tau_E Y_t^{1-\chi} E_{i,t} \\
\Leftrightarrow TC_{i,t} &= \left(\frac{W_t}{T_{t-1}}\right)^{1-\alpha} r_t^{\alpha}\frac{Y_{i,t}}{D_t}\left(\frac{\alpha^{-\alpha}}{(1-\alpha)^{-\alpha}} + \frac{\alpha^{1-\alpha}}{(1-\alpha)^{1-\alpha}}\right) + \tau_E Y_t^{1-\chi} E_{i,t} \\
\Leftrightarrow TC_{i,t} &= \frac{Y_{i,t}}{D_t}\frac{\left(\frac{W_t}{T_{t-1}}\right)^{1-\alpha} r_t^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} + \tau_E Y_t^{1-\chi} A_E \left(1-\theta_1\left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2}\right) Y_{i,t}^{\chi} \qquad (50)
\end{aligned}$$

Hence, the real marginal cost is

$$\begin{aligned}
mc_{i,t} =& \frac{\partial TC_{i,t}}{Y_{i,t}} \\
=& \frac{1}{D_t}\frac{\left(\frac{W_t}{T_{t-1}}\right)^{1-\alpha} r_t^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} + \tau_E A_E\left(\frac{Y_t}{Y_{i,t}}\right)^{1-\chi}\left(\theta_1\theta_2\left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2} + \chi\left[1-\theta_1\left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2}\right]\right) \qquad (51)
\end{aligned}$$

We see that real marginal costs are not the same for all firms.

We restate the the marginal cost function and the labor demand function as follows

$$\begin{aligned}
mc_{i,t} =& \frac{W_t}{T_{t-1}}\frac{1}{D_t}\left(\frac{r_t}{W_t/T_{t-1}}\right)^{\alpha}\frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \\
&+ \tau_E A_E\left(\frac{Y_t}{Y_{i,t}}\right)^{1-\chi}\left(\theta_1\theta_2\left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2} + \chi\left[1-\theta_1\left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2}\right]\right)
\end{aligned}$$

$$
\begin{aligned}
h_{i,t}\frac{D_t}{Y_{i,t}/T_{t-1}} &= \left(\frac{\alpha}{1-\alpha}\frac{W_t/T_{t-1}}{r_t}\right)^{-\alpha} \\
\Leftrightarrow h_{i,t}\frac{D_t}{Y_{i,t}/T_{t-1}} &= \left(\frac{1-\alpha}{\alpha}\frac{r_t}{W_t/T_{t-1}}\right)^{\alpha} \\
\Leftrightarrow h_{i,t}\frac{D_t}{Y_{i,t}/T_{t-1}} &= \left(\frac{r_t}{W_t/T_{t-1}}\right)^{\alpha}\frac{(1-\alpha)^{\alpha}(1-\alpha)^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \\
\Leftrightarrow h_{i,t}\frac{D_t}{Y_{i,t}/T_{t-1}} &= \left(\frac{r_t}{W_t/T_{t-1}}\right)^{\alpha}\frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}(1-\alpha) \\
\Leftrightarrow \frac{h_{i,t}}{Y_{i,t}/T_{t-1}}\frac{D_t}{(1-\alpha)} &= \left(\frac{r_t}{W_t/T_{t-1}}\right)^{\alpha}\frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}
\end{aligned}
$$

Combining these two equations we get

$$
\begin{aligned}
mc_{i,t} &= \frac{W_t}{T_{t-1}}\frac{1}{D_t}\frac{h_{i,t}}{Y_{i,t}/T_{t-1}}\frac{D_t}{(1-\alpha)} \\
&\quad +\tau_E A_E\left(\frac{Y_t}{Y_{i,t}}\right)^{1-\chi}\left(\theta_1\theta_2\left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2}+\chi\left[1-\theta_1\left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2}\right]\right)
\end{aligned}
$$

From (16), we see that

$$
\lambda_{i,t} = \frac{W_t}{T_{t-1}}\frac{1}{D_t}\frac{h_{i,t}}{Y_{i,t}/T_{t-1}}\frac{D_t}{(1-\alpha)}
$$

Hence, we can write

$$
mc_{i,t} = \lambda_{i,t} + \tau_E A_E\left(\frac{Y_t}{Y_{i,t}}\right)^{1-\chi}\left(\theta_1\theta_2\left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2}+\chi\left[1-\theta_1\left(\frac{S_{i,t}}{Y_{i,t}}\right)^{\theta_2}\right]\right) \quad (52)
$$

$$
\Leftrightarrow mc_{i,t} = \frac{P_{i,t}}{P_t} - \mu_{i,t} \quad (53)
$$

(20) is in fact the same as in a standard model of monopolistic competition. Firms charge a price as a mark-up on their marginal costs. However, these marginal costs are higher when there is an emission tax, which produces a larger wedge in the economy. The flxible price equilibrium is given by symmetric firms, thus by $mc_{i,t} = mc_t$

and $P_{i,t} = P_t$. In that case, we obtain the familiar $mc_t = \frac{1}{\mu}$.

Equilibrium

Non-linear non-stationary equilibrium

The non-linear non-stationary (NLNS) competitive equilibrium consists of a set of processes Y_t, C_t, I_t, h_t, K_t, W_t, r_t, Λ_t, D_t, T_t, X_t, E_t, S_t, $g_{T,t}$, and κ_t, that satisfy the NLNS equilibrium equations (1), (2), (4), (5), (3), (9), (10), (11), (14), (15), (16), (18), (20), (21) and (22) given the initial conditions C_{-1}, K_{-1}, I_{-1}, X_{-1}, T_{-1}, Y_{-1}, and S_{-1}.

Non-linear stationary equilibrium

The economy features endogenous growth. This implies that several variables of the model will be non-stationary along the balanced-growth path. Therefore, we need to redefine them in order to get a system of stationary equilibrium equations. To this end, we have to find with which factor the non-stationary variables are cointegrated.

We can show that Y_t, C_t, I_t, K_{t-1}, W_t and S_t grow at the same rate as T_{t-1} does. Hence, the stationary expressions of these variables (denoted in small letters) are

$$y_t = \frac{Y_t}{T_{t-1}},\ c_t = \frac{C_t}{T_{t-1}},\ i_t = \frac{I_t}{T_{t-1}},\ k_{t-1} = \frac{K_{t-1}}{T_{t-1}},\ w_t = \frac{W_t}{T_{t-1}} \text{ and } s_t = \frac{S_t}{T_{t-1}}$$

We can show that Λ_t grows at the same rate as $T_{t-1}^{-\sigma}$. Therefore $\lambda_t = \Lambda_t T_{t-1}^{\sigma}$. We can also show that X_{t-1} and E_t grow at the same rate as T_{t-1}^{χ}

$$x_{t-1} = \frac{X_{t-1}}{T_{t-1}^{\chi}},\ e_t = \frac{E_t}{T_{t-1}^{\chi}}$$

The other variables do not need to be transformed as they are already defined as stationary in the model: h_t, r_t and κ_t.

Here we compute the system of stationary equilibrium equations.

The stationary expression of (9)

$$\begin{aligned} \frac{T_t}{T_{t-1}}\frac{K_t}{T_t} &= (1-\delta)\frac{K_{t-1}}{T_{t-1}}+\frac{I_t}{T_{t-1}} \\ \Leftrightarrow g_{T,t}k_t &= (1-\delta)k_{t-1}+i_t \end{aligned} \tag{54}$$

The stationary expression of (10)

$$\begin{aligned} (1-\tau_w)\frac{W_t}{T_{t-1}} &= \psi\frac{C_t/T_{t-1}}{1-h_t} \\ \Leftrightarrow (1-\tau_w)w_t &= \psi\frac{c_t}{1-h_t} \end{aligned} \tag{55}$$

The stationary expression of (11)

$$\begin{aligned} C_t^{-\sigma}T_{t-1}^{\sigma}(1-h_t)^{\psi(1-\sigma)} &= \beta E_t\left(\frac{T_{t-1}}{T_t}\right)^{\sigma}T_t^{\sigma}C_{t+1}^{-\sigma}(1-h_{t+1})^{\psi(1-\sigma)}(r_{t+1}+1-\delta) \\ \Leftrightarrow c_t^{-\sigma}(1-h_t)^{\psi(1-\sigma)} &= \beta E_t(g_{T,t}c_{t+1})^{-\sigma}(1-h_{t+1})^{\psi(1-\sigma)}(r_{t+1}+1-\delta) \end{aligned} \tag{56}$$

The stationary expression of (14)

$$\begin{aligned} \frac{Y_t}{T_{t-1}} &= z_t d_t A\left(\frac{K_{t-1}}{T_{t-1}}\right)^{\alpha}\left(T_{t-1}\frac{h_t}{T_{t-1}}\right)^{1-\alpha} \\ \Leftrightarrow y_t &= z_t d_t A k_{t-1}^{\alpha}h_t^{1-\alpha} \end{aligned} \tag{57}$$

The stationary expression of (15)

$$\begin{aligned} r_t &= \kappa_t\alpha\frac{Y_t/T_{t-1}}{K_{t-1}/T_{t-1}} \\ \Leftrightarrow r_t &= \kappa_t\alpha\frac{y_t}{k_{t-1}} \end{aligned} \tag{58}$$

The stationary expression of (16)

$$\begin{aligned} \frac{W_t}{T_{t-1}} &= \kappa_t (1-\alpha) \frac{Y_t/T_{t-1}}{h_t} \\ \Leftrightarrow w_t &= \kappa_t (1-\alpha) \frac{y_t}{h_t} \end{aligned} \tag{59}$$

The stationary expression of (18)

$$\left(\frac{s_t}{y_t}\right)^{1-\theta_2} = \tau_E A_E \theta_1 \theta_2 \tag{60}$$

The stationary eqpression of (17)

$$\kappa_t = \frac{1}{\mu} - \tau_E A_E \left(\theta_1 \theta_2 \left(\frac{s_t}{y_t}\right)^{\theta_2} + \chi \left[1 - \theta_1 \left(\frac{s_t}{y_t}\right)^{\theta_2}\right]\right) \tag{61}$$

Equation (2) can be restated with stationary output

$$g_{T,t} = B y_t^{\eta} \tag{62}$$

The stationary expression of (4) is

$$\begin{aligned} &\left(\frac{T_t}{T_{t-1}}\right)^{\chi} \frac{X_t}{T_t^{\chi}} = g_T^{\chi} \left(\rho_x \frac{X_{t-1}}{T_{t-1}^{\chi}} + \frac{E_t}{T_{t-1}^{\chi}}\right) \\ &\Leftrightarrow x_t = \rho_x x_{t-1} + e_t \end{aligned} \tag{63}$$

The stationary expression of (3) is

$$\begin{aligned} &\frac{E_t}{T_{t-1}^{\chi}} = A_E \left(1 - \theta_1 \left(\frac{s_t}{y_t}\right)^{\theta_2}\right) \left(\frac{Y_t}{T_{t-1}}\right)^{\chi} \\ &\Leftrightarrow e_t = A_E \left(1 - \theta_1 \left(\frac{s_t}{y_t}\right)^{\theta_2}\right) y_t^{\chi} \end{aligned} \tag{64}$$

The damage function (5) can be restated with stationary variables

$$d_t = \left(1 - \left(\frac{x_{t-1} - \underline{x}}{\bar{x} - \underline{x}}\right)^{\frac{1}{\gamma}}\right)^{\gamma} \tag{65}$$

The stationary expression of (22)

$$\begin{aligned} \frac{Y_t}{T_{t-1}} &= \frac{C_t}{T_{t-1}} + \frac{I_t}{T_{t-1}} + \frac{s_t}{y_t}\frac{Y_t}{T_t} \\ \Leftrightarrow y_t &= c_t + i_t + \frac{s_t}{y_t} y_t \end{aligned} \tag{66}$$

The non-linear stationary (NLS) competitive equilibrium consists of a set of 13 processes y_t, c_t, i_t, h_t, k_t, w_t, r_t, d_t, $\frac{s_t}{y_t}$, x_t, e_t, κ_t, and $g_{T,t}$, that satisfy the 13 NLS equilibrium equations (54), (55), (56), (57), (58), (59), (61), (62), (63), (64), (65), and (66) given the productivity shocks z_t and the initial conditions k_{-1} and x_{-1}.

Optimal Policy

We define optimal policy as the maximization of household utility by the government, which sets its available instrument, the emission tax. We first rewrite household utility in terms of steady state variables

$$\begin{aligned} U\left(C_t, 1-h_t\right) &= E_0 \Sigma_{t=0}^{\infty} \beta^t \frac{\left[C_t \left(1-h_t\right)^{\psi}\right]^{1-\sigma}}{1-\sigma} \\ \Leftrightarrow U\left(C_t, 1-h_t\right) &= E_0 \Sigma_{t=0}^{\infty} \beta^t T_t^{1-\sigma} \frac{\left[\frac{C_t}{T_t} \left(1-h_t\right)^{\psi}\right]^{1-\sigma}}{1-\sigma} \\ \Leftrightarrow U\left(C_t, 1-h_t\right) &= T_t \frac{\left[\frac{C}{T} \left(1-h\right)^{\psi}\right]^{1-\sigma}}{1-\sigma} E_0 \Sigma_{t=0}^{\infty} \left(\beta g^{1-\sigma}\right)^t \\ \Leftrightarrow U\left(C_t, 1-h_t\right) &= T_t \frac{\left[\frac{C}{T} \left(1-h\right)^{\psi}\right]^{1-\sigma}}{1-\sigma} \frac{1}{1-\beta g^{1-\sigma}} \end{aligned} \tag{67}$$

To derive this we have to assume that $\lim_{t\to\infty}\left(\left(\beta g^{1-\sigma}\right)^{t}\right)=0$, in other words that utility is bounded in a growing economy. This condition is met for a very wide range of parameters, including our choices.

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