# Dual Labour Markets with Search Costs*

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**Abstract**

This paper develops a model of an economy with dual labour markets to understand the dynamics of the informal sector over the business cycle, as well as to analyze the implication of duality for the volatility of output and the persistence of employment.The informal labour market is competitive while the formal labour market is characterized by search costs. The size of each labour market segment depends on labour demand by firms as well as participation decisions of households. The paper shows that the informal sector plays the role of a buffer, expanding in periods of recessions and shrinking when recovery sets in. A second result is that workers switching between the two labour market increases the volatility of output. Finally, labour market segmentation modifies the properties of the search model, as the competitive labour market segment reduces the volatility of employment, unless transition costs are high.

***JEL-Classification:*** E32; E24; J64

# 1 Introduction:

General equilibrium models with non-competitive labour market accounting for the dynamics of wages and employment have become increasingly used. Most of these models have been based upon the search and matching framework where unemployment arises in steady state due to matching frictions. For the most part, however, these general equilibirum models have been applied to advanced economies while the case of developing economies with dual labour market has been mainly addressed in partial equilibrium framework[1]. More generally, labour

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[1]See Sato (2004), Zenou (2008) or Albrecht *et al.* (2007) for instance

market segmentation has played only a limited role in the business cycle debate and remains confined to certain limited academic circles[2].

This paper models an economy with an informal labour market along the line of the search and matching framework and revisits three main issues. The first issue is the dynamics of the informal sector over the business cycle. The traditional view of informality argues that informality acts as a buffer for the formal sector contracting in the upturn and expanding in the downturn[3]. Another view on informality points to the voluntary nature of the entry in the informal sector linked with new opportunities during period of economic prosperity[4]. Empirical studies are mixed but tend to favour the view that informal labour markets act as a buffer against aggregate shocks.

The second issue is the impact of informality on economic volatility. The volatility of output is directly linked to the duality of the labour market, which involves workers transiting between labour markets. Conesa *et al.* (2002) provides empirical evidences showing that countries with small ratios of employment to population experience higher volatility of output. They also model an underground economy, in which population switching between the two sectors amplifies the response of registered output. Higher fluctuations depend on the movement of households between the two sectors, which is driven by the opportunity costs of not participating in the market (the wage differential between registered and non registered activities). On the contrary, Batini *et al.* (2009) point to the fact that informality is potentialy a source of lower output volatility. A benefit attached to informality is the increased flexibility of the economy related to the existence of a non-regulated labour market segment. Informal wages adjust quickly to shocks and reduce business cycle oscillations. Along this line of argument, a standard result of search and matching models is the increase in output volatility with search costs[5].

A third issue is the extent to which the interaction between dual labour markets modifies the standard properties of search models. On the one hand, the formal labour market segment is characterized by a search and matching function. Wages are the result of a Nash bargaining and are not fully flexible. On the other hand, the informal segment is competitive. The model therefore combines both an RBC labour market and a New Keynesian labour market. The existing literature on search and matching models puts forward that search and matching models have a higher explanatory power than RBC models, as they account for the empirically observed volatility of employment. The presence of search costs limits wage volatility and generates persistence in employment dynamic, contrary to RBC models with competitive labour markets[6]. The main question of interest is the impact on employment volatility of interacting

[2]Examples include Saint-Paul (1996) Conesa *et al.* (2002), Batini *et al.* (2009), Arbex and Turdaliev (2008) or Fiess *et al.* (2006)

[3]For a recent review of the literature on the interaction between informality and volatility seeBacchetta *et al.* (2009)

[4]This approach argues that the informal sector is made of self employed, small firms, who prefer to operate informaly to escape taxation and regulation Maloney (2004).

[5]See Andolfatto (1996) for an early example

[6]see Merz (1995); Andolfatto (1996); Gertler and Trigari (2009); Ravenna and Walsh (2008)

dual labour markets.

The model developed in this paper considers informality to increase within firms rather than across sectors. The traditional approach to informality is linked to a sectoral view of informality. Informal workers belong to the agriculture sector, while formal workers are in cities working in manufacturing sectors. In such a framework, the main issue is to understand under what conditions the manufacturing sector can absorb the flow of informal workers migrating into cities (Harris and Todaro, 1970). Over the past decades however, informality has evolved and is now increasingly present within the manufacturing sector itself (see Chen, 2007). Informal workers are employed alongside formal workers in the same production plants. We tackle this issue by modelling a single firm using three inputs capital, formal labour and informal labour, to produce a homogeneous good.

The last distinctive aspect of the model is the formulation of an equilibrium condition for the transition between the two labour markets. Following Zenou (2008), the transition from the informal labour market to the formal labour market segment takes place through a period of unemployment. Informal workers compare the expected value of a formal job with informal wages. In contrast to Zenou (2008), we consider the existence of switching or transition costs that may act as a barrier to formal employment.

The rest of the paper is organized as follows. The model is presented in section 2, which focuses especially on households heterogeneity and the mechanism determining the relative size of the labour markets. Section 3 presents the calibration of the main parameters and the implied steady state values. It also examines the properties of the steady states, and the dynamics of the economy using numerical simulations. Comparative statics concerns the impact of switching costs and search costs on the steady state values of the models. The dynamics of the economy following a productivity shock is assessed with respect to two benchmark models: a RBC model with competitive labour markets and a search and matching model with single labour market. Concluding remarks are in section 4. Lastly, the appendix provides the equations of the complete model and the steady states.

# 2 The Model:

The formal labour market is characterized by search costs following the approach used in Ravenna and Walsh (2008). There has been little attempt to model a dual labour market with matching function. In Batini *et al.* (2009), the formal wage is equal to the informal wage augmented by an institutionally given premium. Wages are set by union in Fiess *et al.* (2006), while wages are flexible in Arbex and Turdaliev (2008). In this model, labour adjusts along the extensive margin (i.e. the number of jobs), while most models also allow for adjustments along the intensive margin (i.e. hours worked). The former approach enables to focus on labour frictions. The later approach implies that marginal costs are mainly driven by the intensive margin and that the properties of these models is sensitive to the marginal rate of substitution between

leisure and consumption.

In our set-up labour market segmentation is not related to a sector approach as in Batini *et al.* (2009), where two firms are assumed to operate in two different sectors, using either formal labour or informal labour. In our model, there is a single firm producing a homogeneous goods using two types of labour inputs. Our approach also differs from models that assume that formal employment are salaried workers, while informality is characterized self employment, as in Conesa *et al.* (2002) and Fiess *et al.* (2006).

This paper relies on Conesa *et al.* (2002) as well as Batini *et al.* (2009) to model heterogeneity. Assuming a perfectly insured market, the competitive equilibrium is characterized by agents purchasing an infinite sequence of lotteries. This approach is similar to assuming that a representative household spends a share of his time on the informal labour market and a share of his time on the formal labour market. Contrary to Conesa *et al.* (2002), however, the participation on each labour markets does not only depend on households decision but also depends on labour demand.

Lastly, the relative size of informal employment is related to two factors. First informality depends on the labour demand by firms, which substitutes the three inputs depending on the relative costs and productivity. Second, households participation into the labour markets is a decision based on relative wages and employment opportunities in both labour markets. We here follow Zenou (2008), which specifies an equilibrium condition for the transition between the two markets. The equilibrium condition includes switching costs which affect the degree of labour market segmentation. Participation in the informal sector is not a pure choice variable as in Conesa *et al.* (2002) and does not result from a tax evasion strategy as in Arbex and Turdaliev (2008).

## 2.1 Unemployment, Vacancies and Matching

There are two labour market segments with different characteristics. The informal labour market is competitive, while the formal labour market is characterized by non-clearing wages. The active population is split between informal employment $n_t^i$, formal employment $n_t^f$ and unemployment $u_t$ in the formal sector; its size is normalized to 1.

$$u_t = 1 - n_t^f - n_t^i$$

The number of formal workers equals the existing stock of employment at the beginning of the period $\rho n_{t-1}^f$ plus new matches $m_t$. The rate of job destruction is a constant $(1-\rho)$:

$$n_t^f = \rho n_{t-1}^f + m_t \tag{1}$$

New matches $m_t$ depend positively on the number of unemployed workers searching a for-

mal jobs $s_t$ and the number of vacancies $v_t$. The function of new matches is similar to a Cobb-Douglas production function, with $\sigma$ the substitution elasticity. The parameter $\sigma_m$ reflects the efficiency of the matching process.

$$m_t = \sigma_m s_t^{\sigma} v_t^{1-\sigma}$$

For convenience, we use the ratio $\theta_t = \frac{v_t}{s_t}$ to measure labour labour market tightness. The probability of firms to fill up a vacancies is denoted $q(\theta_t)$ and is equal to the ratio of matches over the number of vacancies:

$$q_t = \frac{m_t}{v_t} = \sigma_m \theta_t^{-\sigma}$$

Similarly, the probability of an unemployed workers to find a formal job is given by the ratio of new matches over the number of unemployed searching workers:

$$p_t = \frac{m_t}{s_t} = \sigma_m \theta_t^{1-\sigma}$$

Given the above definitions, new formal matches can be expressed as the probability of filling a vacancy times the existing number of vacancies:

$$n_t^f = \rho n_{t-1}^f + q_t v_t$$

## 2.2 Households:

There is a representative household with three types of members depending on their employment status: formal workers, informal workers and unemployed workers:

$$\begin{aligned}
\max_{c_t^f, c_t^i, c_t^u, k_t, x_t} \mathcal{L} = \; & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ n_t^f \frac{\left(c_t^f\right)^{1-\sigma}}{1-\sigma} + n_t^i \frac{(c_t^i)^{1-\sigma}}{1-\sigma} + \left(1 - n_t^f - n_t^i\right) \frac{(c_t^u)^{1-\sigma}}{1-\sigma} \right. \\
& + \lambda_t \left[ x_t - r_t k_{t-1} - n_t^f w_t^f - w^u \left(1 - n_t^f - n_t^i\right) - n_t^i w_t^i + n_t^f c_t^f + n_t^i c_t^i + \left(1 - n_t^f - n_t^i\right) c_t^u - \Pi_t \right] \\
& + \lambda_t \varphi_t \left[ k_t - (1-\delta) k_{t-1} - x_t \left( 1 - \frac{\eta_k}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right)^2 \right) \right] \\
& \left. + \mu_t \left[ n_t^f - \rho n_{t-1}^f - p_t \left(1 - \rho n_{t-1}^f - n_{t-1}^i\right) \right] \right\}
\end{aligned}$$

Households heterogeneity is integrated in a representative agent framework, with composi-

tional effects in the welfare and budget function arising from changes in the relative size of the informal economy (see Conesa *et al.* (2002) and Batini *et al.* (2009) for a similar set-up). The size of the formal and informal labour market segments enters the utility function as well as consumption. Similarly, the total labour income of the representative households is the sum of the wage bill in both sectors. The similarity between the two households simplifies the formulation of the transition between the two labour markets.[7] First order conditions imply that $c_t^f = c_t^i = c_t^u$ at the optimum. The maximization of households can therefore be re-written as follows (making use of a Bellman equation):

$$\begin{aligned}\mathcal{L}_t^h &= \frac{c_t^{1-\sigma}}{1-\sigma} + \beta E_t\left\{H_t\left(n_t^f, n_t^i, k_t, x_t\right)\right\} + \\ &+\lambda_t\left[x_t - r_t k_{t-1} - n_t^f w_t^f - w^u\left(1 - n_t^f - n_t^i\right) - n_t^i w_t^i + c_t - \Pi_t\right] + \\ &+\lambda_t\varphi_t\left[k_t - (1-\delta)k_{t-1} - x_t\left(1 - \frac{\eta_k}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2\right)\right] \\ &+\mu_t\left[n_t^f - \rho n_{t-1}^f - p_t\left(1 - \rho n_{t-1}^f - n_{t-1}^i\right)\right]\end{aligned}$$

Households either consume $c_t$ or accumulate capital $k_t$ through investment $x_t$. The depreciation rate of capital is $\delta$ and capital accumulation is subject to adjustment costs proportional with the rate of change of investment$\left(1 - \frac{\eta_k}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2\right)$ following Christiano *et al.* (2005). The employment constraint (eq 1) faced by formal workers can be expressed as a function of the number of unemployed workers searching a formal job $s_t = 1 - \rho n_{t-1}^f - n_{t-1}^i$. The first order conditions are as follow:

$$\lambda_t = -\frac{1}{c_t^\sigma} \tag{2}$$

$$\varphi_t = E_t\left(\Lambda_{t,t+1}\left[r_{t+1} + \varphi_{t+1}(1-\delta)\right]\right) \tag{3}$$

$$\varphi_t\left[1 - \left(\phi_t + \frac{x_t}{x_{t-1}}\phi_t^{'}\right)\right] = 1 - \beta E_t\left\{\varphi_{t+1}\Lambda_{t,t+1}\left(\frac{x_{t+1}}{x_t}\right)^2\phi_{t+1}^{'}\right\} \tag{4}$$

Equations 2 and 3give the standard Euler equation. Equation 4 gives the optimal conditions for capital and investment, where $\varphi_t$ is the shadow value of a unit of investment. We derive $H_{n,t}$ the representative household's marginal value of having one of its member hired in the labor market rather than unemployed, which enters further below the Nash wage bargaining. $H_{n,t}$ increases with additional income gains expressed in utility from being employed rather than unemployed. $H_{n,t}$ also increases with the expected utility of being still employed in the next

[7] Alternatives are much more complex or much less satisfactory. In Fiess *et al.* (2006), self employed must dis-accumulate their stock of capital before transiting to the formal salaried sector. Galí *et al.* (2007) specify two households, one optimizing and one hand to mouth household. The relative size of the two groups of households is however fixed, as agents with financial assets cannot shift automatically in the group of households with zero savings and assets.

period.

$$H_{n,t} = \lambda_t \left( w_t^f - w^u \right) + \beta \rho E_t \left\{ H_{n,t+1} \left( 1 - p_{t+1} \right) \right\} \tag{5}$$

## 2.3 Firms:

Firms use three inputs, capital $k_{t-1}$, formal labour $n_t^f$ and informal labour $n_t^i$ to produce a homogeneous good $y_t$, using a Cobb-Douglas production function. Firms increase the formal workforce by posting vacancies $v_t$ at a cost $\kappa$. The rate of matching $q_t$ times the number of vacancies give the number of new formal jobs every periods. Firms also pay wages to formal workers $w_t^i$ and informal workers $w_t^f$ as well as the rental rate of capital $r_t$ to households. The bellman equation for firms reads as follow:

$$\begin{aligned} F\left(n_{t-1}^f, n_{t-1}^i, k_{t-1}\right) &= \max_{n_t^f, n_t^i, k_{t-1}} a_t \left(k_{t-1}\right)^{\alpha_k} \left(n_t^f\right)^{\alpha_f} \left(n_t^i\right)^{1-\alpha_k-\alpha_f} - r_t k_{t-1} - w_t^f n_t^f - w_t^i n_t^i - \kappa v_t & (6) \\ & + \beta E_t \left\{ \Lambda_{t,t+1} F\left(n_t^f, n_t^i, k_t\right) \right\} & (7) \end{aligned}$$

The lagrangian of the optimization problem is:

$$\begin{aligned} \max_{v_t, n_t^i, n_t^f, k_{t-1}} \mathcal{L} &= a_t \left(k_{t-1}\right)^{\alpha_k} \left(n_t^f\right)^{\alpha_f} \left(n_t^i\right)^{1-\alpha_k-\alpha_f} - r_t k_{t-1} - w_t^f n_t^f - w_t^i n_t^i - \kappa v_t + \beta E_t \left\{ \Lambda_{t,t+1} F\left(n_t^f, n_t^i, k_t\right) \right\} \\ &+ \psi_t \left[ n_t^f - \rho n_{t-1}^f - q_t v_t \right] \end{aligned}$$

The first two order conditions of the firm's optimization problem with respect to capital $k_{t-1}$ and informal labour $n_t^i$ equate the marginal productivities with their respective prices:

$$r_t = \alpha_k \frac{y_t}{k_{t-1}} \tag{8}$$

$$w_t^i = \left(1 - \alpha_k - \alpha_f\right) \frac{y_t}{n_t^i} \tag{9}$$

Firms then choose the optimal quantity of vacancies $-\psi_t = \frac{\kappa}{q_t}$. Then maximizing profits with respect to formal employment and making use of the envelope condition, we get the equilibrium condition for formal employment:

$$\frac{\kappa}{q_t} = \alpha_f \frac{y_t}{n_t^f} - w_t^f + \beta \rho E_t \left\{ \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} \right\} \tag{10}$$

The equilibrium conditions with segmented labour markets reflect the degree of rigidities of each market. In the informal market, wages are flexible and equal the marginal product of informal labour. Firms increase formal employment to the point where the marginal costs

of hiring an additional workers is equal to its benefit. The benefits of an additional worker is the difference between the marginal productivity of this workers and its wage costs augmented by the expected saving of not having to generate a new match next period. In the absence of search costs, i.e. $\kappa = 0$, the equilibrium condition reduces to that of a competitive labour market. The same occurs if $\sigma_m$ tends to infinity, i.e. when matching efficiency increases without bounds. Equation 8 is the optimal condition for the choice of capital with the rental rate equating the marginal product of capital.

It is necessary to define the value for a firms of an additional formal workers $F_{n,t}$, which enters the wage bargaining in the following section. Using the employment condition for employment and making use of the equilibrium condition for posting vacancies, we get that $F_{n,t} = \frac{\kappa}{q_t}$. Plugging this definition into equation $10$ yields:

$$F_{n,t} = \alpha_f \frac{y_t}{n_t^f} - w_t^f + \beta\rho E_t \left\{\Lambda_{t,t+1} F_{n,t+1}\right\} \tag{11}$$

## 2.4 Nash bargaining in the formal labour market:

Each period, the real wage in the formal labor market is determined through a generalized Nash-bargaining process between the representative firm and the marginal formal worker that was matched with the firm. Formally,

$$w_t^f \equiv max\left\{\left(H_{n,t}\right)^{\eta}\left(F_{n,t}\right)^{1-\eta}\right\}, \quad 0 < \eta < 1 \tag{12}$$

where $\eta$ denotes the bargaining power of the formal workers and where the expressions of $H_{n,t}$ and $F_{n,t}$ are given by (5) and (11), respectively. The first order condition of the Nash-bargaining process is given by

$$\eta F_{n,t} \;\; = \;\; (1-\eta)\frac{H_{n,t}}{\lambda_t} \tag{13}$$

where $\frac{H_{n,t}}{\lambda_t}$ represents the household's marginal value of an additional formal worker expressed in units of consumption goods. The total surplus from an additional match in the formal labor market (or surplus for short), denoted by $S_{n,t}$, is defined as the sum of the firm's marginal value of an additional formal worker and the household's marginal value of an additional formal worker defined in units of consumption goods: $S_{n,t} \equiv F_{n,t} + \frac{H_{n,t}}{\lambda_t}$. One can show that the Nash-bargaining process leads the household and the firm to share that surplus: $F_{n,t} = (1-\eta)S_{n,t}$ and $\frac{H_{n,t}}{\lambda_t} = \eta S_{n,t}$. In addition, the surplus $S_{n,t}$ can also be measured by the size of the gap between firm's reservation wage $\overline{w}_t^f$ and the household's reservation wage $\underline{w_t^f}$:

$$S_{n,t} = \bar{w}_t^f - \underline{w}_t^f \tag{14}$$

The household's formal reservation wage $\underline{w}_t^f$ defines the minimum value of the real wage for which the household is willing to work in the formal labour market. In turn, firm's formal reservation wage $\overline{w}_t^f$ defines the maximum value of the real wage that firms are willing to pay a formal worker. The household's marginal value of an additional worker expressed in units of consumption goods becomes zero $\frac{H_{n,t}}{\lambda_t} = 0$ if the real wage is set equal to the household's reservation wage $w_t^f = \underline{w}_t^f$. In this case, equation (5) becomes:

$$\underline{w}_t^f = w^u - \beta\rho E_t \left\{ \Lambda_{t,t+1} \frac{H_{n,t+1}}{\lambda_{t+1}} (1 - p_{t+1}) \right\} \tag{15}$$

The household's formal reservation wage, $\underline{w}_t^f$, increases with the replacement wage, $w^u$. In turn, $\underline{w}_t^f$ decreases with the household's expected future continuation value of the match, $\beta E_t \Lambda_{t,t+1} \frac{H_{n,t+1}}{\lambda_{t+1}} \rho (1 - p_{t+1})$. Similarly, the firm's marginal value of an additional formal job of a worker is zero $F_{n,t} = 0$, if the formal real wage is set equal to the firm's reservation wage, $w_t^f = \bar{w}_t^f$. In this case, equation (11) becomes:

$$\bar{w}_t^f = \alpha_f \frac{y_t}{n_t^f} + \beta\rho E_t \left\{ \Lambda_{t,t+1} F_{n,t+1} \right\} \tag{16}$$

Firm's formal reservation wage, $\bar{w}_t^f$, increases with the current marginal productivity of labor and with the firm's expected future continuation value of the match. This last element reflects that turn over is costly for firms. The bargained real wage, $w_t^f$, is then obtained by taking the average sum of the two reservation wages, the weights being given by the bargaining powers of firms and households:

$$w_t^f = \eta \bar{w}_t^f + (1 - \eta) \underline{w}_t^f \tag{17}$$

Equation 17 can be rearranged by using equations 15, 16 together with $F_{n,t} = \frac{\kappa}{q_t}$ and $\frac{H_{n,t}}{\lambda_t} = \frac{\eta}{1-\eta} \frac{\kappa}{q_t}$:

$$w_t^f = \eta \alpha_f \frac{y_t}{n_t^f} + (1 - \eta) w^u + \eta\beta\rho\kappa E_t \left\{ \Lambda_{t,t+1} \frac{p_{t+1}}{q_{t+1}} \right\} \tag{18}$$

The real wage is a weighted sum of the marginal productivity of labour and the replacement income at time $t$ and the the expected future state of the labour market at time $t+1$. The weights are made of the bargaining power of firms and workers. We can also compute a recursive expression for the surplus, $S_{n,t}$, by plugging (15) and (16) into (14) and by using the relations between the surplus, $S_{n,t}$, and the marginal values of an additional labor, $H_{n,t}$ and $F_{n,t}$:

$$S_{n,t} = \left( \alpha_f \frac{y_t}{n_t^f} - w^u \right) + \beta\rho E_t \left\{ \Lambda_{t,t+1} S_{n,t+1} (1 - \eta p_{t+1}) \right\} \tag{19}$$

The surplus that arises from the current match is determined by two terms (appearing in the right-hand side of equation (19)). $S_{n,t}$ increases with the gap between the marginal productivity of labor and the replacement wage $w^u$. The current surplus also increases with the expected next period surplus, if the current match is not broken in the following period, $\beta\rho E_t\left\{\Lambda_{t,t+1}S_{n,t+1}\right\}$, net of the expected next period household's marginal value of an additional worker (expressed in units of consumption goods), derived from a new match that would occur in the following period, $\beta\eta\rho E_t\left\{\Lambda_{t,t+1}p_{t+1}S_{n,t+1}\right\} = \beta\rho E_t\left\{\Lambda_{t,t+1}p_{t+1}\frac{H_{n,t+1}}{\lambda_{t+1}}\right\}$.

Recall that a fraction $1-\eta$ of the surplus goes to firms: $F_{n,t} = (1-\eta)\,S_{n,t}$. Combining the latter with equations for $q_t$ and taking into account that $F_{n,t} = \frac{\kappa}{q_t}$, one gets:

$$\frac{\kappa}{q_t} = (1-\eta)\,S_{n,t} \tag{20}$$

By making use of equations (19) and (20), one can get a recursive equation reflecting the dynamic of employment:

$$\frac{\kappa}{q_t} = (1-\eta)\left(\alpha_f\frac{y_t}{n_t^f} - w^u\right) + \beta\rho E_t\left\{\Lambda_{t,t+1}\frac{\kappa}{q_{t+1}}\left(1-\eta p_{t+1}\right)\right\} \tag{21}$$

When either the vacancy posting cost parameter becomes close to zero,$\kappa \to 0$, or the matching efficiency parameter strongly improves, $\alpha_m \to +\infty$, the marginal productivity of labor is equal to the replacement wage in the equilibrium.

## 2.5 Labour market transition and macro closure:

Following Zenou (2008), we make two assumptions about the mobility of workers between the formal and informal labor markets.

1. An employed worker in the informal labor market cannot move directly to the formal labor market. He or she has to become unemployed before seeking a formal job.

2. Formal workers become unemployed if they lose their job but never shifts directly from a formal to an informal job.

Hence, these two assumptions regarding possible job transitions follow from the idea that each labor market requires time and a social network to enter. These restrictions on the mobility of workers facilitate the characterization of the equilibrium in the labor market that includes search and matching frictions.

In order to satisfy these two assumptions, Zenou (2008) defines formally a mobility condition stating that the household's value for unemployment must be equal to the expected intertemporal sum of the informal real wage (see the first equality in equation (3.2) in Zenou (2008) p. 341) plus a (strictly positive) switching cost $\nu$. Hence, unemployed workers are always better of than being informal. The switch from informality to unemployment is sluggish and depends on the size of the switching costs. The mobility condition can be expressed as follows:

$$\begin{aligned}
\frac{\mathcal{U}_t^f}{\lambda_t} &= E_t \sum_{j=0}^{+\infty} \beta^j \Lambda_{t,t+j} w_{t+j}^i + \nu \\
\Leftrightarrow \frac{\mathcal{U}_t^f}{\lambda_t} &= w_t^i + \nu + \beta E_t \left\{ \Lambda_{t,t+1} \frac{\mathcal{U}_{t+1}^f}{\lambda_{t+1}} \right\} \qquad (22)
\end{aligned}$$

where $\frac{\mathcal{U}_t^f}{\lambda_t}$ denotes the household's value for unemployment expressed in units of consumption goods. The value of a position in the unemployment pool, $\mathcal{U}_t^f$ is found by making use of equation 5 and by taking into account that the surplus from employment of a worker is the difference between the employment and the unemployment values: $H_{n,t} = \mathcal{W}_t^f - \mathcal{U}_t^f$. We get the following expressions for $\mathcal{W}_t^f$ and $\mathcal{U}_t^f$:

$$\begin{aligned}
\mathcal{W}_t^f &= \lambda_t w_t^f + \beta E_t \left\{ \rho \left( \mathcal{W}_{t+1}^f - \mathcal{U}_{t+1}^f \right) + \mathcal{U}_{t+1}^f \right\} \\
\mathcal{U}_t^f &= \lambda_t w^u + \beta E_t \left\{ \rho p_{t+1} \left( \mathcal{W}_{t+1}^f - \mathcal{U}_{t+1}^f \right) + \mathcal{U}_{t+1}^f \right\}
\end{aligned}$$

The resource constraint of the artificial economy is derived from the household's budget constraint. In particular, by combining the household's budget constraint, with the expression of the firm's profit. The macroeconomic closure equals private consumption and investment, search costs. Unemployment benefits also appear in the ressource constraint in line with Ravenna and Walsh (2008):

$$y_t = c_t + x_t + \kappa v_t - w^u u_t$$

## 2.6 Equilibrium conditions:

The equilibrium conditions of the model are as follow:

$$\begin{aligned}
q_t &= \sigma_m \theta_t^{-\sigma_s} \\
p_t &= \sigma_m \theta_t^{1-\sigma_s} \\
\theta_t &= \frac{v_t}{1 - \rho n_{t-1}^f - n_{t-1}^i} \\
k_t &= (1-\delta) k_{t-1} + x_t \left( 1 - \frac{\eta_k}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right)^2 \right) \\
y_t &= a_t (k_{t-1})^{\alpha_k} \left( n_t^f \right)^{\alpha_f} \left( n_t^i \right)^{1-\alpha_k-\alpha_f}
\end{aligned}$$

$$
\begin{aligned}
n_t^f &= \rho n_{t-1}^f + q_t v_t \\
\lambda_t &= \frac{1}{c_t^\sigma} \\
\varphi_t &= \left[1 - \beta E_t\left(\varphi_{t+1}\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{x_{t+1}}{x_t}\right)^2 \eta_k\left(\frac{x_{t+1}}{x_t} - 1\right)\right)\right] / \left[1 - \left(\frac{\eta_k}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2 + \frac{x_t}{x_{t-1}}\eta_k\left(\frac{x_t}{x_{t-1}} - 1\right)\right)\right] \\
\varphi_t &= \beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\left[r_{t+1} + \varphi_{t+1}\left(1-\delta\right)\right]\right) \\
r_t &= \alpha_k \frac{y_t}{k_{t-1}} \\
w_t^i &= \left(1 - \alpha_k - \alpha_f\right)\frac{y_t}{n_t^i} \\
y_t &= c_t + x_t + \kappa v_t - w^u u_t \\
w_t^f &= \eta\alpha_f \frac{y_t}{n_t^f} + \left(1-\eta\right)w^u + \eta\beta\kappa E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\theta_{t+1}\right\} \\
\frac{\kappa}{\sigma_m}\theta_t^{\sigma_s} &= \left(1-\eta\right)\left(\alpha_f\frac{y_t}{n_t^f} - w^u\right) + \beta\rho E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\frac{\kappa}{\sigma_m}\left(1-\eta p_{t+1}\right)\theta_{t+1}^{\sigma_s}\right\} \\
\frac{\mathcal{U}_t^f}{\lambda_t} &= w_t^i + \nu + \beta E_t\left\{\Lambda_{t,t+1}\frac{\mathcal{U}_{t+1}^f}{\lambda_{t+1}}\right\} \\
\mathcal{W}_t^f &= \lambda_t w_t^f + \beta E_t\left\{\rho\left(\mathcal{W}_{t+1}^f - \mathcal{U}_{t+1}^f\right) + \mathcal{U}_{t+1}^f\right\} \\
\mathcal{U}_t^f &= \lambda_t w^u + \beta E_t\left\{\rho p_{t+1}\left(\mathcal{W}_{t+1}^f - \mathcal{U}_{t+1}^f\right) + \mathcal{U}_{t+1}^f\right\} \\
a_t &= \rho_a a + \left(1-\rho_a\right)a_{t-1} + \varepsilon_a
\end{aligned}
$$

# 3 Numerical simulations

## 3.1 Calibration:

Table 1: Values of parameters

| | | |
|---|---|---|
| Discount factor | $\beta$ | 0.99 |
| Capital depreciation rate | $\delta$ | 0.04 |
| Production function (capital) | $\alpha_k$ | 0.26 |
| Production function (formal labour) | $\alpha_f$ | 0.2 |
| Elasticities of matches to unemployment | $\sigma$ | 0.5 |
| Matching efficiency parameter | $\sigma_m$ | 0.07 |
| Search costs | $\kappa$ | 0.03 |
| Employment destruction rate | $1-\rho$ | 0.1 |
| Capital adjustment costs | $\eta_k$ | 1.5 |
| Switching costs | $\nu$ | 10 |
| Bargaining power parameter | $\eta$ | 0.4 |
| Technology autoregressive parameter | $\rho_a$ | 0.9 |

The model is calibrated using parameters conventional in the business cycle literature (see table 1). The discount factor of households is $\beta = 0.99$, the rate of depreciation of capital is

$\delta = 0.04$ as in Monacelli *et al.* (2010). The parameters of the production function ($\alpha_k = 0.26$ and $\alpha_f = 0.2$) are chosen such that the relative size of informal employment is comprised between 50 and 60% of total employment, which is reported as being the average size of informal employment in existing studies (see Johannes *et al.*, 2008, Bacchetta et al.). The parameters for the matching function as well as the search costs are chosen such that the level of unemployment is close to 10%. The elasticity of matches to unemployment is the average value used in the literature ($\sigma = 0.5$). Gertler and Trigari (2009) report that plausible values range between 0.4 and 0.7. $\sigma_m$ is set at 0.07 and $\nu$ at 10, as it contributes to generate a relative size of informal employment slightly larger than 50%. Search costs $\kappa = 0.03$ are then chosen such that the rate of unemployment is close to 10%. Lastly, the rate of job destruction is $(1 - \rho) = 0.1$, a standard value in the literature, as in Monacelli *et al.* (2010).

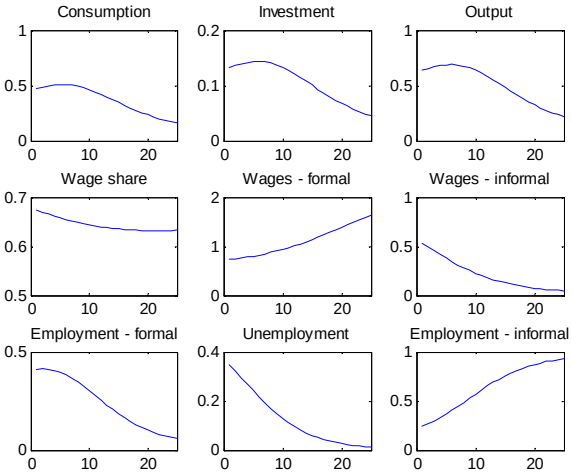
Table 2: Implied steady state values

| | | |
|---|---|---|
| Unemployment rate | $u$ | 0.12 |
| Formal employment | $n^f$ | 0.31 |
| Informal employment | $n^i$ | 0.57 |
| Consumption output ratio | $\frac{C}{y}$ | 0.72 |
| Investment output ratio | $\frac{x}{y}$ | 0.2 |
| Search costs output ratio | $\frac{\kappa v}{y}$ | 0.08 |
| Wage share | $\frac{w^i n^i + w^f n^f}{y}$ | 0.64 |

## 3.2 Comparative statics:

In this subsection, we perform comparative statics on the steady state values of the model to analyse the impact of switching costs and search costs on the long run level of employment and production. The steady states are detailed in the appendix of the model.

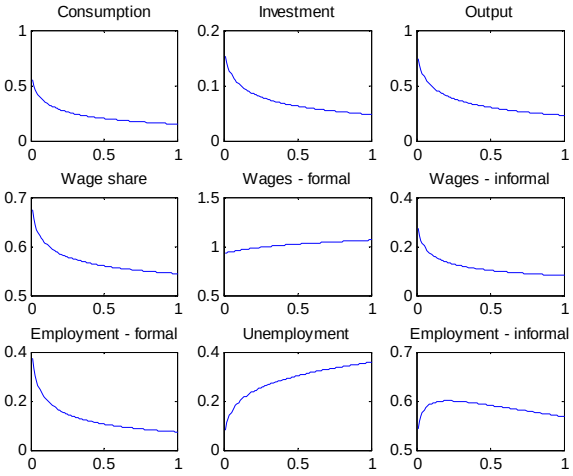
Figure 1: Higher switching costs




The first set of comparative static deals with the role of switching costs for the transition

between the two labour markets. In the model, switching costs $\nu$ enter equation 22 according to which the expected value of being unemployed equals informal wages plus switching costs. At the steady state, switching costs lower informal wages, which increases informal labour demand through the equilibrium condition on the labour market. The size of the informal economy increases relatively to the formal economy when switching costs go up. Given that the informal sector has lower earnings and a lower productivity, consumption, investment and output decrease. Interestingly, the higher the switching costs, the higher the wage gap and productivity gap between the two labour markets (see figure 1).

Figure 2: Higher search costs




The second set of comparative static analyzes the impact of higher search costs on the steady state values of the main macroeconomic variables. Similarly to the standard search model, higher search costs reduce (formal) employment and increase unemployment. Their effects on informal employment is limited, as it increases at first and then decreases slightly. The main transmission channel from the formal sector to the informal sector goes through the equilibrium condition for the transition between the two labour markets. Lower informal wages have limited effects on informal employment as a result of the higher transition costs. In sum, higher search costs increases unemployment, and the relative size of the informal employment. Higher unemployment reduces the wage share, consumption and output (see figure 2).

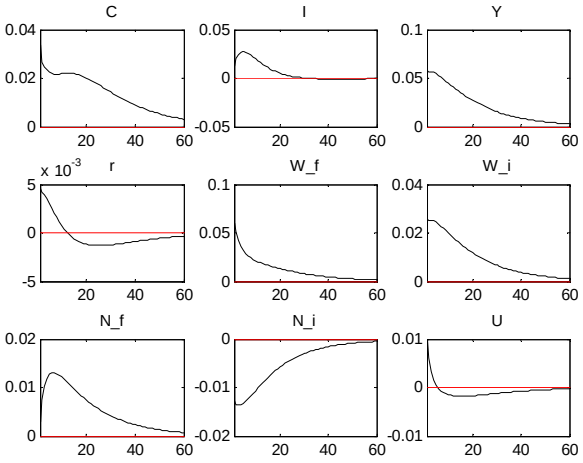
## 3.3 Dynamics

Figure 3 illustrates the key dynamics at work in our model. It displays impulse responses to a productivity shock of one percent of steady state output. Responses are expressed as a percentage deviation from the steady state value of the respective variables. In line with the result of a search model with a single labour market, the main macroeconomic variables (consumption, investment and output) are pro-cyclical. The positive shock on productivity increases the

marginal productivity of labour and capital and their respective prices too (see also correlations in table 3).

The first result is that informality is counter-cyclical, in line with the view of informality as a buffer. Increases in interest rate and formal wages are slower than increases in corresponding marginal productivities due to capital adjustment costs and search costs, which foster an increase in the demand for both inputs.

Figure 3: Informality as a buffer: switching costs




Informal wages increase faster than formal wages in the absence of search costs in the informal sector. Firms therefore substitute informal with formal workers. Formal labour increases at the expense of informal labour. Informal labour plays the role of a buffer and decrease in the upturn. In the long-run, formal employment returns to its steady state, as formal wages adjust downward slower than informal wages.

Table 3 displays statistics summarizing the characteristics of the business cycle for four different nested models. The performances of two variants of our segmented labour market model are assessed against those of an RBC model and a search model. Column 1 is a measure of volatility and displays standard deviation for the variables considered. Column 2 is another measure of volatility and display the standard deviation of a given variable normalized by the standard deviation of output. Column 3 provides a measure of co-movement of each variable with output.

Table 3: Cycle properties: search costs with different level of informality

| | RBC | | | Search model | | | Informality switching costs | | | Informality $\alpha_1 = 0.54$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\sigma(y) = 0.5$ | | | $\sigma(y) = 0.41$ | | | $\sigma(y) = 0.21$ | | | $\sigma(y) = 2.59$ | | |
| | $\sigma(x)$ | $\frac{\sigma(x)}{\sigma(y)}$ | corr(x,y) | $\sigma(x)$ | $\frac{\sigma(x)}{\sigma(y)}$ | corr(x,y) | $\sigma(x)$ | $\frac{\sigma(x)}{\sigma(y)}$ | corr(x,y) | $\sigma(x)$ | $\frac{\sigma(x)}{\sigma(y)}$ | corr(x,y) |
| Consumption | 0.32 | 0.63 | 0.96 | 0.22 | 0.55 | 0.96 | 0.13 | 0.58 | 0.96 | 1.11 | 0.43 | 0.92 |
| Investment | 0.21 | 0.43 | 0.91 | 0.16 | 0.38 | 0.92 | 0.08 | 0.39 | 0.91 | 1.58 | 0.61 | 0.96 |
| Formal employment | 0.03 | 0.06 | 0.71 | 0.05 | 0.13 | 0.97 | 0.05 | 0.24 | 0.96 | 0.03 | 0.01 | 0.56 |
| Unemployment | - | - | - | 0.05 | 0.13 | -0.97 | 0.01 | 0.07 | -0.01 | 0.02 | 0.01 | -0.40 |
| Informal employment | - | - | - | - | - | - | 0.05 | 0.23 | -0.99 | 0.04 | 0.01 | -0.34 |
| Formal wages | 0.35 | 0.70 | 0.99 | 0.22 | 0.54 | 0.99 | 0.13 | 0.62 | 0.96 | 1.10 | 0.43 | 0.97 |
| Informal wages | - | - | - | - | - | - | 0.09 | 0.44 | 0.99 | 1.15 | 0.45 | 0.98 |

It is also worth pointing out that our two benchmark models, the RBC and the search models have conventional properties. In particular, RBC models are unable to account for the observed persistence in employment ($\frac{\sigma(N)}{\sigma(y)}$ is close to zero) as the macroeconomic adjustment fall completely on wages. The innovation of search and matching function is that search costs reduces the volatility of wages and increase that of employment ($\frac{\sigma(N)}{\sigma(y)} = 0.13$ ).

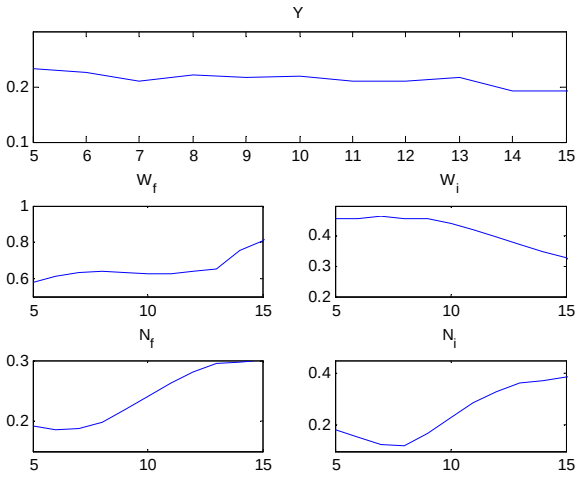
The impact of the informal sector on the persistence of employment and wages is assessed by comparing the characteristics of the RBC and search models against the model with dual labour markets. The model with dual labour markets is analyzed for two different sets of parameters. The first set of parameters corresponds to the parameters discussed in the calibration section. The steady state level of informal employment is calibrated by adjusting the switching costs between the two labour markets. Switching costs $\nu$ are set to 10, while the parameters for the elasticity of capital in the production function $\alpha_k$ is equal to 0.26. The informal sector accounts for 57% of the active population, while the unemployment rate is 13%. The consumption to output ratio also increases to 73%. The corresponding properties of the business cycle following a positive productivity shock are displayed in column 3 of table 3.

The first result is that output volatility drops to 0.21, which is twice as low as that of either the RBC or the search model. This result is consistent with Conesa *et al.* (2002), who show that the volatility of economies with informal labour is related to the transition of population between the two labour markets. In our model, large transition costs reduce the population switching effect.

The presence of both search and switching costs also greatly increase the normalized volatility of employment to 0.24 from 0.13 in the search model with single labour market. The absolute volatility of wages (0.13 and 0.09) is similar to the volatility of the search model with single labour market. The normalized volatility of wages is however much larger in the model with dual labour markets. This result stands in contrast with the conclusion of Batini *et al.* (2009), who argue that the informal labour market increases the overall flexibility of the economy.

In the second set of parameters, the steady state level of informal employment is calibrated by increasing capital elasticity $\alpha_k = 0.54$, while reducing transition costs close to zero $\nu = 0.018$.

Figure 4: Sensitivity analysis, switching costs and employment persistence




The informal sector now accounts for 50% of the active population, while unemployment is smaller at 5%.

A first result is that output volatility ($\sigma(y) = 2.59$) is more than five times larger than that generated by the RBC and search models (see column 4 of table 3). This result is consistent with the previous simulations, which show that the transition of the population between the two sectors of the economy amplifies the business cycle fluctuations.

A second result is that the combination of a large informal sector with small transition costs reduces the impact of search costs on the persistence of employment. An economy with free transition of households between the two labour markets is similar to an economy with a single competitive labour market. Most of the adjustment falls on wages ($\frac{\sigma(x)}{\sigma(y)}$ is 0.43 and 0.45 for formal and informal wages respectively), while employment persistence falls to zero. These results are in line with the conclusions of Batini *et al.* (2009), which argue that the informal sector increases the overall flexibility of both labour markets.

Figure 4 performs sensitivity analysis and displays the standard deviation of output, wages and employment for different values of switching costs ranging from 5 to 15[8]. The sensitivity analysis confirms the results found previously regarding the impact of switching costs on the volatility of output, employment and wages. Results are in line with the above discussion. Increasing switching costs reduce output volatility, as volatility is here generated by households moving between the formal and informal labour markets. Similarly, increasing the switching costs greatly increases the volatility of employment, while the volatility of wages decreases in the informal sector or stays mainly constant in the formal sector.

[8]Standard deviation for wages and employment are normalized by the standard deviation of output.

# 4 Conclusion

Applying the search and matching framework to the case of an economy with dual labour markets allows us to better understand the implication of informality for the dynamics of output, employment and wages in developing economies. We are able to show that informal employment plays the role of a buffer for firms, which substitute informal for formal jobs in periods of economic recessions. We also show that the dynamics of output and employment not only depend on the level of search costs but also depend on the barriers between the two labour markets. When the transition is costly, the existence of an unregulated labour market does not modify the characteristics of the formal labour market, which still behave along the line of a search and matching model. Wages do not adjust instantaenously and employment takes time to return to its steady state values. Furthermore, informal employment also displays persistence despite the frictionless nature of this labour market segment. Results are reversed when transition costs are low. Then, the formal labour market behaves like a Walrasian market with wages absorbing the productivity shocks and employment displaying no or very little persistence.

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