Active labour market policies, search costs and positive fiscal multiplier

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Abstract

This paper presents a model of fiscal policy in which public intervention takes the form of active labour market policy. Although public intervention tends to crowd out private consumption, labour market policy also improves the matching between the unemployed and job vacancies. The model therefore produces small but positive fiscal multipliers on impact and in the short term. This result is similar with that of Monacelli et al. (2010) except that the transmission channel does not depend on the downward adjustment of the reservation wage of workers. The size of the fiscal multiplier is negatively related with the steady state spending to GDP ratio in the presence of diminishing marginal returns on spending. For large value of the multiplier, there is a crowding in of consumption and investment. Lastly, we also show that large fiscal multipliers may take place when active labour market policies are combined with aggregate demand effects in the presence of nominal price rigidities.
1 Non-technical summary and main findings

The effectiveness of fiscal policy is widely debated amongst policy makers. The widespread fiscal stimulus packages attest that fiscal policy is a key instrument in fighting recession, although the large fiscal consolidation that has quickly followed points to the contrary.

An equally controversial issue is the form that fiscal intervention should take, such as tax cuts, government consumption, infrastructure spending, social spending or labour market policies. The analysis of the composition of fiscal stimulus shows that governments have mainly favoured tax cuts and infrastructure spending. Other forms of action such as active labour market policies have been, on the contrary, largely under-utilized despite the profound and long-lasting labour market crisis.

A possible explanation is that the transmission channels associated with public spending are not well understood. Some economists point to the detrimental impact of public spending on private agents and on the crowding-out of their spending. Some other economists points to the positive aggregate demand effects or aggregate supply effects of public spending. The understanding of fiscal policy is much more limited than monetary policy, for which a large consensus exist on the tools and the objectives of the central bank.

This paper addresses the issue of the effectiveness of fiscal policies and the form that fiscal policy should take to be effective. For that purpose, a macroeconomic model is developed to better understand the channel trough which certain forms of fiscal spending affect the labour market and the level of unemployment. The model enables discussion of two aspects associated with fiscal policy. The first aspect concerns the form that fiscal policy may take: public consumption versus active labour market policies. The second aspect deals with the nature of the transmission mechanism: negative and positive supply side effects and aggregate demand effects.

The model is characterized by three types of agents. Households choose consumption to maximize their intertemporal utility function and use their savings to finance investment and to purchase public bonds. They receive labour income when employed; a replacement income when unemployed; interest payments from financing both firms and the government; dividends; and they pay lump sum taxes. Firms use labour and capital to produce homogeneous goods, which are both consumption goods for households and for the government as well as investment goods for firms. Firms pay a wage income and interest payments to households.

In this model, the labour market does not clear at each point in time generating equilibrium unemployment. The labour market aims at matching unemployed workers and firms with job vacancies. However, this process is costly for firms, which pay a cost per vacancy opened. The main consequence is that only a fraction of unemployed and vacancies are matched. Wages are also set following a bargaining between workers and firms. Negotiated wages depend on the minimum wage that workers are willing to accept and the maximum wage that firms are willing to pay. Negotiated wages depend as well as on the relative bargaining power between workers and employers. Regarding the government sector, lump sum taxes are raised on households. Fiscal policy takes either the
form of government consumption or active labour market policy. The difference between spending and income tax is financed by issuing bonds.

There are four main results. First, in the case of flexible price, the model is supply side and higher government consumption crowds out private spending. The decrease in investment and consumption over-balances the impact of higher public spending on aggregate demand. The output multiplier is negative and equal to -0.08 after four semesters.

Second, on the contrary, when public spending takes the form of active labour market policies, the positive impact of government spending on the labour market boosts firms’ production, generating positive fiscal multipliers. Active labour market policies in this framework improves the matching between unemployed and vacancies. The output multiplier is 0.31 on impact and 0.43 after one year.

Third, the fiscal multiplier is larger in countries in which the active labour market spending to GDP ratio is low. For certain values of the labour market spending to GDP ratio, the positive supply side effect is so strong that it crowds in private spending and leads to an increase in consumption and investment. In such a case, the multiplier is much larger at 0.56 on impact for output (0.83 after one year).

Fourth, in the presence of price rigidities, the interaction between the labour market effects and the aggregate demand effects produces a fiscal multiplier close to 2 on impact. In an extension of the model, prices set by firms are characterized by rigidities generating quantity adjustment in the goods market. Following an increase in public spending, firms facing higher aggregate demand are not able to charge higher prices and therefore increase production. There are aggregate demand effects associated with fiscal policy.
2 Introduction

The recent episodes of fiscal stimulus and fiscal consolidation raise the issue of the efficiency of fiscal policy. Empirical estimations of the fiscal multiplier vary across a wide range, with some estimations of a fiscal multiplier close to zero, close but lower than one or larger than one\(^1\). A related question is the form that fiscal intervention should take, for instance tax cuts; infrastructure investment; social transfers; government consumption or labour market policies. The plurality of tools for fiscal intervention make it more difficult to understand their impact and their transmission channels, whereas monetary policy for instance focuses mainly on one type of instrument, the short-term interest rate.

This paper discusses the ability of fiscal intervention to stimulate output and employment. In particular, this paper models active labour market policies as an instrument for fiscal policy, and shows that the transmission channel between fiscal policy and output goes through the labour market. In the model below, active labour market policies play a key role in supporting new jobs, by easing the matching process between searching workers and firms with job vacancies. For that purpose, the traditional matching function is extended to incorporate labour market spending. The Cobb-Douglas matching function is now made of three elements: searching workers, vacancies and active labour market spending. There are ambivalent transmission channels. Fiscal spending crowds out private consumption and investment in line with the Ricardian properties associated with intertemporal optimizing private agents. The increase in the interest rate following fiscal expansion also affects negatively the discounted value of an additional match and reduces the incentive for firms to hire an additional worker. Labour market spending however improves the functioning of the labour market and increases the rate of matching.

Various forms of matching functions are discussed. The benchmark matching function has constant return to scale on the three inputs. Alternatives specifications consider the case where the matching function has constant return to scale on searching workers and vacancies with government spending being nested with one of the two other inputs. We also discuss the impact of the steady state value of public spending on the size of the multiplier and on the outcome of the main macroeconomic variables. Diminishing marginal return on spending implies that low steady state spending to GDP ratio generates large fiscal multiplier and a crowding in of consumption and investment. Public spending crowds out private consumption through the resource constraint. On the other hand, the positive supply-side effect associated with higher matching counter-balances the tendency of private consumption to fall. Another extension discusses the size of the multiplier when active labour market policies are combined with nominal price rigidities. Sticky prices produce a quantity effect associated with increases in aggregate demand and a hiring effect through the search and matching framework.

\(^1\)There are three main methodologies to estimate the size of the fiscal multiplier empirically: the dummy approach Ramey and Shapiro (1998), structural VAR such as Blanchard and Perotti (2002), Monacelli and Perotti (2008) or Gali et al. (2007) or sign restriction Mountford and Uhlig (2009). There are large evidence that the output increases on impact although the long term effects is subject to controversies. The empirical results on wages and investment are inconclusive while there is a consensus that consumption increases following fiscal expansion (see Kuhn (2010) for an overview).
There is growing related literature. Monacelli et al. (2010) also focus on the transmission of fiscal spending through the labour market. They use a utility function à la Shimer (2005) where the negative wealth effect associated with higher public spending enters negatively the surplus from an additional match. Workers lower their reservation wage, which increases the incentive for firms to hire. We use instead the search and matching framework developed by Ravenna and Walsh (2008) in which the reservation wage of workers depends on unemployment benefits. We are therefore able to isolate the contribution of active labour market spending in improving employment.

Related papers include Ganelli (2003), which introduces non-separability between private and public consumption to discuss the size of the fiscal multiplier in an open economy framework. Baxter and King (1993) consider the case of private firms using public investment as an input for production. Gali et al. (2007) use non-optimizing households to break the Ricardian equivalence. Monacelli and Perotti (2008) combine a GHH utility function with sticky prices to produce positive fiscal multipliers, while Ravn et al. (2006) make use of deep habit formation and sticky price for a similar purpose. Lastly, Fernández-Villaverde (2010) shows that fiscal multipliers are positive when public consumption takes place in the presence of financial frictions.

The paper is organized as follow. Section 3 presents the model and the main assumptions. Section 4 discusses the steady states and the calibration, while section 5 details the properties of the model by use of numerical simulations. Section 6 concludes.

3 The model

3.1 Unemployment, Vacancies and Matching

At the beginning of each period, the workforce \(L\), which is normalized to 1, is divided between employed workers \(n_t\) and unemployed workers \(u_t\).

\[
 u_t = 1 - n_t
\]

The number of workers currently employed at time \(t\) is equal to the existing stock of employment at the beginning of the period \(\rho n_{t-1}\) plus new matches \(m_t\). The rate of job survival is a constant \(\rho\):

\[
 n_t = \rho n_{t-1} + m_t
\]

New matches \(m_t\) depend positively on the number of searching workers \(s_t = 1 - \rho n_{t-1}\) and the number of vacancies \(v_t\). The innovative feature of this model is to introduce labour market spending \(g_t\) into the matching function. The main motivation is that active labour market policies aim at improving the matching between searching workers and vacancies. The matching function is similar to a Cobb-
Douglas production function, with $\sigma_i (i = s, v, g)$ the elasticities of substitution between the different inputs. The parameter $\sigma_m$ reflects the efficiency of the matching process. The value of the different elasticities of substitution matters for the dynamic of the economy, as they influence the return to scale of the matching function. It is assumed throughout the paper that the matching function has constant return to scale. The sum of the elasticities of substitution is equal to one $\sigma_v = 1 - \sigma_s - \sigma_g$.

Two alternative specifications of the matching function will be considered against this benchmark case further below. First, labour market spending may be associated with vacancies: $m_t = \sigma_m \sigma_s \sigma_v \sigma_s v_t \sigma_v g_t \sigma_g t$. Second, labour market spending may be associated with unemployment: $m_t = \sigma_m \sigma_s \sigma_v \sigma_s v_t \sigma_v g_t \sigma_g t$ with $\sigma_v = 1 - \sigma_s$.

For convenience, we use the ratio $\theta_s,t = \frac{v_t}{s_t}$ to measure labour market tightness. The ratio $\theta_g,t = \frac{g_t}{v_t}$ measures labour market spending per vacancy. The probability of firms to fill up a vacancies is denoted $q(\theta_s,t; \theta_g,t)$ and is equal to the ratio of matches over the number of vacancies:

$$q_t = \frac{m_t}{v_t} = \sigma_m \sigma_s \sigma_v \sigma_s v_t \sigma_v g_t \sigma_g t$$

Similarly, the probability of an unemployed workers to find a job is given by the ratio of new matches over the number of searching workers:

$$p_t = \frac{m_t}{s_t} = \sigma_m \sigma_s \sigma_v \sigma_s v_t \sigma_v g_t \sigma_g t$$

Both probabilities of firms to fill up a vacancies and of searching workers to find a job are increasing with public spending. Given the above definitions, new matches in the equation for $n_t$ can be expressed as the probability of filling a vacancy times the existing number of vacancies:

$$n_t = \rho n_{t-1} + q_t v_t$$

### 3.2 Households:

Households maximize their expected intertemporal utility function $U(c_t) = c_t^{1-\sigma} (1-\sigma)$ by choosing the optimal level of consumption as well as the level of investment of firms $x_t$ and the quantity of public bonds held $b_t$. Employed workers receive real wage $w_t$ and unemployed households receive a replacement income $w_u$, which is a fraction of the steady state value of wages. A large representative household
with a continuum of members of mass one inhabits the artificial economy. All household’s members pool their incomes to be fully insured against unemployment. The intertemporal budget constraint of a household member is given by:

\[ c_t + x_t + b_t \leq w_t n_t + w^u (1 - n_t) + r_{k,t} k_{t-1} + r_{t-1} b_{t-1} - \tau_t + \Pi_t \]  

(3)

where \( r_{k,t} \) denotes the real rental rate of capital \( k_t \), \( r_t \) the interest on public bonds, \( \tau_t \) is a lump-sum tax, \( \Pi_t \) is the profits received from firms. The representative household faces an employment constraint in the labor market, with \( m_t \) being expressed as a function of the probability of unemployed members to find a job and the fraction of searching members.

\[ n_t = \rho n_{t-1} + p_t s_t \]  

(4)

Capital accumulation is subject to the following constraint:

\[ k_t = (1 - \delta) k_{t-1} + x_t (1 - \phi_t) \]  

(5)

where \( \delta \) is a parameter for the rate of capital depreciation, \( \phi_t \equiv \phi \left( \frac{x_t}{x_{t-1}} - 1 \right) \) denotes the capital adjustment costs that are proportional to the rate of change in investment, with \( \phi (0) = \phi' (0) = 0 \). The optimization problem that any member of the household faces is given by:

\[
\max_{c_t, n_t, x_t, k_t, b_t} E_0 \sum_{t=0}^{+\infty} \beta^t U (c_t)
\]

subject to (3), (4) and (5). The agent’s optimization problems stated above focus only on interior solutions, that is all the quantities are supposed to be strictly positive. The representative household’s optimization problem can be restated with a bellman equation as follows:

\[
H (n_{t-1}, k_{t-1}, b_{t-1}, x_{t-1}) = \max_{c_t, n_t, x_t, k_t, b_t} \left( \frac{c_t^{1-\sigma}}{1-\sigma} + \beta E_t \left\{ H (n_t, k_t, b_t, x_t) \right\} \right)
\]

subject to (3), (4) and (5). The Lagrangian of the household’s problem can be written as follows:

\[
L^h_t = \frac{c_t^{1-\sigma}}{1-\sigma} + \beta E_t \left\{ H (n_t, k_t, b_t, x_t) \right\} + \\
+ \lambda_t \left( w_t n_t + w^u (1 - n_t) + r_{k,t} k_{t-1} + r_{t-1} b_{t-1} - \tau_t + \Pi_t - c_t - x_t - b_t \right) + \\
+ \lambda_t \phi_t ((1 - \delta) k_{t-1} + x_t (1 - \phi_t) - k_t) + \mu_t \left( n_t - \rho n_{t-1} - p_t (1 - \rho n_{t-1}) \right)
\]
The first order condition for consumption links the marginal utility of wealth with the marginal utility of consumption:

$$\lambda_t = \frac{1}{c_t^\sigma}$$  \hspace{1cm} (7)$$

where \(\lambda_t\) is Lagrange multiplier associated with the budget constraint (3). The first order conditions for investment and capital read:

$$\phi_t \left[ 1 - \left( \phi_t + \frac{x_t}{x_{t-1}} \phi_t' \right) \right] = 1 - \beta E_t \left\{ \phi_{t+1} \Lambda_{t,t+1} \left( \frac{x_{t+1}}{x_t} \right)^2 \phi_{t+1} \right\}$$  \hspace{1cm} (8)$$

$$\phi_t = \beta E_t \left( \Lambda_{t,t+1} \left[ r_{k,t+1} + \phi_{t+1} (1 - \delta) \right] \right)$$  \hspace{1cm} (9)$$

with \(\Lambda_{t,t+1}\) is defined as \(\frac{\lambda_{t+1}}{\lambda_t}\). \(\phi_t\) is the shadow value of a unit of investment and \(\phi_t'\) the derivative of the capital cost function with respect to its argument \(\frac{x_t}{x_{t-1}} - 1\). The first order condition with respect to public bonds can be expressed as follow:

$$\frac{1}{r_t} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \right\}$$  \hspace{1cm} (10)$$

We derive \(H_{nt}\), the representative household’s marginal value of having one of its member hired in the labor market rather than unemployed, which enters further below the Nash wage bargaining. \(H_{nt}\) increases with additional income gains expressed in utility from being employed rather than unemployed. \(H_{nt}\) also increases with the expected utility of being still employed in the next period.

$$H_{nt} = \lambda_t (w_t - w^u) + \beta \rho E_t \left\{ H_{nt+1} (1 - p_{t+1}) \right\}$$  \hspace{1cm} (11)$$

### 3.3 Firms

The economy is populated by a large number of identical firms of mass one, which operate in a competitive goods market. In order to produce the final goods, \(y_t\), firms use a constant return to scale Cobb-Douglas technology with two inputs labor \(n_t\) and capital \(k_{t-1}\) of the form:

$$y_t = f \left( k_{t-1}, n_t \right) = (k_{t-1})^\xi \left( n_t \right)^{1-\xi}$$  \hspace{1cm} (12)$$

The timing of events in the formal labor market is the same as in MPT. The representative firm posts \(v_t\) vacancies in the beginning of the period in order to increase its quantity of labor input, \(n_t\). We assume that it is costly for the representative firm to post a vacancy. Specifically, the vacancy cost function, \(C(v_t)\), is assumed to be linear in the number of posted vacancies: \(C(v_t) = \kappa v_t\), where \(\kappa > 0\) is a vacancy cost parameter. \(m_t\) new matches come out of the searching mechanism. In the current period, a fraction \(1 - \rho\) of jobs disappears. Workers that have lost their jobs have to wait until the next
period to look for new jobs. Then, firms use the new matches — as well as of the other inputs — to produce. Note that the new matches cannot be broken until the next period. Thus, formally, from the firms side, the employment evolves over time following equation 2: $n_t = \rho n_{t-1} + q_t v_t$. Firms maximize the expected flows of profits $\pi_t$ with respect to employment and capital and subject to the production function and the law of motion of employment:

$$\max_{k_{t-1}, n_t} E_t \sum_{j=0}^{+\infty} \beta^j \Lambda_{t+j} \Pi_{t+j}$$

subject to (12) and (2). Profit $\Pi_t$ is made of the value of output minus labour costs, the cost of renting capital and the costs of vacancies:

$$\Pi_t \equiv y_t - w_t n_t - r_{k,t} k_{t-1} - \kappa v_t$$  \hspace{1cm} (13)

The representative firm’s optimization problem can be restated with a bellman equation:

$$F(n_{t-1}, k_{t-1}) = \max_{n_t, k_{t-1}} \left( (k_{t-1})^{\zeta} (n_t)^{1-\zeta} - w_t n_t - r_{k,t} k_{t-1} - \kappa v_t + \beta E_t \{ \Lambda_{t+1} F(n_t, k_t) \} \right)$$  \hspace{1cm} (14)

subject to (2). The related Lagrangian of the firm’s optimization problem is $\mathcal{L}_t^f$:

$$\mathcal{L}_t^f = (k_{t-1})^{\zeta} (n_t)^{1-\zeta} - w_t n_t - r_{k,t} k_{t-1} - \kappa v_t + \beta E_t \{ \Lambda_{t+1} F(n_t, k_t) \} + \psi_t [n_t - \rho n_{t-1} - q_t v_t]$$

The first order conditions of the firm’s optimization problem with respect to $k_{t-1}$ equates the productivity marginal of capital with the rental rate of capital:

$$r_{k,t} = \frac{\zeta y_t}{k_{t-1}}$$ \hspace{1cm} (15)

Firms first choose the optimal quantity of vacancies $-\psi_t = \frac{\kappa}{q_t}$. Then maximizing profits with respect to employment and making use of the envelope condition, we get the equilibrium condition for employment:

$$\frac{\kappa}{q_t} = a_t - w_t + \beta \rho E_t \left\{ \Lambda_{t+1} \frac{\kappa}{q_{t+1}} \right\}$$ \hspace{1cm} (16)

where $a_t \equiv (1 - \zeta) \frac{y_t}{n_t}$ is the marginal productivity of labour. In equilibrium, the marginal cost of hiring a workers is equal to its marginal benefits. The latter is the difference between the marginal productivity of an additional jobs and its wage costs at time $t$, plus savings from not having to hire an additional workers at time $t + 1$.

It is necessary to define the value for a firms of an additional workers $F_{n,t}$, which enters the wage
bargaining in the following section. Using the employment condition for employment and making use of the equilibrium condition for posting vacancies, we get that $F_{n,t} = \frac{e}{q_t}$. Plugging this definition into equation 16 yields:

$$F_{n,t} = a_t - w_t + \beta \rho E_t \{ \Lambda_{t+1} F_{n,t+1} \}$$  \hspace{1cm} (17)

### 3.3.1 Nash bargaining in the labor market and surplus

Each period, the real wage in the formal labor market is determined through a generalized Nash-bargaining process between the representative firm and the marginal worker that was matched with the firm. Formally,

$$w_t \equiv \max \left\{ (H_{n,t})^\eta (F_{n,t})^{1-\eta} \right\}, \quad 0 < \eta < 1$$  \hspace{1cm} (18)

where $\eta$ denotes the bargaining power of the workers and where the expressions of $H_{n,t}$ and $F_{n,t}$ are given by (11) and (17), respectively. The first order condition of the Nash-bargaining process is given by

$$\eta F_{n,t} = (1-\eta) H_{n,t} \lambda_t$$  \hspace{1cm} (19)

where $\frac{H_{n,t}}{\lambda_t}$ represents the household’s marginal value of an additional worker expressed in units of consumption goods. The total surplus from a marginal match in the labor market (or surplus for short), denoted by $S_{n,t}$, is defined as the sum of the firm’s marginal value of an additional hiring a worker and the household’s marginal value of an additional worker defined in units of consumption goods: $S_{n,t} = F_{n,t} + \frac{H_{n,t}}{\lambda_t}$. Straightforwardly, one can show that the Nash-bargaining process leads the household and the firm to share that surplus: $F_{n,t} = (1-\eta) S_{n,t}$ and $\frac{H_{n,t}}{\lambda_t} = \eta S_{n,t}$. In addition, the surplus $S_{n,t}$ can also be measured by the size of the gap between firm’s reservation wage $\bar{w}_t$ and the household’s reservation wage $w_t$:

$$S_{n,t} = \bar{w}_t - w_t$$  \hspace{1cm} (20)

The household’s reservation wage $w_t$ defines the minimum value of the real wage for which the household is willing to work in the labour market. In turn, firm’s reservation wage $\bar{w}_t$ defines the maximum value of the real wage that firms are willing to pay a worker. The household’s marginal value of an additional worker expressed in units of consumption goods becomes zero $\frac{H_{n,t}}{\lambda_t} = 0$ if the real wage is set equal to the household’s reservation wage $w_t = \bar{w}_t$. In this case, equation (11) becomes:
Similarly, the firm’s marginal value of an additional hiring of a worker is zero if the current match is not broken in the following period, $w$. The replacement wage, $w^\mu$, is set equal to the firm’s reservation wage, $w_t = \bar{w}_t$. In this case, equation (17) becomes:

$$w_t = \eta \bar{w}_t + (1 - \eta) w_f$$  \hspace{1cm} (23)$$

Firm’s reservation wage, $\bar{w}_t$, increases with the current marginal productivity of labor and with the firm’s expected future continuation value of the match. This last element reflects that turn over is costly for firms. The bargained real wage, $w_t$, is then obtained by taking the average sum of the two reservation wages, the weights being given by the bargaining powers of firms and households:

$$w_t = \eta \bar{w}_t + (1 - \eta) w_f$$  \hspace{1cm} (24)$$

Equation 23 can be rearrange by using equations 21, 22 together with $F_{n,t} = \kappa q_t$ and $H_{n,t} = \frac{\eta}{1 - \eta} q_t$:

$$w_t = \eta a_t + (1 - \eta) w^\mu + \eta \beta \rho \kappa \Lambda_{t+1} \left\{ \frac{p_{t+1}}{q_{t+1}} \right\}$$  \hspace{1cm} (25)$$

The real wage is a weighted sum of the marginal productivity of labour and the replacement income at time $t$ and the the expected future state of the labour market at time $t + 1$. The weights are made of the bargaining power of firms and workers. We can also compute a recursive expression for the surplus, $S_{n,t}$, by plugging (21) and (22) into (20) and by using the relations between the surplus, $S_{n,t}$, and the marginal values of an additional labor, $H_{n,t}$ and $F_{n,t}$:

$$S_{n,t} = (a_t - w^\mu) + \beta \rho E_t \left\{ \Lambda_{t+1} S_{n,t+1} (1 - \eta p_{t+1}) \right\}$$  \hspace{1cm} (26)$$

The surplus that arises from the current match is determined by two terms (appearing in the right-hand side of equation (25)). $S_{n,t}$ increases with the gap between the marginal productivity of labor and the replacement wage $w^\mu$. The current surplus also increases with the expected next period surplus, if the current match is not broken in the following period, $\beta \rho E_t \left\{ \Lambda_{t+1} S_{n,t+1} \right\}$, net of the expected next period household’s marginal value of an additional worker (expressed in units of consumption goods), derived from a new match that would occur in the following period, $\beta \eta \rho E_t \left\{ \Lambda_{t+1} p_{t+1} S_{n,t+1} \right\} = \beta \rho E_t \left\{ \Lambda_{t+1} p_{t+1} \frac{H_{n,t+1}}{\lambda_{t+1}} \right\}$.

Recall that a fraction $1 - \eta$ of the surplus goes to firms: $F_{n,t} = (1 - \eta) S_{n,t}$. Combining the latter with equations 1 and taking into account that $F_{n,t} = \kappa q_t$, one gets:
By making use of equations (25) and (26), one can get a recursive equation reflecting the dynamic of employment:

\[
\frac{\kappa}{q_t} = (1 - \eta) (a_t - w^u) + \beta \rho E_t \left\{ \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} (1 - \eta p_{t+1}) \right\}
\]

(27)

When either the vacancy posting cost parameter becomes close to zero, \( \kappa \to 0 \), or the matching efficiency parameter strongly improves, \( \alpha_m \to +\infty \), the marginal productivity of labor is equal to the replacement wage in the equilibrium.

This expression slightly differs from the corresponding equation in MPT to the extent that the marginal value of non-work activities is replaced by \( w^u \). This difference comes from the choice of utility function. In this model, employment does not enter the utility function negatively, while MPT uses a utility function similar to that of Shimer (2005). In MPT, the lower value of non-work activities following a spending shock reduces the reservation wage of workers. The incentive for firm to hire increases and lead to a positive fiscal multiplier. The absence of this transmission channel between fiscal policy and employment in this model enables to control that public spending affects employment directly through the new formulation of the matching function.

3.3.2 Government policy and resource constraint:

The government issue bonds \( b_t \) to finance the difference between tax income and spending. Public debt increases with the last period stock of debt and the associated interest payments \( r_{t-1} b_{t-1} \) and with the primary deficit \( d_t \). The primary deficit is the difference between tax income and labour market spending. Taxes are lump sum in this simple version of the model. Public spending are made of active labour market spending \( g_t \) as well as unemployment benefits \( w^u (1 - n_t) \).

\[
b_t = r_{t-1} b_{t-1} + d_t
\]

\[
d_t = g_t + w^u (1 - n_t) - \tau_t
\]

Public consumption \( g_t \) follows an auto-regressive process. Taxes \( \tau_t \) also follow an auto-regressive process and adjust as well to the level of public debt.

\[
g_t = (1 - \rho_g) g + \rho_g g_{t-1} + \epsilon_{t,t}
\]

\[
\tau_t = (1 - \rho_\tau) \tau + \rho_\tau \tau_{t-1} + \tau_b (b_{t-1} - b)
\]

The resource constraint states that aggregate demand equals the sum of private consumption,
investment, search costs and labour market spending. In the absence of nominal price stickiness, \( y_t \) is determined by the supply side. The resource constraint implies that labour market spending crowds out private consumption and investment.

\[
y_t = c_t + x_t + \kappa v_t + g_t
\]

### 3.4 Equilibrium conditions

The equilibrium condition of the model are as follow:

\[
\begin{align*}
q_t &= \sigma_m \theta_{s,t}^{\sigma_s} \theta_{g,t}^{\sigma_g} \\
p_t &= \sigma_m \theta_{s,t}^{1-\sigma_s} \theta_{g,t}^{\sigma_g} \\
\theta_{s,t} &= \frac{g_t}{v_t} \\
\theta_{g,t} &= \frac{gr_t}{v_t} \\
k_t &= (1 - \delta) k_{t-1} + x_t \left( 1 - \frac{\eta_k}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right)^2 \right) \\
y_t &= k_{t-1}^\zeta n_t^{1-\zeta} \\
n_t &= \rho n_{t-1} + g_t v_t \\
\lambda_t &= \frac{1}{c_t^\beta} \\
\varphi_t &= \left( 1 - \beta E_t \left( \frac{\lambda_t + 1}{\lambda_t} \left( \frac{x_{t+1}}{x_t} \right)^2 \eta_k \left( \frac{x_{t+1}}{x_t} - 1 \right) \right) \right) / \left( 1 - \left( \frac{\eta_k}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right)^2 + \frac{x_t}{x_{t-1}} \eta_k \left( \frac{x_t}{x_{t-1}} - 1 \right) \right) \right) \\
\varphi_t &= \beta E_t \left( \frac{\lambda_t + 1}{\lambda_t} \left[ r_{k,t+1} + \varphi_t + (1 - \delta) \right] \right) \\
r_{k,t} &= \frac{\zeta}{k_{t-1}} \\
a_t &= (1 - \zeta) \frac{y_t}{n_t} \\
y_t &= c_t + x_t + \kappa v_t + g_t \\
w_t &= \eta a_t + (1 - \eta) w^u + \eta \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \theta_{s,t+1} \right\} \\
\frac{\kappa}{\sigma_m} \theta_{s,t}^{\sigma_s} \theta_{g,t}^{\sigma_g} &= (1 - \eta) (a_t - w^u) + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa}{\sigma_m} (1 - \eta p_{t+1}) \theta_{s,t+1} \theta_{g,t+1} \right\} \\
b_t &= r_{t-1} b_{t-1} + d_t \\
d_t &= g_t + w^u (1 - n_t) - \tau_t \\
g_t &= (1 - \rho_g) g + \rho_g g_{t-1} + \epsilon_{g,t}
\end{align*}
\]
\[
\tau_t = (1 - \rho \tau) \tau + \rho \tau_{t-1} + \tau_b (b_{t-1} - b)
\]
\[
\frac{1}{r_t} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \right\}
\]

4 Steady states and calibration

The model contains 16 parameters, gathered in the set
\[
\Theta = \{ \beta, \delta, \zeta, \eta_k, \rho_g, \rho, \sigma_s, \eta, \sigma, \sigma_m, \kappa, \alpha_u, \rho \tau, \rho_b, w^u \}
\]
and 20 variables, which steady state levels are gathered in the set
\[
\Delta = \{ y, c, x, n, k, w, r_k, v, p, q, \theta_s, \theta_g, a, \lambda, \varphi, g, b, r, d, \tau \}
\]

In order to assign a value to each parameter and steady state variable of the model, we solve a system formed by the deterministic steady-state expressions of 19 equilibrium equations (20 equations minus auto-correlation of the public spending shock) and 17 additional restrictions. The deterministic steady state expressions of the equilibrium equations are:

\[
\varphi = 1
\]
\[
r_k = 1/\beta - 1 - \delta
\]
\[
r = 1/\beta
\]
\[
\theta_s = 0.5
\]
\[
\theta_g = 0.45
\]
\[
p = 0.45
\]
\[
\sigma_m = \frac{p}{\theta_s^{1-\sigma_s} \theta_g^{\sigma_g}}
\]
\[
q = \sigma_m \theta_s^{1-\sigma_s} \theta_g^{\sigma_g}
\]
\[
v = \frac{\theta_k (1 - \rho)}{1 - \rho + \rho q \theta_s}
\]
\[
n = \frac{qv}{1 - \rho}
\]
\[
y = \frac{r_k}{\zeta}
\]
\[
k = n \left( \frac{y}{k} \right)^{\frac{1}{\zeta - 1}}
\]
\[
y = \left( \frac{y}{k} \right) k
\]
\[
x = \delta k
\]
\[ a = (1 - \zeta) \frac{y}{n} \]

At this stage, we get the steady state values of \( w^u \) and \( \kappa \) by solving the equations 24 and 27. We then have:

\[
egin{align*}
   w &= \alpha_u w^u \\
   g &= \frac{v}{\theta_g} \\
   c &= y - x - \kappa v - g \\
   \lambda &= \frac{1}{c} \\
   b &= 0.6y \\
   \tau &= g + w^u (1 - n) - (1 - r) b \\
   d &= b / (1 - r)
\end{align*}
\]

The 17 additional restrictions are put on a subset of steady state variable levels, \( \Delta \equiv \{ p, \theta_s, \theta_g \} \), on a subset of the model parameters \( \Theta \equiv \{ \beta, \delta, \zeta, \eta_k, \rho_g, \rho, \sigma_g, \sigma_s, \eta, \sigma, \alpha_u, \rho_\tau, \rho_b \} \), and finally on a steady state ratio, \( \Gamma \equiv \left\{ \frac{b}{y} \right\} \). We assume that the time unit is a semester, similarly to Ravenna and Walsh (2008). The calibration of \( \Delta, \Theta, \) and \( \Gamma \) which is based on postwar U.S. data, is discussed below.

The calibration follows Ravenna and Walsh (2008) for the parameters of the labour market. The calibration of the discount factor is set so that the annual real interest rate is approximately 4 percent on average. The capital is assumed to depreciate at the rate of 10 percent on an annual basis. Around one third of the economy’s income goes to capital owners. Specifically, the discount factor, the capital depreciation rate and the capital share in the production function are set to \( \beta = 0.99 \), \( \delta = 0.025 \), and \( \zeta = \frac{1}{3} \). These values are standard in the business cycle literature. The parameter for the adjustment of capital cost is set to \( \eta_k = 0 \) such that we ignore the cost of capital adjustment in a first step. The survival rate in the labor market is set to \( \rho = 1 - 0.1 \). The elasticity of matches to searching workers parameter is set to \( \sigma_s = 0.6 \), in line with existing estimations ranging between 0.5 and 0.7 in the literature. The bargaining power between firms and households is symmetric \( \eta = 0.6 \) to meet the Hosios condition for efficiency. We set \( p = 0.45 \) to meet the estimates of the job finding rates and \( \theta_s = 0.5 \) to meet the measures of US vacancies Shimer (2005). We calibrate \( \theta_g \) to 0.45 such that the spending to GDP ratio corresponds to 1.2% the average labour market spending to GDP ratio in OECD countries\(^2\). The debt to GDP ratio is also fixed at 60% and the auto-regressive parameter of the government spending and taxes stochastic process are set to \( \rho_g = \rho_\tau = 0.9 \). Table 1 below summarized the restrictions on the model parameters and on the model steady states.

\(^2\)We use the active labour market spending data from the social expenditure database published by the OECD
Table 1: Parameters and main steady state values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital depreciation rate in the production function</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Capital share</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>Capital adjustment cost parameter</td>
<td>$\eta_k$</td>
</tr>
<tr>
<td>Government auto regressive parameters</td>
<td>$\rho_g, \rho_\tau$</td>
</tr>
<tr>
<td>Survival rate</td>
<td>$\rho$</td>
</tr>
<tr>
<td>Elasticity of matches to unemployment</td>
<td>$\sigma_s$</td>
</tr>
<tr>
<td>Elasticity of matches to public spending</td>
<td>$\sigma_g$</td>
</tr>
<tr>
<td>Bargaining power</td>
<td>$\eta$</td>
</tr>
<tr>
<td>Probability of finding a job</td>
<td>$p$</td>
</tr>
<tr>
<td>Labor market tightness</td>
<td>$\theta_s$</td>
</tr>
<tr>
<td>Spending per vacancy</td>
<td>$\theta_g$</td>
</tr>
<tr>
<td>Government spending share in output</td>
<td>$g_y$</td>
</tr>
<tr>
<td>Debt to GDP ratio</td>
<td>$b_y$</td>
</tr>
<tr>
<td>Consumption to GDP ratio</td>
<td>$c_y$</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>$u$</td>
</tr>
</tbody>
</table>

5 Results

This section rely on numerical simulations to investigate the properties of the model. We study in particular the dynamic of the economy following a positive shock on public spending and for different configurations of the economy. The impulse response function are displayed in percentage deviation from steady state following a spending shock equal to 1 percent of GDP. Employment as well as labour market tightness and job finding probabilities are in absolute term.

5.1 Baseline dynamic results

Figure 1 displays the dynamic of the economy following a positive shocks on labour market policy spending (solid line). The effects of active labour market spending are a mix between the resource effect associated with higher spending and a productivity shock on matching efficiency. For the sake of comparison, the case corresponding to an increase in government spending (no labour market spending, $\sigma_g = 0$, dashed line) and the case corresponding to a matching efficiency shock (no spending, dot line) are also represented in the figure. The share of spending in output is 1.2
percent at the steady state in line with the average level of active labour market spending in OECD countries. The spending shock amounts to 1 percent of GDP leading spending to increase to 2.2 percent of GDP at time 1.

An increase in labour market spending produces a positive output multiplier effect and an increase in employment. The output multiplier is positive and smaller than one. It is 0.31 on impact and 0.43 after one year. The employment multiplier is positive too and increases after four semesters. The multiplier are larger after 4 semester than on impact since matching takes time. Positive effects on employment and output are related to the improved efficiency of the labour markets induced by labour market spending. Labour market spending increases the number of matches and the overall level of employment. In fact, both labour market tightness and the job finding probability increases. There is as well an increase in wages. The Ricardian effect through which higher government spending crowds out private consumption is still at work as consumption drops on impact at -0.11. This experiment however shows that active labour market policies over balance the crowding out of private consumption through an increased efficiency of the labour market. This result is similar with the baseline result of Monacelli et al. (2010) except that the rise in employment is not linked to the specificities of the wage bargaining. In Monacelli et al. (2010), the positive multiplier effect is related to the crowding out of consumption, which reduces the dis-utility of work activities and lead workers to accept a lower wage.

In line with empirical evidence, wages, labour market tightness and the probability of finding a job increases by 0.2 percent, 1 percent and 2 percent respectively. These increases result from the positive effect on employment of higher spending and the reduction in searching workers. Contrastingly, the number of vacancies increases on impact before to decline. This result is related to the increases
efficiency of matching, whose side effect is to reduce vacancy posting.

The result must be evaluated against the benchmark case of general government spending, which can be simulated assuming $\sigma_{g} = 0$ (broken line in Figure 1). An increase in government consumption produces the usual crowding out of private consumption and output in line with the properties of a Ricardian economy. The increase in the interest rate reduces investment and employment leading to a decline in the quantity of goods supplied. The output multiplier is negative at -0.08 after one year.

A matching efficiency shock ($\gamma_{m}$ dot line in Figure 1) also generates an increase in output and employment, as well as an increase in labour market tightness and the probability of finding a job. The main difference with active labour market spending is the absence of the resource constraint effect. It follows that both consumption and investment increases following the shock.

In the appendix, it is shown that the positive effect of output on the fiscal multiplier holds true for different specification of the matching function. In particular, two cases are taken into consideration. In the first case, labour market spending are associated with unemployment: $m_{t} = \sigma_{m} \left( \sigma_{g} s_{t} \right)^{\sigma_{v}} v_{t}^{\sigma_{s}}$ with $\sigma_{v} = 1 - \sigma_{s}$. In the second case, labour market spending are associated with vacancies in the matching function: $m_{t} = \sigma_{m} \sigma_{s} \left( \sigma_{g} s_{t} v_{t} \right)^{\sigma_{v}}$ with $\sigma_{v} = 1 - \sigma_{s}$. These two sub-cases capture the idea that labour market spending may target specifically either searching workers or firms. Using the same set of parameters, the output and employment multipliers are both positive. They are larger in the case of spending nested with unemployed workers given that $\sigma_{s}$ is equal to 0.6. There are both however smaller than in the case of constant return to scale as the elasticities of matching to spending are smaller (see Figure 4).

### 5.2 Diminishing marginal return on labour market spending and crowding-in of consumption:

Figure 2 displays the dynamic of the main macroeconomic variables following a positive shock on labour market spending when the steady state level of active labour market spending to GDP is decreased from 1.2% to 0.7%, which correspond to the value of this ratio for the USA. For this purpose we keep $\sigma_{g}$ constant at 0.05 and decrease $\theta_{g}$ to 0.25 down from 0.45. It follows that the steady state spending to GDP ratio declines from 1.2 to 0.7 percent.

The fiscal multiplier is now larger 0.56 on impact and 0.83 after one year. This result follows from the diminishing marginal return, which implies that lower steady state spending to GDP ratio increases the size of the multiplier. Furthermore, consumption is now responding positively to public spending. Despite a negative reaction on impact, consumption turns positive after three semesters. It seems that the increase in output generated by higher employment crowds in private consumption. The positive impact of public spending on employment boosts output through a supply side effect. The positive impact on consumption that follows over balances the negative impact of higher public spending on private consumption. The same mechanism takes place regarding investment, which increases after two semesters. This feedback channel stands in contrast with existing models, which
produce an increase in consumption either by assuming non separability between employment (or hours worked) and consumption or by relying on a wealth effect as in the perpetual youth model.

5.3 Nominal price rigidities and aggregate demand effects

Nominal price rigidities are central when discussing fiscal policy to the extent that they generate quantity adjustment. Following fiscal policy, firms respond to the surge in aggregate demand by raising prices. In the presence of price rigidities, firms must also increase output to meet excess demand. Ravn et al. (2006) for instance use price stickiness (together with deep habit formation) to produce a positive fiscal multiplier. Adding price rigidities should trigger an increase in private consumption given that firms increase labour inputs and that the real wage increases. Nominal price rigidities also limit the crowding out of private consumption. Nominal price rigidities also impact the incentive of firms to hire. The markup now enters the marginal productivity of labour in the surplus equation from an additional match (see equation 27). The fall in the markup raises the marginal productivity of labour and contributes to the improvement of the labour market alongside labour market policies. The resulting effect should be a larger multiplier effect.

We follow Ravenna and Walsh (2008) to model price rigidities. The program of the firms described in the previous part of the model now corresponds to the intermediate goods producers. Intermediate goods producers face search costs in the labour market but sell their goods in a competitive market. Intermediate goods are then combined to produce a final good, which is sold on a non-competitive market. Introducing prices requires to modify the equilibrium condition presented

\[ \text{See also Trigari (2006) and Trigari (2009)} \]
in section 4. The new set of equation is presented in the appendix 7.2. The probability that firms cannot adjust their prices to the perfect competition prices is set at \( \omega = 0.75 \). The price elasticity in the demand function for retail goods is conventional and equal to \( \varepsilon = 10 \). The model is closed by specifying monetary policy (see equation 28).

\[
\frac{r^n_t}{r^n} = \left( \frac{r^n_{t-1}}{r^n} \right)^\rho_m \left( \frac{E_t \{ \pi_{t+1} \}}{\pi} \right) \phi_\pi (1-\rho_m) \left( \frac{y_t}{y_{t-1}} \right) \phi_y (1-\rho_m)
\]  

(28)

The Central Bank adjusts the interest rate to the forward looking inflation at a speed \( \phi_\pi = 1.1 \), while the sensitivity of the interest rate to the output gap is \( \phi_y = 0.675 \). \( \rho_m \) is the coefficient for interest rate rigidities and is equal to 0.9. The main result is that the fiscal multiplier is now much larger and almost equal to 2 on impact. Accordingly, the employment multiplier is also larger at 0.8. Consumption increases and is hump-shaped. Consumption reaches a peak of 0.035 after period 17.

6 Conclusion

This paper has discussed the efficiency of public spending using a DSGE model with search and matching function. The main objective was to show that despite the crowding out of resources, increases in spending in the form of active labour market policies are likely to yield positive fiscal multipliers. The multiplier are positive on impact and reach their maximum after 1 year. Active labour market spending produces positive supply side effect by easing the matching between unemployed workers and vacancies and increasing labour inputs. The paper showed further that the size of the
multiplier is larger in countries with a limited steady state spending to GDP ratio. It follows that in these countries, larger value of the multiplier generates a crowding in of consumption and output.

The mechanism displayed in this paper could be further detailed as active labour market policies encompass a large range of measures from subsidies to vacancy posting, training of unemployed workers, search incentives or public employment system. Two extensions could be considered. First, spending could take the form of subsidies of vacancies. Second, the matching model could include search or training effort, which would be subsidized by public authorities.
Appendix

7.1 Nested matching function

This appendix discusses the impact on the fiscal multiplier of different types of matching function. The baseline matching function has constant return to scale in \( s_t, v_t \) and \( g_t \) with \( \sigma_s + \sigma_v + \sigma_g = 1 \). We here briefly discuss the impact of two alternative specification. In a first case, spending are nested with searching workers and captures the idea that active labour market spending may be directed towards unemployed:

\[
m_t = \sigma_m \left( \frac{\sigma_s}{g_t s_t} \right)^{\sigma_s} \left( \frac{\sigma_s}{g_t s_t} \right)^{1-\sigma_s} v_t^{1-\sigma_s}
\]

with \( \sigma_v = 1 - \sigma_s \). The matching function has constant return on unemployment and vacancies and decreasing return on active labour market policies. For convenience, we use the ratio \( \theta_{s,f} = \frac{\nu}{\theta_s} \) to measure labour market tightness:

\[
q_t = \frac{m_t}{v_t} = \sigma_m g_t \sigma_s \sigma_s \sigma_s v_t^{1-\sigma_s} = \sigma_m \theta_{s,f} \sigma_s \sigma_s g_t \sigma_s
\]

\[
p_t = \frac{m_t}{s_t} = \sigma_m g_t \sigma_s \sigma_s \sigma_s v_t^{1-\sigma_s} = \sigma_m \theta_{s,f} \sigma_s \sigma_s g_t \sigma_s
\]

In a second case, spending are nested with vacancies to capture the idea that labour market policies also target firms:

\[
m_t = \sigma_m \sigma_f \left( \frac{\sigma_s}{g_t v_t} \right)^{1-\sigma_s}
\]

with \( \sigma_v = 1 - \sigma_s \). The matching function has constant return on unemployment and vacancies and decreasing return on active labour market policies. For convenience, we use the ratio \( \theta_{s,f} = \frac{\nu}{\theta_s} \) to measure labour market tightness:

\[
q_t = \frac{m_t}{v_t} = \sigma_m g_t \sigma_s (1-\sigma_s) \sigma_s \sigma_s v_t^{1-\sigma_s} = \sigma_m \theta_{s,f} \sigma_s \sigma_s g_t \sigma_s
\]

\[
p_t = \frac{m_t}{s_t} = \sigma_m g_t \sigma_s (1-\sigma_s) \sigma_s \sigma_s v_t^{1-\sigma_s} = \sigma_m \theta_{s,f} \sigma_s \sigma_s g_t \sigma_s (1-\sigma_s)
\]

The results are displayed in Figure 4 with the solid line being associated with active labour market spending nested with searching workers. Note that the set of parameters used in both cases is the same. The output and employment multiplier are both positive but they are larger in the case
of spending nested with unemployed workers. There are both however smaller than in the case of constant return to scale as the elasticities of matching to spending are smaller.

7.2 Nominal price rigidities and aggregate demand effects

Introducing nominal price rigidities modifies the system of equations of the model, which now appears as follow:

\[ q_t = \sigma_m \theta_{s,t}^{\sigma_s} \theta_{g,t}^{\sigma_g} \]
\[ p_t = \sigma_m \theta_{s,t}^{1-\sigma_s} \theta_{g,t}^{\sigma_g} \]
\[ \theta_{s,t} = \frac{v_t}{1 - \rho n_{t-1}} \]
\[ \theta_{g,t} = \frac{g_t}{v_t} \]
\[ k_t = (1 - \delta) r_{k,t-1} + x_t \left( 1 - \frac{\eta_k}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right)^2 \right) \]
\[ y_t^w = k_t^{1-\zeta} n_t \]
\[ n_t = \rho n_{t-1} + q_t v_t \]
\[ \lambda_t = \frac{1}{c_r} \]
\[ \varphi_t = \left[ 1 - \beta E_t \left( \varphi_{t+1} + \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{x_{t+1}}{x_t} \right)^2 \eta_k \left( \frac{x_{t+1}}{x_t} - 1 \right) \right) \right] / \left[ 1 - \left( \frac{\eta_k}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right)^2 + \frac{x_t}{x_{t-1}} \eta_k \left( \frac{x_t}{x_{t-1}} - 1 \right) \right) \right] \]
\[ \varphi_t = \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \left[ r_{k,t+1} + \varphi_{t+1} (1 - \delta) \right] \right) \]
\[ r_{k,t} = \zeta m c_t \frac{y_t^w}{k_t} \]
\[a_t = (1 - \zeta) mc_t \frac{y_t^{1.0}}{n_t}\]

\[w_t = \eta a_t + (1 - \eta) w^{\mu} + \eta \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \theta_{s,t+1} \right\}\]

\[\frac{\kappa}{\sigma_m} \theta_{s,t}^{\sigma_t} \theta_{g,t}^{-\sigma_g} = (1 - \eta) (a_t - w^{\mu}) + \beta \rho E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa}{\sigma_m} (1 - \eta p_{t+1}) \theta_{s,t+1}^{\sigma_t} \theta_{g,t+1}^{-\sigma_g} \right\}\]

\[y_t = c_t + x_t + \kappa v_t + g_t\]

\[y_t^{1.0} = s_t y_t\]

\[s_t = (1 - \omega) \bar{p}_t^{-\varepsilon} + \omega \pi_t^{1.0} s_{t-1}\]

\[b_t = \frac{r_t-1}{r_t} - b_{t-1} + d_t\]

\[d_t = g_t + w^{\mu} (1 - n_t) - \tau_t\]

\[g_t = (1 - \rho_g) g + \rho_g s_{t-1} + \epsilon_{l,t}\]

\[\tau_t = (1 - \rho_\tau) \tau + \rho_\tau \tau_{t-1} + \tau (b_{t-1} - b)\]

\[\frac{\pi_{t+1}}{r_t} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \right\}\]

\[f_t^1 = \frac{\varepsilon - 1}{\varepsilon} f_t^2\]

\[f_t^1 = \bar{p}_t^{-\varepsilon} y_t mc_t + \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \pi_t^{1.0} \left( \frac{\bar{p}_t}{\bar{p}_{t+1}} \right)^{-1-\varepsilon} f_t^1 \right)\]

\[f_t^2 = \bar{p}_t^{-\varepsilon} y_t + \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \pi_t^{1.0} \left( \frac{\bar{p}_t}{\bar{p}_{t+1}} \right)^{-\varepsilon} f_t^2 \right)\]

\[r_t = \left( \frac{r_{t-1}}{r} \right) \rho_m \left( \frac{E_t \pi_{t+1}}{\pi} \right) \phi(1 - \rho_m) \left( \frac{y_t}{y_{t-1}} \right) \phi((1 - \rho_m)\frac{1}{\phi(1 - \rho_m)}\right)\]

\[1 = \omega \pi_t^{1.0} + (1 - \omega) \bar{p}_t^{-\varepsilon}\]
References


