22 THE TREATMENT OF SEASONAL PRODUCTS

Introduction

22.1 The existence of seasonal commodities poses some significant challenges for price statisticians. *Seasonal commodities* are commodities which are either: (a) not available in the marketplace during certain seasons of the year, or (b) are available throughout the year, but there are regular fluctuations in prices or quantities that are synchronized with the season or the time of the year.[1] A commodity that satisfies (a) is termed a *strongly seasonal commodity*, whereas a commodity that satisfies (b) is called a *weakly seasonal commodity*. It is strongly seasonal commodities that create the biggest problems for price statisticians in the context of producing a monthly or quarterly consumer price index (CPI) because if a commodity price is available in only one of the two months (or quarters) being compared, then obviously it is not possible to calculate a relative price for the commodity and traditional bilateral index number theory breaks down. In other words, if a commodity is present in one month but not the next, how can the month-to-month amount of price change for that commodity be computed?[2] In this chapter, a solution to this problem is presented which "works", even if the commodities consumed are entirely different for each month of the year.[3]

22.2 There are two main sources of seasonal fluctuations in prices and quantities: (a) climate, and (b) custom.[4] In the first category, fluctuations in temperature, precipitation and hours of daylight cause fluctuations in the demand or supply for many commodities; for example, summer versus winter clothing, the demand for light and heat, holidays, etc. With respect to custom and convention as a cause of seasonal fluctuations, consider the following quotation:

> Conventional seasons have many origins—ancient religious observances, folk customs, fashions, business practices, statute law… Many of the conventional seasons have considerable effects on economic behaviour. We can count on active retail buying before Christmas, on the Thanksgiving demand for turkeys, on the first of July demand for fireworks, on the preparations for June weddings, on heavy dividend and interest payments at the beginning of each quarter, on an increase in bankruptcies in January, and so on (Mitchell (1927, p. 237)).

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[1] This classification of seasonal commodities corresponds to Balk's narrow and wide sense seasonal commodities; see Balk (1980a, p. 7; 1980b, p. 110; 1980c, p. 68). Diewert (1998b, p. 457) used the terms type 1 and type 2 seasonality.

[2] Victor Zarnowitz (1961, p. 238) was perhaps the first to note the importance of this problem: "But the main problem introduced by the seasonal change is precisely that the market basket is different in the consecutive months (seasons), not only in weights but presumably often also in its very composition by commodities. This is a general and complex problem which will have to be dealt with separately at later stages of our analysis.

[3] The same commodities must, however, reappear each year for each separate month.

[4] This classification dates back to Wesley C. Mitchell (1927, p. 236) at least: "Two types of seasons produce annually recurring variations in economic activity – those which are due to climates and those which are due to conventions".

22.3 Examples of important seasonal commodities are: many food items; alcoholic beverages; many clothing and footwear items; water; heating oil; electricity; flowers and garden supplies; vehicle purchases; vehicle operation; many entertainment and recreation expenditures; books; insurance expenditures; wedding expenditures; recreational equipment; toys and games; software; air travel and tourism expenditures. For a "typical" country, seasonal expenditures will often amount to one-fifth to one-third of all consumer expenditures.[5]

22.4 In the context of producing a monthly or quarterly CPI, it must be recognized that there is no completely satisfactory way of dealing with strongly seasonal commodities. If a commodity is present in one month but missing from the marketplace in the next month, then none of the index number theories that were considered in Chapters 15 to 20 can be applied because all these theories assumed that the dimensionality of the commodity space was constant for the two periods being compared. However, if seasonal commodities are present in the market during each season, then, in theory, traditional index number theory can be applied in order to construct month-to-month or quarter-to-quarter price indices. This "traditional" approach to the treatment of seasonal commodities will be followed in paragraphs 22.78 to 22.90. The reason why this straightforward approach is deferred to the end of the chapter is twofold:

- The approach that restricts the index to commodities that are present in every period often does not work well in the sense that systematic biases can occur.
- The approach is not fully representative; i.e., it does not make use of information on commodities that are not present in every month or quarter.

22.5 In the next section, a modified version of Turvey's (1979) artificial data set is introduced. This data set will be used in order to evaluate numerically all the index number formulae suggested in this chapter. It will be seen in paragraphs 22.63 to 22.77 that very large seasonal fluctuations in volumes, combined with systematic seasonal changes in price, can make month-to-month or quarter-to-quarter price indices behave rather poorly.

22.6 Even though existing index number theory cannot deal satisfactorily with seasonal commodities in the context of constructing month-to-month indices of consumer prices, it can deal satisfactorily with seasonal commodities if the focus is changed from month-to-month CPIs to CPIs that compare the prices of one month with the prices of the *same* month in a previous year. Thus, in paragraphs 22.16 to 22.34, year-over-year monthly CPIs are studied. Turvey's seasonal data set is used to evaluate the performance of these indices and they are found to perform quite well.

22.7 In paragraphs 22.35 to 22.44, the year-over-year monthly indices defined in paragraphs 23.16 to 23.34 are aggregated into an annual index that compares all the monthly prices in a given calendar year with the corresponding monthly prices in a base year. In paragraphs 22.45 to 22.54, this idea of comparing the prices of a current calendar year with the corresponding prices in a base year is extended to annual indices that compare the prices of the last 12 months with the corresponding prices in the 12 months of a base year. The

[5] Alterman, Diewert and Feenstra (1999, p. 151) found that over the 40 months between September 1993 and December 1996, somewhere between 23 and 40 per cent of United States imports and exports exhibited seasonal variations in quantities, whereas only about 5 per cent of United States export and import prices exhibited seasonal fluctuations.

resulting *rolling year indices* can be regarded as seasonally adjusted price indices. The modified Turvey data set is used to test out these year-over-year indices, and they are found to work very well on this data set.

22.8 The rolling year indices can provide an accurate gauge of the movement of prices in the current rolling year compared to the base year. This measure of price inflation can, however, be regarded as a measure of inflation for a year that is centered around a month six months prior to the last month in the current rolling year. Hence for some policy purposes, this type of index is not as useful as an index that compares the prices of the current month to the previous month, so that more up-to-date information on the movement of prices can be obtained. In paragraphs 22.55 to 22.62, it will nevertheless be shown that under certain conditions, the year-over-year monthly index for the current month, along with the year-over-year monthly index for last month, can successfully predict or forecast a rolling year index that is centered around the current month.

22.9 The year-over-year indices defined in paragraphs 22.16 to 22.34, and their annual averages studied in paragraphs 22.35 to 22.54, offer a theoretically satisfactory method for dealing with strongly seasonal commodities; i.e., commodities that are available only during certain seasons of the year. These methods rely on the year-over-year comparison of prices and hence cannot be used in the month-to-month or quarter-to-quarter type of index, which is typically the main focus of a consumer price programme. Thus there is a need for another type of index, which may not have very strong theoretical foundations, but which can deal with seasonal commodities in the context of producing a month-to-month index. In paragraphs 22.63 to 22.77, such an index is introduced and it is implemented using the artificial data set for the commodities that are available during each month of the year. Unfortunately, because of the seasonality in both prices and quantities of the always available commodities, this type of index can be systematically biased. This bias shows up for the modified Turvey data set.

22.10 Since many CPIs are month-to-month indices that use *annual basket quantity weights*, this type of index is studied in paragraphs 22.78 to 22.84. For months when the commodity is not available in the marketplace, the last available price is carried forward and used in the index. In paragraphs 22.85 and 22.86, an annual quantity basket is again used but instead of carrying forward the prices of seasonally unavailable items, an imputation method is used to fill in the missing prices. The annual basket type indices defined in paragraphs 22.78 to 22.84 are implemented using the artificial data set. Unfortunately, the empirical results are not satisfactory in that the indices show tremendous seasonal fluctuations in prices, so they would not be suitable for users who wanted up-to-date information on trends in general inflation.

22.11 In paragraphs 22.87 to 22.90, the artificial data set is used in order to evaluate another type of month-to-month index that is frequently suggested in the literature on how to deal with seasonal commodities; namely the *Bean and Stine Type C* (1924) or *Rothwell* (1958) index. Again, this index does not get rid of the tremendous seasonal fluctuations that are present in the modified Turvey data set.

22.12 Paragraphs 22.78 to 22.84 show that the annual basket type indices with carry forward of missing prices or imputation of missing prices do not get rid of seasonal fluctuations in prices. However, in paragraphs 22.91 to 22.96, it is shown how seasonally adjusted versions of these annual basket indices can be used successfully to forecast rolling year indices that are centered on the current month. In addition, the results show how these annual basket type indices can be seasonally adjusted (using information obtained from rolling year indices from prior periods or by using traditional seasonal adjustment procedures), and hence these seasonally adjusted annual basket indices could be used as successful indicators of general inflation on a timely basis.

22.13 Paragraph 23.97 outlines some conclusions.

A seasonal commodity data set

22.14 It is useful to illustrate the index number formulae defined in subsequent sections by computing them for an actual data set. Turvey (1979) constructed an artificial data set for five seasonal commodities (apples, peaches, grapes, strawberries and oranges) for four years by month so that there are 5 × 4 × 12 = 240 observations in all. At certain times of the year, peaches and strawberries (commodities 2 and 4) are unavailable, so in Tables 22.1 and 22.2 the prices and quantities for these two commodities are entered as zeros.[6] The data in Tables 22.1 and 22.2 are essentially the same as the data set constructed by Turvey except that a number of adjustments have been made to it in order to illustrate various points. The two most important adjustments are:

- The data for commodity 3 (grapes) have been adjusted so that the annual Laspeyres and Paasche indices (defined in paragraphs 22.35 to 22.44) would differ more than in the original data set.[7]
- After the above adjustments were made, each price in the last year of data was escalated by the monthly inflation factor 1.008 so that month-to-month inflation for the last year of data would be at an approximate monthly rate of 1.6 per cent per month compared to about 0.8 per cent per month for the first three years of data.[8]

Table 22.1 An artificial seasonal data set: Prices

Year t	**Month** m	$p_1^{t,m}$	$p_2^{t,m}$	$p_3^{t,m}$	$p_4^{t,m}$	$p_5^{t,m}$
1970	1	1.14	0	2.48	0	1.30
	2	1.17	0	2.75	0	1.25

[6] The corresponding prices are not necessarily equal to zero (the commodities may be offered for sale at certain prices but there are no purchasers at those prices), but they are entered as zeros for convenience in programming the various indices.

[7] After the first year, the price data for grapes has been adjusted downward by 30 per cent each year and the corresponding volume has been adjusted upward by 40 per cent each year. In addition, the quantity of oranges (commodity 5) for November 1971 has been changed from 3,548 to 8,548 so that the seasonal pattern of change for this commodity is similar to that of other years. For similar reasons, the price of oranges in December 1970 has been changed from 1.31 to 1.41 and in January 1971 from 1.35 to 1.45.

[8] Pierre Duguay of the Bank of Canada, while commenting on a preliminary version of this chapter, observed that rolling year indices would not be able to detect the *magnitude* of systematic changes in the month-to-month inflation rate. The original Turvey data set was roughly consistent with a month-to-month inflation rate of 0.8 per cent per month; i.e., prices grew roughly at the rate of 1.008 each month over the four years of data. This second major adjustment of the Turvey data was introduced to illustrate Duguay's observation, which is quite correct: the centred rolling year indices pick up the correct magnitude of the new inflation rate only after a lag of half a year or so. They do, however, quickly pick up the direction of change in the inflation rate.

Year t	Month m	$p_1^{t,m}$	$p_2^{t,m}$	$p_3^{t,m}$	$p_4^{t,m}$	$p_5^{t,m}$
	3	1.17	0	5.07	0	1.21
	4	1.40	0	5.00	0	1.22
	5	1.64	0	4.98	5.13	1.28
	6	1.75	3.15	4.78	3.48	1.33
	7	1.83	2.53	3.48	3.27	1.45
	8	1.92	1.76	2.01	0	1.54
	9	1.38	1.73	1.42	0	1.57
	10	1.10	1.94	1.39	0	1.61
	11	1.09	0	1.75	0	1.59
	12	1.10	0	2.02	0	1.41
1971	1	1.25	0	2.15	0	1.45
	2	1.36	0	2.55	0	1.36
	3	1.38	0	4.22	0	1.37
	4	1.57	0	4.36	0	1.44
	5	1.77	0	4.18	5.68	1.51
	6	1.86	3.77	4.08	3.72	1.56
	7	1.94	2.85	2.61	3.78	1.66
	8	2.02	1.98	1.79	0	1.74
	9	1.55	1.80	1.28	0	1.76
	10	1.34	1.95	1.26	0	1.77
	11	1.33	0	1.62	0	1.76
	12	1.30	0	1.81	0	1.50
1972	1	1.43	0	1.89	0	1.56
	2	1.53	0	2.38	0	1.53
	3	1.59	0	3.59	0	1.55
	4	1.73	0	3.90	0	1.62
	5	1.89	0	3.56	6.21	1.70
	6	1.98	4.69	3.51	3.98	1.78
	7	2.07	3.32	2.73	4.30	1.89
	8	2.12	2.29	1.65	0	1.91
	9	1.73	1.90	1.15	0	1.92
	10	1.56	1.97	1.15	0	1.95
	11	1.56	0	1.46	0	1.94
	12	1.49	0	1.73	0	1.64

Year t	Month m	$p_1^{t,m}$	$p_2^{t,m}$	$p_3^{t,m}$	$p_4^{t,m}$	$p_5^{t,m}$
1973	1	1.68	0	1.62	0	1.69
	2	1.82	0	2.16	0	1.69
	3	1.89	0	3.02	0	1.74
	4	2.00	0	3.45	0	1.91
	5	2.14	0	3.08	7.17	2.03
	6	2.23	6.40	3.07	4.53	2.13
	7	2.35	4.31	2.41	5.19	2.22
	8	2.40	2.98	1.49	0	2.26
	9	2.09	2.21	1.08	0	2.22
	10	2.03	2.18	1.08	0	2.31
	11	2.05	0	1.36	0	2.34
	12	1.90	0	1.57	0	1.97

Table 22.2 An artificial seasonal data set: Quantities

Year t	Month m	$q_1^{t,m}$	$q_2^{t,m}$	$q_3^{t,m}$	$q_4^{t,m}$	$q_5^{t,m}$
1970	1	3086	0	82	0	10266
	2	3765	0	35	0	9656
	3	4363	0	9	0	7940
	4	4842	0	8	0	5110
	5	4439	0	26	700	4089
	6	5323	91	75	2709	3362
	7	4165	498	82	1970	3396
	8	3224	6504	1490	0	2406
	9	4025	4923	2937	0	2486
	10	5784	865	2826	0	3222
	11	6949	0	1290	0	6958
	12	3924	0	338	0	9762
1971	1	3415	0	119	0	10888
	2	4127	0	45	0	10314
	3	4771	0	14	0	8797
	4	5290	0	11	0	5590
	5	4986	0	74	806	4377
	6	5869	98	112	3166	3681
	7	4671	548	132	2153	3748
	8	3534	6964	2216	0	2649

Year *t*	**Month** *m*	$q_1^{t,m}$	$q_2^{t,m}$	$q_3^{t,m}$	$q_4^{t,m}$	$q_5^{t,m}$
	9	4509	5370	4229	0	2726
	10	6299	932	4178	0	3477
	11	7753	0	1831	0	8548
	12	4285	0	496	0	10727
1972	1	3742	0	172	0	11569
	2	4518	0	67	0	10993
	3	5134	0	22	0	9621
	4	5738	0	16	0	6063
	5	5498	0	137	931	4625
	6	6420	104	171	3642	3970
	7	5157	604	202	2533	4078
	8	3881	7378	3269	0	2883
	9	4917	5839	6111	0	2957
	10	6872	1006	5964	0	3759
	11	8490	0	2824	0	8238
	12	5211	0	731	0	11827
1973	1	4051	0	250	0	12206
	2	4909	0	102	0	11698
	3	5567	0	30	0	10438
	4	6253	0	25	0	6593
	5	6101	0	220	1033	4926
	6	7023	111	252	4085	4307
	7	5671	653	266	2877	4418
	8	4187	7856	4813	0	3165
	9	5446	6291	8803	0	3211
	10	7377	1073	8778	0	4007
	11	9283	0	4517	0	8833
	12	4955	0	1073	0	12558

22.15 Ralph Turvey sent his artificial data set to statistical agencies around the world, asking them to use their normal techniques to construct monthly and annual average price indices. About 20 countries replied, and Turvey (1979, p. 13) summarized the responses as follows: “It will be seen that the monthly indices display very large differences, e.g., a range of 129.12 -169.50 in June, while the range of simple annual means is much smaller. It will also be seen that the indices vary as to the peak month or year.”
The above (modified) data are used to test out various index number formulae in subsequent sections.

Year-over-year monthly indices

22.16 It can be seen that the existence of seasonal commodities that are present in the marketplace in one month but not the next causes the accuracy of a month-to-month index to fall.[9] A way of dealing with these strongly seasonal commodities is to change the focus from short-term month-to-month price indices and instead focus on making year-over-year price comparisons for each month of the year. In the latter type of comparison, there is a good chance that seasonal commodities that appear, say, in February will also appear in subsequent Februarys so that the overlap of commodities will be maximized in these year-over-year monthly indices.

22.17 For over a century, it has been recognized that making year-over-year comparisons[10] provides the simplest method for making comparisons that are free from the contaminating effects of seasonal fluctuations. According to W. Stanley Jevons (1884, p. 3):

> In the daily market reports, and other statistical publications, we continually find comparisons between numbers referring to the week, month, or other parts of the year, and those for the corresponding parts of a previous year. The comparison is given in this way in order to avoid any variation due to the time of the year. And it is obvious to everyone that this precaution is necessary. Every branch of industry and commerce must be affected more or less by the revolution of the seasons, and we must allow for what is due to this cause before we can learn what is due to other causes.

22.18 The economist A.W. Flux and the statistician G. Udny Yule also endorsed the idea of making year-over-year comparisons to minimize the effects of seasonal fluctuations:

> Each month the average price change compared with the corresponding month of the previous year is to be computed. … The determination of the proper seasonal variations of weights, especially in view of the liability of seasons to vary from year to year, is a task from which, I imagine, most of us would be tempted to recoil (Flux (1921, pp. 184-185)).

> My own inclination would be to form the index number for any month by taking ratios to the corresponding month of the year being used for reference, the year before presumably, as this would avoid any difficulties with seasonal commodities. I should then form the annual average by the geometric mean of the monthly figures (Yule (1921, p. 199)).

In more recent times, Victor Zarnowitz (1961, p. 266) also endorsed the use of year-over-year monthly indices:

> There is of course no difficulty in measuring the average price change between the same months of successive years, if a month is our unit "season", and if a constant seasonal market basket can be used, for traditional methods of price index construction can be applied in such comparisons.

22.19 In the remainder of this section, it is shown how year-over-year Fisher indices and approximations to them can be constructed.[11] For each month $m = 1,2,...,12$, let $S(m)$ denote the set of commodities that are available in the marketplace for each year $t = 0,1,...,T$. For t =

[9] At the limit, if each commodity appeared in only one month of the year, then a month-to-month index would break down completely.

[10] In the seasonal price index context, this type of index corresponds to Bean and Stine's (1924, p. 31) Type D index.

[11] Diewert (1996b, p. 17-19; 1999a, p. 50) noted various separability restrictions on consumer preferences that would justify these year-over-year monthly indices from the viewpoint of the economic approach to index number theory.

0,1,...,T and m = 1,2,...,12, let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of commodity n that is in the marketplace in month m of year t, where n belongs to $S(m)$. Let $p^{t,m}$ and $q^{t,m}$ denote the month m and year t price and quantity vectors, respectively. Then *the year-over-year monthly Laspeyres, Paasche and Fisher indices* going from month m of year t to month m of year t+1 can be defined as follows:

$$P_L\left(p^{t,m}, p^{t+1,m}, q^{t,m}\right) = \frac{\sum_{n\in S(m)} p_n^{t+1,m} q_n^{t,m}}{\sum_{n\in S(m)} p_n^{t,m} q_n^{t,m}} \quad m = 1,2,...12 \qquad (22.1)$$

$$P_P\left(p^{t,m}, p^{t+1,m}, q^{t+1,m}\right) = \frac{\sum_{n\in S(m)} p_n^{t+1,m} q_n^{t+1,m}}{\sum_{n\in S(m)} p_n^{t,m} q_n^{t+1,m}} \quad m = 1,2,...12 \qquad (22.2)$$

$$P_F\left(p^{t,m}, p^{t+1,m}, q^{t,m}, q^{t+1,m}\right) \equiv \sqrt{P_L\left(p^{t,m}, p^{t+1,m}, q^{t,m}\right) P_P\left(p^{t,m}, p^{t+1,m}, q^{t+1,m}\right)} \quad m = 1,2,...,12 \qquad (22.3)$$

22.20 The above formulae can be rewritten in price relative and monthly expenditure share form as follows:

$$P_L\left(p^{t,m}, p^{t+1,m}, s^{t,m}\right) = \sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m} / p_n^{t,m}\right) \quad m = 1,2,...12 \qquad (22.4)$$

$$P_P\left(p^{t,m}, p^{t+1,m}, s^{t+1,m}\right) = \left[\sum_{n\in S(m)} s_n^{t+1,m}\left(p_n^{t+1,m} / p_n^{t,m}\right)^{-1}\right]^{-1} \quad m = 1,2,...12 \qquad (22.5)$$

$$P_F\left(p^{t,m}, p^{t+1,m}, s^{t,m}, s^{t+1,m}\right) \equiv \sqrt{P_L\left(p^{t,m}, p^{t+1,m}, s^{t,m}\right) P_P\left(p^{t,m}, p^{t+1,m}, s^{t+1,m}\right)} \qquad (22.6)$$

$$= \sqrt{\sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m} / p_n^{t,m}\right)} \sqrt{\left[\sum_{n\in S(m)} s_n^{t+1,m}\left(p_n^{t+1,m} / p_n^{t,m}\right)^{-1}\right]^{-1}} \quad m = 1,2,...12$$

where the monthly expenditure share for commodity $n \in S(m)$ for month m in year t is defined as:

$$s_n^{t,m} = \frac{p_n^{t,m} q_n^{t,m}}{\sum_{i\in S(m)} p_i^{t,m} q_i^{t,m}}; \quad m = 1,2,...,12 \quad n \in S(m) \quad t = 0,1,...,T \qquad (22.7)$$

and $s^{t,m}$ denotes the vector of month m expenditure shares in year t, $[s_n^{t,m}]$ for $n \in S(m)$.

22.21 Current period expenditure shares $s_n^{t,m}$ are not likely to be available. Hence it will be necessary to approximate these shares using the corresponding expenditure shares from a base year 0.

22.22 Use the base period monthly expenditure share vectors $s^{0,m}$ in place of the vector of month m and year t expenditure shares $s^{t,m}$ in equation (22.4), and use the base period monthly expenditure share vectors $s^{0,m}$ in place of the vector of month m and year t+1 expenditure shares $s^{t+1,m}$ in equation (22.5). Similarly, replace the share vectors $s^{t,m}$ and $s^{t+1,m}$ in equation (22.6) by the base period expenditure share vector for month m, $s^{0,m}$. The resulting *approximate year-over-year monthly Laspeyres, Paasche and Fisher indices* are defined by equations (22.8) to (22.10):[12]

$$P_{AL}\left(p^{t,m},p^{t+1,m},s^{0,m}\right)=\sum_{n\in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right) \qquad m = 1,2,...12 \qquad (22.8)$$

$$P_{AP}\left(p^{t,m},p^{t+1,m},s^{0,m}\right)=\left[\sum_{n\in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1} \qquad m = 1,2,...12 \qquad (22.9)$$

$$P_{AF}\left(p^{t,m},p^{t+1,m},s^{0,m},s^{0,m}\right)\equiv\sqrt{P_{AL}\left(p^{t,m},p_n^{t+1,m},s^{0,m}\right)P_{AP}\left(p^{t,m},p^{t+1,m},s^{0,m}\right)}$$

$$=\sqrt{\sum_{n\in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}}\times\sqrt{\left[\sum_{n\in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}} \qquad (22.10)$$

22.23 The approximate Fisher year-over-year monthly indices defined by equation (22.10) will provide adequate approximations to their true Fisher counterparts defined by equation (22.6) only if the monthly expenditure shares for the base year 0 are not too different from their current year *t* and *t*+1 counterparts. Hence, it will be useful to construct the true Fisher indices on a delayed basis in order to check the adequacy of the approximate Fisher indices defined by equation (22.10).

22.24 The year-over-year monthly approximate Fisher indices defined by equation (22.10) will normally have a certain amount of upward bias, since these indices cannot reflect long-term substitution of consumers towards commodities that are becoming relatively cheaper over time. This reinforces the case for computing true year-over-year monthly Fisher indices defined by equation (22.6) on a delayed basis so that this substitution bias can be estimated.

22.25 Note that the approximate year-over-year monthly Laspeyres and Paasche indices, P_{AL} and P_{AP} defined by equations (22.8) and (22.9) above, satisfy the following inequalities:

$$P_{AL}\left(p^{t,m},p^{t+1,m},s^{0,m}\right)P_{AL}\left(p^{t+1,m},p^{t,m},s^{0,m}\right)\geq 1 \qquad m = 1,2,...,12 \qquad (22.11)$$

$$P_{AP}\left(p^{t,m},p^{t+1,m},s^{0,m}\right)P_{AP}\left(p^{t+1,m},p^{t,m},s^{0,m}\right)\leq 1 \qquad m = 1,2,...,12 \qquad (22.12)$$

with strict inequalities if the monthly price vectors $p^{t,m}$ and $p^{t+1,m}$ are not proportional to each other. [13] The inequality (22.11) says that the approximate year-over-year monthly Laspeyres index fails the time reversal test with an upward bias, while the inequality (22.12) says that the approximate year-over-year monthly Paasche index fails the time reversal test with a downward bias. Hence the fixed weight approximate Laspeyres index P_{AL} has a built-in upward bias and the fixed weight approximate Paasche index P_{AP} has a built-in downward

[12] If the monthly expenditure shares for the base year, $s_n^{0,m}$, are all equal, then the approximate Fisher index defined by equation (22.10) reduces to Fisher's (1922, p. 472) formula 101. Fisher (1922, p. 211) observed that this index was empirically very close to the unweighted geometric mean of the price relatives, while Dalén (1992, p. 143) and Diewert (1995a, p. 29) showed analytically that these two indices approximated each other to the second order. The equally weighted version of equation (22.10) was recommended as an elementary index by Carruthers, Sellwood and Ward (1980, p. 25) and Dalén (1992, p. 140).

[13] See Hardy, Littlewood and Pólya (1934, p. 26).

bias. Statistical agencies should avoid the use of these formulae. The formulae can, however, be combined as in the approximate Fisher formula (22.10) and the resulting index should be free from any systematic formula bias (but there still could be some substitution bias).

22.26 The year-over-year monthly indices defined in this section are illustrated using the artificial data set given in Tables 22.1 and 22.2. Although fixed base indices are not formally defined in this section, these indices have similar formulae to the year-over-year indices except that the variable base year *t* is replaced by the fixed base year 0. The resulting 12 year-over-year monthly fixed base Laspeyres, Paasche and Fisher indices are listed in Tables 22.3 to 22.5.

Table 22.3 Year-over-year monthly fixed base Laspeyres indices

Year	**Month**										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**1**
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1
1972	1.2060	1.2442	1.3062	1.2783	1.2184	1.1734	1.2364	1.1827	1.1049	1.1809	1
1973	1.3281	1.4028	1.4968	1.4917	1.4105	1.3461	1.4559	1.4290	1.2636	1.4060	1

Table 22.4 Year-over-year monthly fixed base Paasche indices

Year	**Month**										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**1**
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1
1971	1.1074	1.1070	1.1471	1.1486	1.1115	1.0827	1.1075	1.0699	1.0414	1.0762	1
1972	1.2023	1.2436	1.3038	1.2773	1.2024	1.1657	1.2307	1.1455	1.0695	1.1274	1
1973	1.3190	1.4009	1.4912	1.4882	1.3715	1.3266	1.4433	1.3122	1.1664	1.2496	1

Table 22.5 Year-over-year monthly fixed base Fisher indices

Year	**Month**										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**1**
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1
1971	1.1080	1.1069	1.1474	1.1487	1.1137	1.0835	1.1089	1.0741	1.0453	1.0831	1
1972	1.2041	1.2439	1.3050	1.2778	1.2104	1.1695	1.2336	1.1640	1.0870	1.1538	1
1973	1.3235	1.4019	1.4940	1.4900	1.3909	1.3363	1.4496	1.3694	1.2140	1.3255	1

22.27 Comparing the entries in Tables 22.3 and 22.4, it can be seen that the year-over-year monthly fixed base Laspeyres and Paasche price indices do not differ substantially for the early months of the year, but that there are substantial differences between the indices for the last five months of the year by the time the year 1973 is reached. The largest percentage difference between the Laspeyres and Paasche indices is 12.5 per cent for month 10 in 1973 (1.4060/1.2496 = 1.125). However, all the year-over-year monthly series show a smooth year-over-year trend.

22.28 Approximate fixed base year-over-year Laspeyres, Paasche and Fisher indices can be constructed by replacing current month expenditure shares for the five commodities by the corresponding base year monthly expenditure shares on the five commodities. The resulting approximate Laspeyres indices are equal to the original fixed base Laspeyres indices so there is no need to present the approximate Laspeyres indices in a table. The approximate year-over-year Paasche and Fisher indices do, however, differ from the fixed base Paasche and Fisher indices found in Tables 22.4 and 22.5, so these new approximate indices are listed in Tables 22.6 and 22.7.

Table 22.6 Year-over-year approximate monthly fixed base Paasche indices

Year	**Month**										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**1**
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1
1971	1.1077	1.1057	1.1468	1.1478	1.1135	1.0818	1.1062	1.0721	1.0426	1.0760	1
1972	1.2025	1.2421	1.3036	1.2757	1.2110	1.1640	1.2267	1.1567	1.0788	1.1309	1
1973	1.3165	1.3947	1.4880	1.4858	1.3926	1.3223	1.4297	1.3315	1.1920	1.2604	1

Table 22.7 Year-over-year approximate monthly fixed base Fisher indices

Year	**Month**										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**1**
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1
1971	1.1081	1.1063	1.1472	1.1483	1.1147	1.0831	1.1082	1.0752	1.0459	1.0830	1
1972	1.2043	1.2432	1.3049	1.2770	1.2147	1.1687	1.2316	1.1696	1.0918	1.1557	1
1973	1.3223	1.3987	1.4924	1.4888	1.4015	1.3341	1.4428	1.3794	1.2273	1.3312	1

22.29 Comparing Table 22.4 with Table 22.6, it can be seen that, with a few exceptions, the entries correspond fairly closely. One of the bigger differences is the 1973 entry for the fixed base Paasche index for month 9, which is 1.1664, while the corresponding entry for the approximate fixed base Paasche index is 1.1920, for a 2.2 per cent difference (1.1920 /1.1664 = 1.022). In general, the approximate fixed base Paasche indices are somewhat bigger than the true fixed base Paasche indices, as could be expected, since the approximate indices have some substitution bias built into them as their expenditure shares are held fixed at the 1970 levels.

22.30 Turning now to the chained year-over-year monthly indices using the artificial data set, the resulting 12 year-over-year monthly chained Laspeyres, Paasche and Fisher indices, P_L, P_P and P_F, where the month-to-month links are defined by equations (22.4) to (22.6), are listed in Tables 22.8 to 22.10.

Table 22.8 Year-over-year monthly chained Laspeyres indices

Year	**Month**										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**1**
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1

1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1
1972	1.2058	1.2440	1.3058	1.2782	1.2154	1.1720	1.2357	1.1753	1.0975	1.1690	1
1973	1.3274	1.4030	1.4951	1.4911	1.4002	1.3410	1.4522	1.3927	1.2347	1.3593	1

Table 22.9 Year-over-year monthly chained Paasche indices

Year	**Month**										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**1**
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1
1971	1.1074	1.1070	1.1471	1.1486	1.1115	1.0827	1.1075	1.0699	1.0414	1.0762	1
1972	1.2039	1.2437	1.3047	1.2777	1.2074	1.1682	1.2328	1.1569	1.0798	1.1421	1
1973	1.3243	1.4024	1.4934	1.4901	1.3872	1.3346	1.4478	1.3531	1.2018	1.3059	1

Table 22.10 Year-over-year monthly chained Fisher indices

Year	**Month**										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**1**
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1
1971	1.1080	1.1069	1.1474	1.1487	1.1137	1.0835	1.1089	1.0741	1.0453	1.0831	1
1972	1.2048	1.2438	1.3052	1.2780	1.2114	1.1701	1.2343	1.1660	1.0886	1.1555	1
1973	1.3258	1.4027	1.4942	1.4906	1.3937	1.3378	1.4500	1.3728	1.2181	1.3323	1

22.31 Comparing the entries in Tables 22.8 and 22.9, it can be seen that the year-over-year monthly chained Laspeyres and Paasche price indices have smaller differences than the corresponding fixed base Laspeyres and Paasche price indices in Tables 22.3 and 22.4. This is a typical pattern, as found in Chapter 19: the use of chained indices tends to reduce the spread between Paasche and Laspeyres indices compared to their fixed base counterparts. The largest percentage difference between corresponding entries for the chained Laspeyres and Paasche indices in Tables 22.8 and 22.9 is 4.1 per cent for month 10 in 1973 (1.3593/1.3059 = 1.041). Recall that the fixed base Laspeyres and Paasche indices differed by 12.5 per cent for the same month, so that chaining does tend to reduce the spread between these two equally plausible indices.

22.32 The chained year-over-year Fisher indices listed in Table 22.10 are regarded as the "best" estimates of year-over-year inflation using the artificial data set.

22.33 The year-over-year chained Laspeyres, Paasche and Fisher indices listed in Tables 22.8 to 22.10 can be approximated by replacing current period commodity expenditure shares for each month by the corresponding base year monthly commodity expenditure shares. The resulting 12 year-over-year monthly approximate chained Laspeyres, Paasche and Fisher indices, P_{AL}, P_{AP} and P_{AF}, where the monthly links are defined by equations (22.8) to (22.10), are listed in Tables 22.11 to 22.13.

Table 22.11 Year-over-year monthly approximate chained Laspeyres indices

Year	**Month**

	1	2	3	4	5	6	7	8	9	10	1
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1
1972	1.2056	1.2440	1.3057	1.2778	1.2168	1.1712	1.2346	1.1770	1.0989	1.1692	1
1973	1.3255	1.4007	1.4945	1.4902	1.4054	1.3390	1.4491	1.4021	1.2429	1.3611	1

Table 22.12 Year-over-year monthly approximate chained Paasche indices

Year	**Month**										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**1**
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1
1971	1.1077	1.1057	1.1468	1.1478	1.1135	1.0818	1.1062	1.0721	1.0426	1.0760	1
1972	1.2033	1.2424	1.3043	1.2764	1.2130	1.1664	1.2287	1.1638	1.0858	1.1438	1
1973	1.3206	1.3971	1.4914	1.4880	1.3993	1.3309	1.4386	1.3674	1.2183	1.3111	1

Table 22.13 Year-over-year monthly approximate chained Fisher indices

Year	**Month**									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1081	1.1063	1.1472	1.1483	1.1147	1.0831	1.1082	1.0752	1.0459	1.0830
1972	1.2044	1.2432	1.3050	1.2771	1.2149	1.1688	1.2317	1.1704	1.0923	1.1565
1973	1.3231	1.3989	1.4929	1.4891	1.4024	1.3349	1.4438	1.3847	1.2305	1.3358

22.34 The year-over-year chained indices listed in Tables 22.11 to 22.13 approximate their true chained counterparts listed in Tables 22.8 to 22.10 very closely. For the year 1973, the largest discrepancies are for the Paasche and Fisher indices for month 9: the chained Paasche is 1.2018, while the corresponding approximate chained Paasche is 1.2183 for a difference of 1.4 per cent, and the chained Fisher is 1.2181, while the corresponding approximate chained Fisher is 1.2305 for a difference of 1.0 per cent. It can be seen that for the modified Turvey data set, the approximate year-over-year monthly approximate Fisher indices listed in Table 22.13 approximate the theoretically preferred (but in practice unfeasible in a timely fashion) Fisher chained indices listed in Table 22.10 quite satisfactorily. Since the approximate Fisher indices are just as easy to compute as the approximate Laspeyres and Paasche indices, it may be useful to ask that statistical agencies make available to the public these approximate Fisher indices along with the approximate Laspeyres and Paasche indices.

Year-over-year annual indices

22.35 Assuming that each commodity in each season of the year is a separate “annual” commodity is the simplest and theoretically most satisfactory method for dealing with seasonal commodities when the goal is to construct annual price and quantity indices. This idea can be traced back to Bruce D. Mudgett in the consumer price context and to Richard Stone in the producer price context:

The basic index is a yearly index and as a price or quantity index is of the same sort as those about which books and pamphlets have been written in quantity over the years (Mudgett (1955, p. 97)). The existence of a regular seasonal pattern in prices which more or less repeats itself year after year suggests very strongly that the varieties of a commodity available at different seasons cannot be transformed into one another without cost and that, accordingly, in all cases where seasonal variations in price are significant, the varieties available at different times of the year should be treated, in principle, as separate commodities (Stone (1956, pp. 74-75)).

22.36 Using the notation introduced in the previous section, the *Laspeyres, Paasche and Fisher annual (chain link) indices* comparing the prices of year t with those of year t+1 can be defined as follows:

$$P_L\left(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};q^{t,1},...,q^{t,12}\right)\equiv\frac{\sum_{m=1}^{12}\sum_{n\in S(m)}p_n^{t+1,m}q_n^{t,m}}{\sum_{m=1}^{12}\sum_{n\in S(m)}p_n^{t,m}q_n^{t,m}} \quad (22.13)$$

$$P_P\left(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};q^{t+1,1},...,q^{t+1,12}\right)\equiv\frac{\sum_{m=1}^{12}\sum_{n\in S(m)}p_n^{t+1,m}q_n^{t+1,m}}{\sum_{m=1}^{12}\sum_{n\in S(m)}p_n^{t,m}q_n^{t+1,m}} \quad (22.14)$$

$$P_L\left(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};q^{t,1},...,q^{t,12};q^{t+1,1},...,q^{t+1,12}\right)\equiv$$
$$\sqrt{P_L\left(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};q^{t,1},...,q^{t,12}\right)}\times\sqrt{P_P\left(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};q^{t+1,1},...,q^{t+1,12}\right)}$$
(22.15)

22.37 The above formulae can be rewritten in price relative and monthly expenditure share form as follows:

$$P_L\left(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};\sigma_1^t s^{t,1},...,\sigma_{12}^t s^{t,12}\right)$$
$$\equiv\sum_{m=1}^{12}\sum_{n\in S(m)}\sigma_m^t s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right) \quad (22.16)$$
$$=\sum_{m=1}^{12}\sigma_m^t P_L\left(p^{t,m},p^{t+1,m},s^{t,m}\right)$$

$$P_P\left(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};\sigma_1^{t+1}s^{t+1,1},...,\sigma_{12}^{t+1}s^{t+1,12}\right)$$
$$\equiv\left[\sum_{m=1}^{12}\sum_{n\in S(m)}\sigma_m^{t+1}s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1} \quad (22.17)$$
$$=\left[\sum_{m=1}^{12}\sigma_m^{t+1}\sum_{n\in S(m)}s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}$$
$$=\left[\sum_{m=1}^{12}\sigma_m^{t+1}\left[P_P\left(p^{t,m},p^{t+1,m},s^{t+1,m}\right)\right]^{-1}\right]^{-1}$$

$$P_F\left(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};\sigma_1^t s^{t,1},...,\sigma_{12}^t s^{t,12};\sigma_1^{t+1}s^{t+1,1},...,\sigma_{12}^{t+1}s^{t+1,12}\right)$$
$$\equiv\sqrt{\sum_{m=1}^{12}\sum_{n\in S(m)}\sigma_m^t s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)\left[\sum_{m=1}^{12}\sum_{n\in S(m)}\sigma_m^{t+1}s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}} \quad (22.18)$$

$$= \sqrt{\sum_{m=1}^{12} \sigma_m^t \left[P_L\left(p^{t,m}, p^{t+1,m}, s^{t,m}\right)\right] \left[\sum_{m=1}^{12} \sigma_m^{t+1} \left[P_P\left(p^{t,m}, p^{t+1,m}, s^{t+1,m}\right)\right]^{-1}\right]^{-1}}$$

where the expenditure share for month m in year t is defined as:

$$\sigma_m^t = \frac{\sum_{n=S(m)} p_n^{t,m} q_n^{t,m}}{\sum_{i=1}^{12} \sum_{j \in S(i)} p_j^{t,i} p_j^{t,i}} \quad m = 1,2,...,12;\ t = 0,1,...,T \qquad (22.19)$$

and the year-over-year monthly Laspeyres and Paasche (chain link) price indices $P_L(p^{t,m},p^{t+1,m},s^{t,m})$ and $P_P(p^{t,m},p^{t+1,m},s^{t+1,m})$ are defined by equations (22.4) and (22.5), respectively. As usual, the annual chain link Fisher index P_F defined by equation (22.18), which compares the prices in every month of year t with the corresponding prices in year t+1, is the geometric mean of the annual chain link Laspeyres and Paasche indices, P_L and P_P, defined by equations (22.16) and (22.17). The last equations in (22.16), (22.17) and (22.18) show that these annual indices can be defined as (monthly) share-weighted averages of the year-over-year monthly chain link Laspeyres and Paasche indices, $P_L(p^{t,m},p^{t+1,m},s^{t,m})$ and $P_P(p^{t,m},p^{t+1,m},s^{t+1,m})$, defined by equations (22.4) and (22.5). Hence once the year-over-year monthly indices defined above have been calculated numerically, it is easy to calculate the corresponding annual indices.

22.38 Fixed base counterparts to the formulae defined by equations (22.16) to (22.18) can readily be defined: simply replace the data pertaining to period t by the corresponding data pertaining to the base period 0.

22.39 The annual fixed base Laspeyres, Paasche and Fisher indices, as calculated using the data from the artificial data set tabled in paragraphs 22.14 and 22.15, are listed in Table 22.14.

Table 22.14 Annual fixed base Laspeyres, Paasche and Fisher price indices

Year	P_L	P_P	P_F
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0961	1.0984
1972	1.2091	1.1884	1.1987
1973	1.4144	1.3536	1.3837

Table 22.14 shows that by 1973, the annual fixed base Laspeyres index exceeds its Paasche counterpart by 4.5 per cent. Note that each series increases steadily.

22.40 The annual fixed base Laspeyres, Paasche and Fisher indices can be approximated by replacing any current shares by the corresponding base year shares. The resulting annual approximate fixed base Laspeyres, Paasche and Fisher indices are listed in Table 22.15. Also listed in the last column of Table 22.15 is the fixed base geometric Laspeyres annual index, P_{GL}. This is the weighted geometric mean counterpart to the fixed base Laspeyres index, which is equal to a base period weighted arithmetic average of the long-term price relatives; see Chapter 19. It can be shown that P_{GL} approximates the approximate fixed base Fisher index, P_{AF}, to the second order around a point where all the long-term price relatives are equal to unity.[14] It can be seen that the entries for the Laspeyres price indices are exactly the same in Tables 22.14 and 22.15. This is as it should be, because the fixed base Laspeyres price index uses only expenditure shares from the base year 1970; hence the approximate fixed base Laspeyres index is equal to the true fixed base Laspeyres index. Comparing the columns labelled P_P and P_F in Table 22.14 with the columns P_{AP} and P_{AF} in Table 22.15 shows that the approximate Paasche and approximate Fisher indices are quite close to the corresponding annual Paasche and Fisher indices. Hence, for the artificial data set, the true annual fixed base Fisher index can be very closely approximated by the corresponding approximate Fisher index, P_{AF} (or the geometric Laspeyres index, P_{GL}), which, of course, can be computed using the same information set that is normally available to statistical agencies.

Table 22.15 Annual approximate fixed base Laspeyres, Paasche, Fisher and geometric Laspeyres indices

Year	P_{AL}	P_{AP}	P_{AF}	P_{GL}
1970	1.0000	1.0000	1.0000	1.0000
1971	1.1008	1.0956	1.0982 1	.0983
1972	1.2091	1.1903	1.1996	1.2003
1973	1.4144	1.3596	1.3867	1.3898

22.41 Using the artificial data set in Tables 22.1 and 22.2, the annual chained Laspeyres, Paasche and Fisher indices can readily be calculated, using the formulae (22.16) to (22.18) for the chain links. The resulting indices are listed in Table 22.16, which shows that the use of chained indices has substantially narrowed the gap between the Paasche and Laspeyres indices. The difference between the chained annual Laspeyres and Paasche indices in 1973 is only 1.5 per cent (1.3994 versus 1.3791), whereas from Table 22.14, the difference between the fixed base annual Laspeyres and Paasche indices in 1973 is 4.5 per cent (1.4144 versus 1.3536). Thus the use of chained annual indices has substantially reduced the substitution (or representativity) bias of the Laspeyres and Paasche indices. Comparing Tables 22.14 and 22.16, it can be seen that for this particular artificial data set, the annual fixed base Fisher indices are very close to their annual chained Fisher counterparts. The annual chained Fisher indices should, however, normally be regarded as the more desirable target index to approximate, since this index will normally give better results if prices and expenditure shares are changing substantially over time.[15]

Table 22.16 Annual chained Laspeyres, Paasche and Fisher price indices

[14] See footnote 12.

[15] The gap between the Laspeyres and Paasche indices will be normally be reduced using chained indices under these circumstances. Of course, if there are no substantial trends in prices, so that prices are just changing randomly, then it will generally be preferable to use the fixed base Fisher index.

Year	P_L	P_P	P_F
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0961	1.0984
1972	1.2052	1.1949	1.2001
1973	1.3994	1.3791	1.3892

22.42 Obviously, the current year weights, $s_n^{t,m}$ and σ_m^t and $s_n^{t+1,m}$ and σ_m^{t+1}, which appear in the chain link formulae (22.16) to (22.18), can be approximated by the corresponding base year weights, $s_n^{0,m}$ and σ_m^0. This leads to the annual approximate chained Laspeyres, Paasche and Fisher indices listed in Table 22.17.

Table 22.17 Annual approximate chained Laspeyres, Paasche and Fisher price indices

Year	P_{AL}	P_{AP}	P_{AF}
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0956	1.0982
1972	1.2051	1.1952	1.2002
1973	1.3995	1.3794	1.3894

22.43 Comparing the entries in Tables 22.16 and 22.17 shows that the approximate chained annual Laspeyres, Paasche and Fisher indices are extremely close to the corresponding true chained annual Laspeyres, Paasche and Fisher indices. Hence, for the artificial data set, the true annual chained Fisher index can be very closely approximated by the corresponding approximate Fisher index, which can be computed using the same information set that is normally available to statistical agencies.

22.44 The approach to computing annual indices outlined in this section, which essentially involves taking monthly expenditure share-weighted averages of the 12 year-over-year monthly indices, should be contrasted with the approach that simply takes the arithmetic mean of the 12 monthly indices. The problem with the latter approach is that months where expenditures are below the average (e.g., February) are given the same weight in the unweighted annual average as months where expenditures are above the average (e.g., December).

Rolling year annual indices

22.45 In the previous section, the price and quantity data pertaining to the 12 months of a calendar year were compared to the 12 months of a base calendar year. There is, however, no need to restrict attention to calendar-year comparisons: any 12 consecutive months of price and quantity data could be compared to the price and quantity data of the base year, provided that the January data in the non-calendar year are compared to the January data of the base year, the February data of the non-calendar year are compared to the February data of the base year, and so on, up to the December data of the non-calendar year being compared to the

December data of the base year.[16] Alterman, Diewert and Feenstra (1999, p. 70) called the resulting indices *rolling year* or *moving year* indices.[17]

22.46 In order to theoretically justify the rolling year indices from the viewpoint of the economic approach to index number theory, some restrictions on preferences are required. The details of these assumptions can be found in Diewert (1996b, pp. 32-34; 1999a, pp. 56-61).

22.47 The problems involved in constructing rolling year indices for the artificial data set are now considered. For both fixed base and chained rolling year indices, the first 13 index number calculations are the same. For the year that ends with the data for December of 1970, the index is set equal to 1 for the Laspeyres, Paasche and Fisher moving year indices. The base year data are the 44 non-zero price and quantity observations for the calendar year 1970. When the data for January 1971 become available, the three non-zero price and quantity entries for January of calendar year 1970 are dropped and replaced by the corresponding entries for January 1971. The data for the remaining months of the comparison year remain the same; i.e., for February to December of the comparison year, the data for the rolling year are set equal to the corresponding entries for February to December 1970. Thus the Laspeyres, Paasche or Fisher rolling year index value for January 1971 compares the prices and quantities of January 1971 with the corresponding prices and quantities of January 1970. For the remaining months of this first moving year, the prices and quantities of February to December 1970 are simply compared with exactly the same prices and quantities of February to December 1970. When the data for February 1971 become available, the three non-zero price and quantity entries for February for the last rolling year (which are equal to the three non-zero price and quantity entries for February 1970) are dropped and replaced by the corresponding entries for February 1971. The resulting data become the price and quantity data for the second rolling year. The Laspeyres, Paasche or Fisher rolling year index value for February 1971 compares the prices and quantities of January and February 1971 with the corresponding prices and quantities of January and February 1970. For the remaining months of this first moving year, the prices and quantities of March to December 1970 are compared with exactly the same prices and quantities of March to December 1970. This process of exchanging the price and quantity data of the current month in 1971 with the corresponding data of the same month in the base year 1970 in order to form the price and quantity data for the latest rolling year continues until December 1971 is reached, when the current rolling year becomes the calendar year 1971. Thus the Laspeyres, Paasche and Fisher rolling year indices for December 1971 are equal to the corresponding fixed base (or chained) annual Laspeyres, Paasche and Fisher indices for 1971, listed in Tables 22.14 or 22.16.

22.48 Once the first 13 entries for the rolling year indices have been defined as indicated above, the remaining fixed base rolling year Laspeyres, Paasche and Fisher indices are constructed by taking the price and quantity data of the last 12 months and rearranging the

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[16] Diewert (1983c) suggested this type of comparison and termed the resulting index a "split year" comparison.

[17] Crump (1924, p. 185) and Mendershausen (1937, p. 245), respectively, used these terms in the context of various seasonal adjustment procedures. The term "rolling year" seems to be well established in the business literature in the United Kingdom.

data so that the January data in the rolling year are compared to the January data in the base year, the February data in the rolling year are compared to the February data in the base year, and so on, up to the December data in the rolling year being compared to the December data in the base year. The resulting fixed base rolling year Laspeyres, Paasche and Fisher indices for the artificial data set are listed in Table 22.18.

22.49 Once the first 13 entries for the fixed base rolling year indices have been defined as indicated above, the remaining chained rolling year Laspeyres, Paasche and Fisher indices are constructed by taking the price and quantity data of the last 12 months and comparing these data to the corresponding data of the rolling year of the 12 months preceding the current rolling year. The resulting chained rolling year Laspeyres, Paasche and Fisher indices for the artificial data set are listed in the last three columns of Table 22.18. Note that the first 13 entries of the fixed base Laspeyres, Paasche and Fisher indices are equal to the corresponding entries for the chained Laspeyres, Paasche and Fisher indices. It will also be noted that the entries for December (month 12) of 1970, 1971, 1972 and 1973 for the fixed base rolling year Laspeyres, Paasche and Fisher indices are equal to the corresponding fixed base annual Laspeyres, Paasche and Fisher indices listed in Table 22.14. Similarly, the entries in Table 22.18 for December (month 12) of 1970, 1971, 1972 and 1973 for the chained rolling year Laspeyres, Paasche and Fisher indices are equal to the corresponding chained annual Laspeyres, Paasche and Fisher indices listed in Table 22.16.

Table 22.18 Rolling year Laspeyres, Paasche and Fisher price indices

Year	**Month**	P_L **(fixed)**	P_P **(fixed)**	P_F **(fixed)**	P_L **(chain)**	P_P **(chain)**	P_F **(chain)**
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0082	1.0087	1.0085	1.0082	1.0087	1.0085
	2	1.0161	1.0170	1.0165	1.0161	1.0170	1.0165
	3	1.0257	1.0274	1.0265	1.0257	1.0274	1.0265
	4	1.0344	1.0364	1.0354	1.0344	1.0364	1.0354
	5	1.0427	1.0448	1.0438	1.0427	1.0448	1.0438
	6	1.0516	1.0537	1.0527	1.0516	1.0537	1.0527
	7	1.0617	1.0635	1.0626	1.0617	1.0635	1.0626
	8	1.0701	1.0706	1.0704	1.0701	1.0706	1.0704
	9	1.0750	1.0740	1.0745	1.0750	1.0740	1.0745
	10	1.0818	1.0792	1.0805	1.0818	1.0792	1.0805
	11	1.0937	1.0901	1.0919	1.0937	1.0901	1.0919
	12	1.1008	1.0961	1.0984	1.1008	1.0961	1.0984
1972	1	1.1082	1.1035	1.1058	1.1081	1.1040	1.1061
	2	1.1183	1.1137	1.1160	1.1183	1.1147	1.1165
	3	1.1287	1.1246	1.1266	1.1290	1.1260	1.1275
	4	1.1362	1.1324	1.1343	1.1366	1.1342	1.1354

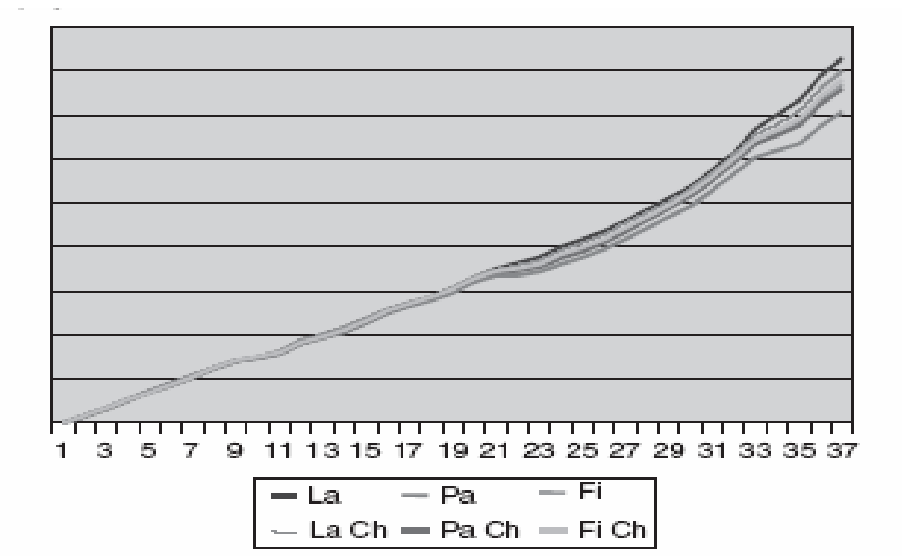
Year	Month	P_L (fixed)	P_P (fixed)	P_F (fixed)	P_L (chain)	P_P (chain)	P_F (chain)
	5	1.1436	1.1393	1.1414	1.1437	1.1415	1.1426
	6	1.1530	1.1481	1.1505	1.1528	1.1505	1.1517
	7	1.1645	1.1595	1.1620	1.1644	1.1622	1.1633
	8	1.1757	1.1670	1.1713	1.1747	1.1709	1.1728
	9	1.1812	1.1680	1.1746	1.1787	1.1730	1.1758
	10	1.1881	1.1712	1.1796	1.1845	1.1771	1.1808
	11	1.1999	1.1805	1.1901	1.1962	1.1869	1.1915
	12	1.2091	1.1884	1.1987	1.2052	1.1949	1.2001
1973	1	1.2184	1.1971	1.2077	1.2143	1.2047	1.2095
	2	1.2300	1.2086	1.2193	1.2263	1.2172	1.2218
	3	1.2425	1.2216	1.2320	1.2393	1.2310	1.2352
	4	1.2549	1.2341	1.2444	1.2520	1.2442	1.2481
	5	1.2687	1.2469	1.2578	1.2656	1.2579	1.2617
	6	1.2870	1.2643	1.2756	1.2835	1.2758	1.2797
	7	1.3070	1.2843	1.2956	1.3038	1.2961	1.3000
	8	1.3336	1.3020	1.3177	1.3273	1.3169	1.3221
	9	1.3492	1.3089	1.3289	1.3395	1.3268	1.3331
	10	1.3663	1.3172	1.3415	1.3537	1.3384	1.3460
	11	1.3932	1.3366	1.3646	1.3793	1.3609	1.3700
	12	1.4144	1.3536	1.3837	1.3994	1.3791	1.3892

22.50 Table 22.18 shows that the rolling year indices are very smooth and free from seasonal fluctuations. For the fixed base indices, each entry can be viewed as a *seasonally adjusted annual consumer price index* that compares the data of the 12 consecutive months that end with the year and month indicated with the corresponding price and quantity data of the 12 months in the base year, 1970. Thus rolling year indices offer statistical agencies an objective and reproducible method of seasonal adjustment that can compete with existing time series methods of seasonal adjustment.[18]

[18] For discussions on the merits of econometric or time series methods versus index number methods of seasonal adjustment, see Diewert (1999a, pp. 61-68) and Alterman, Diewert and Feenstra (1999,
pp. 78-110). The basic problem with time series methods of seasonal adjustment is that the target seasonally adjusted index is very difficult to specify in an unambiguous way; i.e., there are an infinite number of possible target indices. For example, it is impossible to identify a temporary increase in inflation within a year from a changing seasonal factor. Hence different econometricians will tend to generate different seasonally adjusted series, leading to a lack of reproducibility.

22.51 Table 22.18 shows that the use of chained indices has substantially narrowed the gap between the fixed base moving year Paasche and Laspeyres indices. The difference between the rolling year chained Laspeyres and Paasche indices in December 1973 is only 1.5 per cent (1.3994 versus 1.3791), whereas the difference between the rolling year fixed base Laspeyres and Paasche indices in December 1973 is 4.5 per cent (1.4144 versus 1.3536). Thus, the use of chained indices has substantially reduced the substitution (or representativity) bias of the Laspeyres and Paasche indices. As in the previous section, the chained Fisher rolling year index is regarded as the target seasonally adjusted annual index when seasonal commodities are in the scope of the CPI. This type of index is also a suitable index for central banks to use for inflation targeting purposes.[19] The six series in Table 22.18 are charted in Figure 22.1. The fixed base Laspeyres index is the highest one, followed by the chained Laspeyres, the two Fisher indices (which are virtually indistinguishable), and the chained Paasche. Finally, the fixed base Paasche is the lowest index. An increase in the slope of each graph can clearly be seen for the last eight months, reflecting the increase in the month-to-month inflation rates that was built into the data for the last 12 months of the data set.[20]

Figure 22.1 Rolling year fixed base and chained Laspeyres, Paasche and Fisher indices



22.52 As in the previous section, the current year weights, $s_n^{t,m}$ and σ_m^{t} and $s_n^{t+1,m}$ and σ_m^{t+1}, which appear in the chain link formulae (22.16) to (22.18) or in the corresponding fixed base formulae, can be approximated by the corresponding base year weights, $s_n^{0,m}$ and σ_m^{0}. This

[19] See Diewert (2002c) for a discussion of the measurement issues involved in choosing such an index.

[20] The arithmetic average of the 36 month-over-month inflation rates for the rolling year fixed base Fisher indices is 1.0091; the average of these rates for the first 24 months is 1.0076, for the last 12 months is 1.0120 and for the last 2 months is 1.0156. Hence, the increased month-to-month inflation rates for the last year are not fully reflected in the rolling year indices until a full 12 months have passed. However, the fact that inflation has increased for the last 12 months of data compared to the earlier months is picked up almost immediately.

leads to the annual approximate fixed base and chained rolling year Laspeyres, Paasche and Fisher indices listed in Table 22.19.

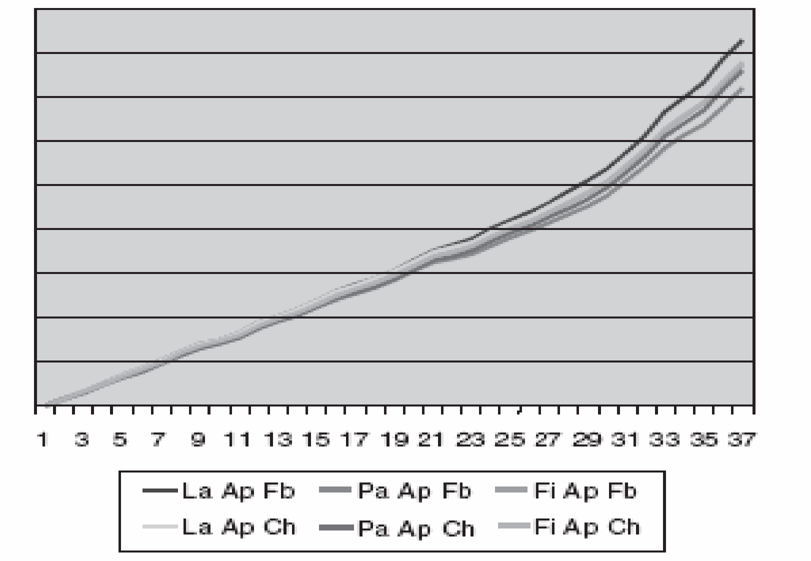
Table 22.19 Rolling year approximate Laspeyres, Paasche and Fisher price indices

Year	**Month**	P_{AL} **(fixed)**	P_{AP} **(fixed)**	P_{AF} **(fixed)**	P_{AL} **(chain)**	P_{AP} **(chain)**	P_{AF} **(chain)**
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0082	1.0074	1.0078	1.0082	1.0074	1.0078
	2	1.0161	1.0146	1.0153	1.0161	1.0146	1.0153
	3	1.0257	1.0233	1.0245	1.0257	1.0233	1.0245
	4	1.0344	1.0312	1.0328	1.0344	1.0312	1.0328
	5	1.0427	1.0390	1.0409	1.0427	1.0390	1.0409
	6	1.0516	1.0478	1.0497	1.0516	1.0478	1.0497
	7	1.0617	1.0574	1.0596	1.0617	1.0574	1.0596
	8	1.0701	1.0656	1.0679	1.0701	1.0656	1.0679
	9	1.0750	1.0702	1.0726	1.0750	1.0702	1.0726
	10	1.0818	1.0764	1.0791	1.0818	1.0764	1.0791
	11	1.0937	1.0881	1.0909	1.0937	1.0881	1.0909
	12	1.1008	1.0956	1.0982	1.1008	1.0956	1.0982
1972	1	1.1082	1.1021	1.1051	1.1083	1.1021	1.1052
	2	1.1183	1.1110	1.1147	1.1182	1.1112	1.1147
	3	1.1287	1.1196	1.1241	1.1281	1.1202	1.1241
	4	1.1362	1.1260	1.1310	1.1354	1.1268	1.1311
	5	1.1436	1.1326	1.1381	1.1427	1.1336	1.1381
	6	1.1530	1.1415	1.1472	1.1520	1.1427	1.1473
	7	1.1645	1.1522	1.1583	1.1632	1.1537	1.1584
	8	1.1757	1.1620	1.1689	1.1739	1.1642	1.1691
	9	1.1812	1.1663	1.1737	1.1791	1.1691	1.1741
	10	1.1881	1.1710	1.1795	1.1851	1.1747	1.1799
	11	1.1999	1.1807	1.1902	1.1959	1.1855	1.1907
	12	1.2091	1.1903	1.1996	1.2051	1.1952	1.2002
1973	1	1.2184	1.1980	1.2082	1.2142	1.2033	1.2087
	2	1.2300	1.2074	1.2187	1.2253	1.2133	1.2193
	3	1.2425	1.2165	1.2295	1.2367	1.2235	1.2301
	4	1.2549	1.2261	1.2404	1.2482	1.2340	1.2411
	5	1.2687	1.2379	1.2532	1.2615	1.2464	1.2540

Year	Month	P_{AL} (fixed)	P_{AP} (fixed)	P_{AF} (fixed)	P_{AL} (chain)	P_{AP} (chain)	P_{AF} (chain)
	6	1.2870	1.2548	1.2708	1.2795	1.2640	1.2717
	7	1.3070	1.2716	1.2892	1.2985	1.2821	1.2903
	8	1.3336	1.2918	1.3125	1.3232	1.3048	1.3139
	9	1.3492	1.3063	1.3276	1.3386	1.3203	1.3294
	10	1.3663	1.3182	1.3421	1.3538	1.3345	1.3441
	11	1.3932	1.3387	1.3657	1.3782	1.3579	1.3680
	12	1.4144	1.3596	1.3867	1.3995	1.3794	1.3894

22.53 Comparing the indices in Tables 22.18 and 22.19, it can be seen that the approximate rolling year fixed base and chained Laspeyres, Paasche and Fisher indices listed in Table 22.19 are very close to their true rolling year counterparts listed in Table 22.18. In particular, the approximate chain rolling year Fisher index (which can be computed using just base year expenditure share information, along with current information on prices) is very close to the preferred target index, the rolling year chained Fisher index. In December 1973, these two indices differ by only 0.014 per cent (1.3894/1.3892 = 1.00014). The indices in Table 22.19 are charted in Figure 22.2. It can be seen that Figures 22.1 and 22.2 are very similar; in particular, the Fisher fixed base and chained indices are virtually identical in both figures.

Figure 22.2 Rolling year approximate fixed base and chained Laspeyres, Paasche and Fisher indices



22.54 From the above tables, it can be seen that year-over-year monthly indices and their generalizations to rolling year indices perform very well using the modified Turvey data set; like is compared to like, and the existence of seasonal commodities does not lead to erratic fluctuations in the indices. The only drawback to the use of these indices is that it seems that they cannot give any information on short-term, month-to-month fluctuations in prices. This is most evident if seasonal baskets are totally different for each month since in this case, there is no possibility of comparing prices on a month-to-month basis. In the following section, it is shown how a current period year-over-year monthly index can be used to predict a rolling year index that is centered on the current month.

Predicting a rolling year index using a current period year-over-year monthly index

22.55 It might be conjectured that under a regime where the long-run trend in prices is smooth, changes in the year-over-year inflation rate for a particular month compared to the previous month could give valuable information about the long-run trend in price inflation. For the modified Turvey data set, this conjecture turns out to be true, as seen below.

22.56 The basic idea is illustrated using the fixed base Laspeyres rolling year indices listed in Table 22.18 and the year-over-year monthly fixed base Laspeyres indices listed in Table 22.3. In Table 22.18, the fixed base Laspeyres rolling year entry for December of 1971 compares the 12 months of price and quantity data pertaining to 1971 with the corresponding prices and quantities pertaining to 1970. This index number, P_L, is the first entry in Table 22.20. Thus the P_{LRY} column of Table 22.20 shows the fixed base rolling year Laspeyres index, taken from Table 22.18, starting at December 1971 and carrying through to December 1973, which is 24 observations in all. Looking at the first entry of this column, it can be seen that the index is a weighted average of year-over-year price relatives over all 12 months in 1970 and 1971. Thus this index is an average of year-over-year monthly price changes, centered between June and July of the two years for which prices are being compared. Hence, an approximation to this annual index could be obtained by taking the arithmetic average of the June and July year-over-year monthly indices pertaining to the years 1970 and 1971 (see the entries for months 6 and 7 for the year 1971 in Table 22.3, 1.0844 and 1.1103).[21] The next rolling year fixed base Laspeyres index corresponds to the January 1972 entry in Table 22.18. An approximation to this rolling year index, P_{ARY}, could be obtained by taking the arithmetic average of the July and August year-over-year monthly indices pertaining to the years 1970 and 1971 (see the entries for months 7 and 8 for the year 1971 in Table 22.3, 1.1103 and 1.0783). These arithmetic averages of the two year-over-year monthly indices that are in the middle of the corresponding rolling year are listed in the P_{ARY} column of Table 22.20. From Table 22.20, it can be seen that the P_{ARY} column does not approximate the P_{LRY} column particularly well, since the approximate indices in the P_{ARY} column are seen to have some pronounced seasonal fluctuations, whereas the rolling year indices in the P_{LRY} column are free from seasonal fluctuations.

22.57 Some seasonal adjustment factors (*SAF*) are listed in Table 22.20. For the first 12 observations, the entries in the *SAF* column are simply the ratios of the entries in the P_{LRY} column, divided by the corresponding entries in the P_{ARY} column; i.e., for the first 12 observations, the seasonal adjustment factors are simply the ratio of the rolling year indices starting at December 1971, divided by the arithmetic average of the two year-over-year monthly indices that are in the middle of the corresponding rolling year.[22] The initial 12

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[21] Obviously, if an average of the year-over-year monthly indices for May, June, July and August were taken, a better approximation to the annual index could be obtained, and if an average of the year-over-year monthly indices for April, May, June, July, August and September were taken, an even better approximation could be obtained to the annual index, and so on.

[22] Thus if *SAF* is greater than one, this means that the two months in the middle of the corresponding rolling year have year-over-year rates of price increase that average out to a

(continued)

seasonal adjustment factors are then just repeated for the remaining entries for the *SAF* column.

22.58 Once the seasonal adjustment factors have been defined, then the approximate rolling year index P_{ARY} can be multiplied by the corresponding seasonal adjustment factor *SAF* in order to form a *seasonally adjusted approximate rolling year index*, P_{SAARY}, as listed in Table 22.20.

Table 22.20 Rolling year fixed base Laspeyres and seasonally adjusted approximate rolling year price indices

Year	**Month**	P_{LRY}	P_{SAARY}	P_{ARY}	***SAF***
1971	12	1.1008	1.1008	1.0973	1.0032
1972	1	1.1082	1.1082	1.0943	1.0127
	2	1.1183	1.1183	1.0638	1.0512
	3	1.1287	1.1287	1.0696	1.0552
	4	1.1362	1.1362	1.1092	1.0243
	5	1.1436	1.1436	1.1066	1.0334
	6	1.1530	1.1530	1.1454	1.0066
	7	1.1645	1.1645	1.2251	0.9505
	8	1.1757	1.1757	1.2752	0.9220
	9	1.1812	1.1812	1.2923	0.9141
	10	1.1881	1.1881	1.2484	0.9517
	11	1.1999	1.1999	1.1959	1.0033
	12	1.2091	1.2087	1.2049	1.0032
1973	1	1.2184	1.2249	1.2096	1.0127
	2	1.2300	1.2024	1.1438	1.0512
	3	1.2425	1.2060	1.1429	1.0552
	4	1.2549	1.2475	1.2179	1.0243
	5	1.2687	1.2664	1.2255	1.0334
	6	1.2870	1.2704	1.2620	1.0066
	7	1.3070	1.2979	1.3655	0.9505
	8	1.3336	1.3367	1.4498	0.9220
	9	1.3492	1.3658	1.4943	0.9141

number below the overall average of the year-over-year rates of price increase for the entire rolling year, and above the overall average if *SAF* is less than one.

Year	Month	P_{LRY}	P_{SAARY}	P_{ARY}	*SAF*
	10	1.3663	1.3811	1.4511	0.9517
	11	1.3932	1.3828	1.3783	1.0033
	12	1.4144	1.4055	1.4010	1.0032

22.59 Comparing the P_{LRY} and P_{SAARY} columns in Table 22.20, the rolling year fixed base Laspeyres index, P_{LRY}, and the seasonally adjusted approximate rolling year index, P_{SAARY}, are identical for the first 12 observations, which follows by construction since P_{SAARY} equals the approximate rolling year index, P_{ARY}, multiplied by the seasonal adjustment factor *SAF* which in turn is equal to the rolling year Laspeyres index, P_{LRY}, divided by P_{ARY}. However, starting at December 1972, the rolling year index, P_{LRY}, differs from the corresponding seasonally adjusted approximate rolling year index, P_{SAARY}. It can be seen that for these last 13 months, P_{SAARY} is surprisingly close to P_{LRY}.[23]

Figure 22.3 Fixed base Laspeyres, seasonally adjusted approximate and approximate rolling year indices

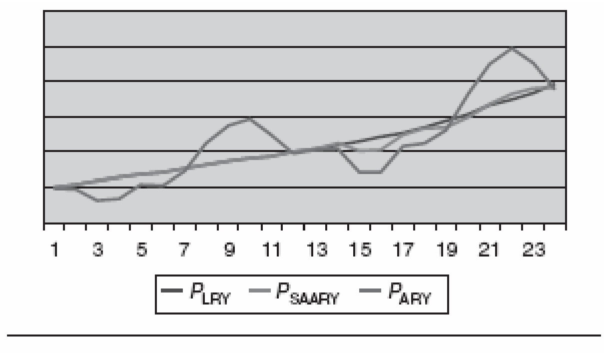

Figure 22.3 shows P_{LRY}, P_{SAARY} and P_{ARY} graphically. Because of the acceleration in the monthly inflation rate for the last year of data, it can be seen that the seasonally adjusted approximate rolling year series, P_{SAARY}, does not pick up this accelerated inflation rate for the first few months of the last year (it lies well below P_{LRY} for February and March 1973) but, in general, it predicts the corresponding centered year quite well.

22.60 The above results for the modified Turvey data set are quite encouraging. If these results can be replicated for other data sets, then it means that statistical agencies can use the latest information on year-over-year monthly inflation to predict reasonably well the (seasonally adjusted) rolling year inflation rate for a rolling year that is centered around the last two months. Thus policy-makers and other interested users of the CPI can obtain a reasonably accurate forecast of trend inflation (centered around the current month) some six months in advance of the final estimates being calculated.

[23] The means for the last 13 observations in columns P_{LRY} and P_{ARY} of Table 22.20 are 1.2980 and 1.2930. A regression of P_L on P_{SAARY} leads to an R^2 of 0.9662 with an estimated variance of the residual of .000214.

22.61 The method of seasonal adjustment used in this section is rather crude compared to some of the sophisticated econometric or statistical methods that are available. Thus, these more sophisticated methods could be used in order to improve the forecasts of trend inflation. If improved forecasting methods are used, however, it is useful to use the rolling year indices as targets for the forecasts, rather than using a statistical package that simultaneously seasonally adjusts current data and calculates a trend rate of inflation. What is being suggested here is that the rolling year concept can be used in order to eliminate the lack of reproducibility in the estimates of trend inflation that existing statistical methods of seasonal adjustment generate.[24]

22.62 In this section and the previous ones, all the suggested indices have been based on year-over-year monthly indices and their averages. In the subsequent sections of this chapter, attention will be turned to more traditional price indices that attempt to compare the prices in the current month with the prices in a previous month.

Maximum overlap month-to-month price indices

22.63 A reasonable method for dealing with seasonal commodities in the context of picking a target index for a month-to-month CPI is the following:[25]

- Determine the set of commodities that is present in the marketplace in both months of the comparison.
- For this maximum overlap set of commodities, calculate one of the three indices recommended in previous chapters; i.e., calculate the Fisher, Walsh or Törnqvist–Theil index.[26]

Thus the bilateral index number formula is applied only to the subset of commodities that is present in both periods.[27]

22.64 The question now arises: should the comparison month and the base month be adjacent months (thus leading to chained indices) or should the base month be fixed (leading to fixed base indices)? It seems reasonable to prefer chained indices over fixed base indices for two reasons:

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[24] The operator of a statistical seasonal adjustment package has to make somewhat arbitrary decisions on many factors. For example, are the seasonal factors additive or multiplicative? How long should the moving average be and what type of average should be calculated? Thus different operators of the seasonal adjustment package will tend to produce different estimates of the trend and the seasonal factors.

[25] For more on the economic approach and the assumptions on consumer preferences that can justify month-to-month maximum overlap indices, see Diewert (1999a, pp. 51-56).

[26] For simplicity, only the Fisher index is considered in detail in this chapter.

[27] Keynes (1930, p. 95) called this the highest common factor method for making bilateral index number comparisons. Of course, this target index drops those strongly seasonal commodities that are not present in the marketplace during one of the two months being compared. Thus the index number comparison is not completely comprehensive. Mudgett (1955, p. 46) called the "error" in an index number comparison that is introduced by the highest common factor method (or maximum overlap method) the "homogeneity error".

- The set of seasonal commodities which overlaps during two consecutive months is likely to be much larger than the set obtained by comparing the prices of any given month with a fixed base month (such as January of a base year). Hence the comparisons made using chained indices will be more comprehensive and accurate than those made using a fixed base.
- In many economies, on average 2 or 3 per cent of price quotes disappear each month because of the introduction of new commodities and the disappearance of older ones. This rapid sample attrition means that fixed base indices rapidly become unrepresentative. Hence it seems preferable to use chained indices which can more closely follow marketplace developments.[28]

22.65 It will be useful to review the notation at this point and define some new notation. Let there be N commodities that are available in some month of some year and let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of commodity n that is in the marketplace [29] in month m of year t (if the commodity is unavailable, define $p_n^{t,m}$ and $q_n^{t,m}$ to be 0). Let $p^{t,m} \equiv [p_1^{t,m},p_2^{t,m},...,p_N^{t,m}]$ and $q^{t,m} \equiv [q_1^{t,m},q_2^{t,m},...,q_N^{t,m}]$ be the month m and year t price and quantity vectors, respectively. Let $S(t,m)$ be the set of commodities that is present in month m of year t and the following month. Then the maximum overlap Laspeyres, Paasche and Fisher indices going from month m of year t to the following month can be defined as follows:[30]

$$P_L\left(p^{t,m},p^{t,m+1},q^{t,m},S(t,m)\right)=\frac{\sum_{n\in S(t,m)} p_n^{t,m+1}q_n^{t,m}}{\sum_{n\in S(t,m)} p_n^{t,m}q_n^{t,m}} \qquad m=1,2,\ldots 11 \qquad (22.20)$$

$$P_P\left(p^{t,m},p^{t,m+1},q^{t,m+1},S(t,m)\right)=\frac{\sum_{n\in S(t,m)} p_n^{t,m+1}q_n^{t,m+1}}{\sum_{n\in S(t,m)} p_n^{t,m}q_n^{t,m+1}} \qquad m=1,2,\ldots 11 \qquad (22.21)$$

$$P_F\left(p^{t,m},p^{t,m+1},q^{t,m},q^{t,m+1},S(t,m)\right) \equiv \sqrt{P_L\left(p^{t,m},p^{t,m+1},q^{t,m},S(t,m)\right)P_P\left(p^{t,m},p^{t,m+1},q^{t,m+1},S(t,m)\right)} \qquad (22.22)$$

m = 1,2,...,11

Note that P_L, P_P and P_F depend on the two (complete) price and quantity vectors pertaining to months m and m+1 of year t, $p^{t,m},p^{t,m+1},q^{t,m},q^{t,m+1}$, but they also depend on the set $S(t,m)$, which is the set of commodities that are present in both months. Thus, the commodity indices n that are in the summations on the right-hand sides of equations (22.20) to (22.22) include

[28] This rapid sample degradation essentially forces some form of chaining at the elementary level in any case.

[29] As was seen in Chapter 20, it is necessary to have a target concept for the individual prices and quantities $p_n^{t,m}$ and $q_n^{t,m}$ at the finest level of aggregation. Under most circumstances, these target concepts can be taken to be unit values (for prices) and total quantities consumed (for quantities).

[30] The formulae are slightly different for the indices that go from December to January of the following year.

indices n that correspond to commodities that are present in both months, which is the meaning of $n \in S(t,m)$; i.e., n belongs to the set $S(t,m)$.

22.66 In order to rewrite the definitions (22.20) to (22.22) in expenditure share and price relative form, some additional notation is required. Define the expenditure shares of commodity n in month m and m+1 of year t, using the set of commodities that are present in month m of year t and the subsequent month, as follows:

$$s_n^{t,m}(t,m) = \frac{p_n^{t,m} q_n^{t,m}}{\sum_{i \in S(t,m)} p_i^{t,m} q_i^{t,m}} \qquad n \in S(t,m) \qquad m = 1,2,...,11 \qquad (22.23)$$

$$s_n^{t,m+1}(t,m) = \frac{p_n^{t,m+1} q_n^{t,m+1}}{\sum_{i \in S(t,m)} p_i^{t,m+1} q_i^{t,m+1}} \qquad n \in S(t,m) \qquad m = 1,2,...,11 \qquad (22.24)$$

The notation in equations (22.23) and (22.24) is rather messy because $s_n^{t,m+1}(t,m)$ has to be distinguished from $s_n^{t,m+1}(t,m+1)$. The expenditure share $s_n^{t,m+1}(t,m)$ is the share of commodity n in month m+1 of year t where n is restricted to the set of commodities that are present in month m of year t and the subsequent month, whereas $s_n^{t,m+1}(t,m+1)$ is the share of commodity n in month m+1 of year t but where n is restricted to the set of commodities that are present in month m+1 of year t and the subsequent month. Thus, the set of superscripts t,m+1 in $s_n^{t,m+1}(t,m)$ indicates that the expenditure share is calculated using the price and quantity data of month m+1 of year t and (t,m) indicates that the set of admissible commodities is restricted to the set of commodities that are present in both month m of year t and the subsequent month.

22.67 Now define vectors of expenditure shares. If commodity n is present in month m of year t and the following month, define $s_n^{t,m}(t,m)$ using equation (22.23); if this is not the case, define $s_n^{t,m}(t,m) = 0$. Similarly, if commodity n is present in month m of year t and the following month, define $s_n^{t,m+1}(t,m)$ using equation (22.24); if this is not the case, define $s_n^{t,m+1}(t,m) = 0$. Now define the N-dimensional vectors $s^{t,m}(t,m) \equiv [s_1^{t,m}(t,m), s_2^{t,m}(t,m),...,s_N^{t,m}(t,m)]$ and $s^{t,m+1}(t,m) \equiv [s_1^{t,m+1}(t,m), s_2^{t,m+1}(t,m),...,s_N^{t,m+1}(t,m)]$. Using these share definitions, the month-to-month Laspeyres, Paasche and Fisher formulae (22.20) to (22.22) can also be rewritten in expenditure share and price form as follows:

$$P_L\left(p^{t,m}, p^{t,m+1}, s^{t,m}(t,m)\right) \equiv \sum_{n \in S(t,m)} s_n^{t,m}(t,m) \left(p_n^{t,m+1} / p_n^{t,m}\right) \quad m = 1,2,...11 \qquad (22.25)$$

$$P_P\left(p^{t,m}, p^{t,m+1}, s^{t,m+1}(t,m)\right) \equiv \left[\sum_{n \in S(t,m)} s_n^{t,m+1}(t,m) \left(p_n^{t,m+1} / p_n^{t,m}\right)^{-1}\right]^{-1} \quad m = 1,2,...11 \qquad (22.26)$$

$$P_F\left(p^{t,m}, p^{t,m+1}, s^{t,m}(t,m), s^{t,m+1}(t,m)\right) \equiv \sqrt{\frac{\sum_{n \in S(t,m)} s_n^{t,m}(t,m)\left(p_n^{t,m+1} / p_n^{t,m}\right)}{\sum_{n \in S(t,m)} s_n^{t,m+1}(t,m)\left(p_n^{t,m+1} / p_n^{t,m}\right)^{-1}}} \quad m = 1,2,...,11 \qquad (22.27)$$

22.68 It is important to recognize that the expenditure shares $s_n^{t,m}(t,m)$ that appear in the maximum overlap month-to-month Laspeyres index defined by equation (22.25) are not the expenditure shares that could be taken from a consumer expenditure survey for month m of year t: instead, they are the shares that result after expenditures on seasonal commodities that are present in month m of year t, but are not present in the following month, are dropped. Similarly, the expenditure shares $s_n^{t,m+1}(t,m)$ that appear in the maximum overlap month-to-month Paasche index defined by equation (22.26) are not the expenditure shares that could be taken from a consumer expenditure survey for month m+1 of year t: instead, they are the shares that result after expenditures on seasonal commodities that are present in month m+1 of year t, but are not present in the preceding month, are dropped.[31] The maximum overlap month-to-month Fisher index defined by equation (22.27) is the geometric mean of the Laspeyres and Paasche indices defined by equations (22.25) and (22.26).

22.69 Table 22.21 lists the maximum overlap chained month-to-month Laspeyres, Paasche and Fisher price indices for the data listed in Tables 22.1 and 22.2. These indices are defined by equations (22.25), (22.26) and (22.27).

Table 22.21 Month-to-month maximum overlap chained Laspeyres, Paasche and Fisher price indices

Year	**Month**	P_L	P_P	P_F
1970	1	1.0000	1.0000	1.0000
	2	0.9766	0.9787	0.9777
	3	0.9587	0.9594	0.9590
	4	1.0290	1.0534	1.0411
	5	1.1447	1.1752	1.1598
	6	1.1118	1.0146	1.0621
	7	1.1167	1.0102	1.0621
	8	1.1307	0.7924	0.9465
	9	1.0033	0.6717	0.8209
	10	0.9996	0.6212	0.7880
	11	1.0574	0.6289	0.8155
	12	1.0151	0.5787	0.7665
1971	1	1.0705	0.6075	0.8064
	2	1.0412	0.5938	0.7863
	3	1.0549	0.6005	0.7959
	4	1.1409	0.6564	0.8654
	5	1.2416	0.7150	0.9422
	6	1.1854	0.6006	0.8438
	7	1.2167	0.6049	0.8579
	8	1.2230	0.4838	0.7692

[31] It is important that the expenditure shares used in an index number formula add up to unity. The use of unadjusted expenditure shares from a household expenditure survey would lead to a systematic bias in the index number formula.

Year	Month	P_L	P_P	P_F
	9	1.0575	0.4055	0.6548
	10	1.0497	0.3837	0.6346
	11	1.1240	0.3905	0.6626
	12	1.0404	0.3471	0.6009
1972	1	1.0976	0.3655	0.6334
	2	1.1027	0.3679	0.6369
	3	1.1291	0.3765	0.6520
	4	1.1974	0.4014	0.6933
	5	1.2818	0.4290	0.7415
	6	1.2182	0.3553	0.6579
	7	1.2838	0.3637	0.6833
	8	1.2531	0.2794	0.5916
	9	1.0445	0.2283	0.4883
	10	1.0335	0.2203	0.4771
	11	1.1087	0.2256	0.5001
	12	1.0321	0.1995	0.4538
1973	1	1.0866	0.2097	0.4774
	2	1.1140	0.2152	0.4897
	3	1.1532	0.2225	0.5065
	4	1.2493	0.2398	0.5474
	5	1.3315	0.2544	0.5821
	6	1.2594	0.2085	0.5124
	7	1.3585	0.2160	0.5416
	8	1.3251	0.1656	0.4684
	9	1.0632	0.1330	0.3760
	10	1.0574	0.1326	0.3744
	11	1.1429	0.1377	0.3967
	12	1.0504	0.1204	0.3556

22.70 The chained maximum overlap Laspeyres, Paasche and Fisher indices for December 1973 are 1.0504, 0.1204 and 0.3556, respectively. Comparing these results to the year-over-year results listed in Tables 22.3, 22.4 and 22.5 (page 398) indicates that the results in Table 22.21 are not at all realistic. These hugely different direct indices compared with the last row of Table 22.21 indicate that the maximum overlap indices suffer from a serious downward bias for the artificial data set.

22.71 What are the factors that can explain this downward bias? It is evident that part of the problem has to do with the seasonal pattern of prices for peaches and strawberries (commodities 2 and 4). These are the commodities that are not present in the marketplace for each month of the year. When these commodities first become available, they come into the marketplace at relatively high prices and then, in subsequent months, their prices drop substantially. The effects of these initially high prices (compared to the relatively low prices that prevailed in the last month that the commodities were available in the previous year) are not captured by the maximum overlap month-to-month indices, so the resulting indices build up a tremendous downward bias. The downward bias is most pronounced in the Paasche indices, which use the quantities or volumes of the current month. Those volumes are relatively large compared to the volumes in the initial month when the commodities become available, reflecting the effects of lower prices as the quantity dumped in the market increases.

22.72 Table 22.22 lists the results using chained Laspeyres, Paasche and Fisher indices for the artificial data set where the strongly seasonal commodities 2 and 4 are dropped from each comparison of prices. Thus, the indices in Table 22.22 are the usual chained Laspeyres, Paasche and Fisher indices restricted to commodities 1, 3 and 5, which are available in each season. The indices derived using these three commodities are labeled $P_L(3)$, $P_P(3)$ and $P_F(3)$.

Table 22.22 Month-to-month chained Laspeyres, Paasche and Fisher price indices

Year	**Month**	$\boldsymbol{P_L(3)}$	$\boldsymbol{P_P(3)}$	$\boldsymbol{P_F(3)}$	$\boldsymbol{P_L(2)}$	$\boldsymbol{P_P(2)}$	$\boldsymbol{P_F(2)}$
1970	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2	0.9766	0.9787	0.9777	0.9751	0.9780	0.9765
	3	0.9587	0.9594	0.9590	0.9522	0.9574	0.9548
	4	1.0290	1.0534	1.0411	1.0223	1.0515	1.0368
	5	1.1447	1.1752	1.1598	1.1377	1.1745	1.1559
	6	1.2070	1.2399	1.2233	1.2006	1.2424	1.2214
	7	1.2694	1.3044	1.2868	1.2729	1.3204	1.2964
	8	1.3248	1.1537	1.2363	1.3419	1.3916	1.3665
	9	1.0630	0.9005	0.9784	1.1156	1.1389	1.1272
	10	0.9759	0.8173	0.8931	0.9944	1.0087	1.0015
	11	1.0324	0.8274	0.9242	0.9839	0.9975	0.9907
	12	0.9911	0.7614	0.8687	0.9214	0.9110	0.9162
1971	1	1.0452	0.7993	0.9140	0.9713	0.9562	0.9637
	2	1.0165	0.7813	0.8912	0.9420	0.9336	0.9378
	3	1.0300	0.7900	0.9020	0.9509	0.9429	0.9469
	4	1.1139	0.8636	0.9808	1.0286	1.0309	1.0298
	5	1.2122	0.9407	1.0679	1.1198	1.1260	1.1229
	6	1.2631	0.9809	1.1131	1.1682	1.1763	1.1723
	7	1.3127	1.0170	1.1554	1.2269	1.2369	1.2319
	8	1.3602	0.9380	1.1296	1.2810	1.2913	1.2861

Year	Month	$P_L(3)$	$P_P(3)$	$P_F(3)$	$P_L(2)$	$P_P(2)$	$P_F(2)$
	9	1.1232	0.7532	0.9198	1.1057	1.0988	1.1022
	10	1.0576	0.7045	0.8632	1.0194	1.0097	1.0145
	11	1.1325	0.7171	0.9012	1.0126	1.0032	1.0079
	12	1.0482	0.6373	0.8174	0.9145	0.8841	0.8992
1972	1	1.1059	0.6711	0.8615	0.9652	0.9311	0.9480
	2	1.1111	0.6755	0.8663	0.9664	0.9359	0.9510
	3	1.1377	0.6912	0.8868	0.9863	0.9567	0.9714
	4	1.2064	0.7371	0.9430	1.0459	1.0201	1.0329
	5	1.2915	0.7876	1.0086	1.1202	1.0951	1.1075
	6	1.3507	0.8235	1.0546	1.1732	1.1470	1.1600
	7	1.4091	0.8577	1.0993	1.2334	1.2069	1.2201
	8	1.4181	0.7322	1.0190	1.2562	1.2294	1.2427
	9	1.1868	0.5938	0.8395	1.1204	1.0850	1.1026
	10	1.1450	0.5696	0.8076	1.0614	1.0251	1.0431
	11	1.2283	0.5835	0.8466	1.0592	1.0222	1.0405
	12	1.1435	0.5161	0.7682	0.9480	0.8935	0.9204
1973	1	1.2038	0.5424	0.8081	1.0033	0.9408	0.9715
	2	1.2342	0.5567	0.8289	1.0240	0.9639	0.9935
	3	1.2776	0.5755	0.8574	1.0571	0.9955	1.0259
	4	1.3841	0.6203	0.9266	1.1451	1.0728	1.1084
	5	1.4752	0.6581	0.9853	1.2211	1.1446	1.1822
	6	1.5398	0.6865	1.0281	1.2763	1.1957	1.2354
	7	1.6038	0.7136	1.0698	1.3395	1.2542	1.2962
	8	1.6183	0.6110	0.9944	1.3662	1.2792	1.3220
	9	1.3927	0.5119	0.8443	1.2530	1.1649	1.2081
	10	1.3908	0.5106	0.8427	1.2505	1.1609	1.2049
	11	1.5033	0.5305	0.8930	1.2643	1.1743	1.2184
	12	1.3816	0.4637	0.8004	1.1159	1.0142	1.0638

22.73 The chained Laspeyres, Paasche and Fisher indices (using only the three always present commodities) for January 1973 are 1.2038, 0.5424 and 0.8081, respectively. From Tables 22.8, 22.9 and 22.10, the chained year-over-year Laspeyres, Paasche and Fisher indices for January 1973 are 1.3274, 1.3243 and 1.3258, respectively. Thus the chained indices using the always present commodities which are listed in Table 22.22 evidently suffer from substantial downward biases.

22.74 If the data in Tables 22.1 and 22.2 are examined, it can be seen that the quantities of grapes (commodity 3) on the marketplace varies tremendously over the course of a year, with substantial increases in price for the months when grapes are almost out of season. Thus the price of grapes decreases substantially as the quantity in the marketplace increases during the last half of each year, but the annual substantial increase in the price of grapes takes place in the first half of the year when quantities in the market are small. This pattern of seasonal price and quantity changes will cause the overall index to take on a downward bias.[32] To verify that this conjecture is true, see the last three columns of Table 22.22 where chained Laspeyres, Paasche and Fisher indices are calculated using only commodities 1 and 5. These indices are labeled $P_L(2)$, $P_P(2)$ and $P_F(2)$, respectively, and for January 1973 they are equal to 1.0033, 0.9408 and 0.9715, respectively. These estimates based on two always present commodities are much closer to the chained year-over-year Laspeyres, Paasche and Fisher indices for January 1973, which were 1.3274, 1.3243 and 1.3258, respectively, than the estimates based on the three always present commodities. It can be seen that the chained Laspeyres, Paasche and Fisher indices restricted to commodities 1 and 5 still have very substantial downward biases for the artificial data set. Basically, the problems are caused by the high volumes associated with low or declining prices, and the low volumes caused by high or rising prices. These weight effects make the seasonal price declines bigger than the seasonal price increases using month-to-month index number formulae with variable weights.[33]

22.75 In addition to the downward biases that show up in Tables 22.21 and 22.22, all of these month-to-month chained indices show substantial seasonal fluctuations in prices over the course of a year. Hence these month-to-month indices are of little use to policy-makers who are interested in short-term inflationary trends. Thus, if the purpose of the month-to-month CPI is to indicate changes in general inflation, then statistical agencies should be cautious about including commodities that show strong seasonal fluctuations in prices in the month-to-month index.[34] If seasonal commodities are included in a month-to-month index

[32] Andrew Baldwin (1990, p. 264) used the Turvey data to illustrate various treatments of seasonal commodities and discusses what causes various month-to-month indices to behave badly: "It is a sad fact that for some seasonal commodity groups, monthly price changes are not meaningful, whatever the choice of formula".

[33] This remark has an application to Chapter 20 on elementary indices where irregular sales during the course of a year could induce a similar downward bias in a month-to-month index that used monthly weights. Another problem with month-to-month chained indices is that purchases and sales of individual commodities can become quite irregular as the time period becomes shorter and shorter, and the problem of zero purchases and sales becomes more pronounced. Feenstra and Shapiro (2003, p. 125) find an upward bias for their chained weekly indices for canned tuna compared to a fixed base index; their bias was caused by variable weight effects resulting from the timing of advertising expenditures. In general, these drift effects of chained indices can be reduced by lengthening the time period, so that the trends in the data become more prominent than the high frequency fluctuations.

[34] If the purpose of the index is to compare the prices that consumers actually face in two consecutive months, ignoring the possibility that the consumer may regard a seasonal good as being qualitatively different in the two months, then the production of a month-to-month CPI that has large seasonal fluctuations can be justified.

that is meant to indicate general inflation, then a seasonal adjustment procedure should be used to remove these strong seasonal fluctuations. Some simple types of seasonal adjustment procedures are considered in paragraphs 22.91 to 22.96.

22.76 The rather poor performance of the month-to-month indices listed in Tables 22.21 and 22.22 does not always occur in the context of seasonal commodities. In the context of calculating import and export price indices using quarterly data for the United States, Alterman, Diewert and Feenstra (1999) found that maximum overlap month-to-month indices worked reasonably well.[35] Statistical agencies should check that their month-to-month indices are at least approximately consistent with the corresponding year-over-year indices.

22.77 Obviously the various Paasche and Fisher indices computed in this section could be approximated by indices that replaced all current period expenditure shares by the corresponding expenditure shares from the base year. These approximate Paasche and Fisher indices will not be reproduced here since they resemble their "true" counterparts and hence are also subject to tremendous downward bias.

Annual basket indices with carry forward of unavailable prices

22.78 Recall that the Lowe (1823) index defined in earlier chapters had two reference periods:[36]

- a reference period for the vector of quantity weights;
- a reference period for the base period prices.

The Lowe index for month m is defined by the following formula:

$$P_{Lo}(p^o, p^m, q) \equiv \sum_{n=1}^{N} p_n^m q_n \Big/ \sum_{n=1}^{N} p_n^0 q_n \qquad (22.28)$$

where $p^0 \equiv [p_1^0,\ldots,p_N^0]$ is the base month price vector, $p^m \equiv [p_1^m,\ldots,p_N^m]$ is the current month m price vector, and $q \equiv [q_1,\ldots,q_N]$ is the base year reference quantity vector. For the purposes of this section, where the modified Turvey data set is used to illustrate the index numerically, the base year will be taken to be 1970. The resulting base year quantity vector turns out to be:

$$q \equiv [q_1,\ldots,q_5] = [53889, 12881, 9198, 5379, 68653] \qquad (22.29)$$

The base period for the prices will be taken to be December 1970. For prices that are not available in the current month, the last available price is carried forward. The resulting Lowe index with carry forward of missing prices using the modified Turvey data set can be found in the P_{Lo} column of Table 22.23 on p. 412.

22.79 Andrew Baldwin's (1990, p. 258) comments on this type of annual basket (AB) index are worth quoting at length:

> For seasonal goods, the AB index is best considered an index partially adjusted for seasonal variation. It is based on annual quantities, which do not reflect the seasonal fluctuations in the volume of

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[35] They checked the validity of their month-to-month indices by cumulating them for four quarters and comparing them to the corresponding year-over-year indices, and found only relatively small differences. However, note that irregular high-frequency fluctuations will tend to be smaller for quarters than for months, and hence chained quarterly indices can be expected to perform better than chained monthly or weekly indices.

[36] In the context of seasonal price indices, this type of index corresponds to Bean and Stine's (1924, p. 31) Type A index.

purchases, and on raw monthly prices, which do incorporate seasonal price fluctuations. Zarnowitz (1961, pp. 256-257) calls it an index of "a hybrid sort". Being neither of sea nor land, it does not provide an appropriate measure either of monthly or 12 month price change. The question that an AB index answers with respect to price change from January to February say, or January of one year to January of the next, is "What would the change in consumer prices have been if there were no seasonality in purchases in the months in question, but prices nonetheless retained their own seasonal behaviour?" It is hard to believe that this is a question that anyone would be interested in asking. On the other hand, the 12 month ratio of an AB index based on seasonally adjusted prices would be conceptually valid, if one were interested in eliminating seasonal influences.

Despite Baldwin's somewhat negative comments on the Lowe index, it is the index that is preferred by many statistical agencies, so it is necessary to study its properties in the context of strongly seasonal data.

22.80 Recall that the Young (1812) index was defined in earlier chapters as follows:

$$P_Y(p^0, p^m, s) \equiv \sum_{n=1}^{N} s_n (p_n^m / p_n^0) \qquad (22.30)$$

where $s \equiv [s_1,\ldots,s_N]$ is the base year reference vector of expenditure shares. For the purposes of this section, where the modified Turvey data set is used to numerically illustrate the index, the base year will be taken to be 1970. The resulting base year expenditure share vector turns out to be:

$$s \equiv [s_1,\ldots,s_5] = [0.3284, 0.1029, 0.0674, 0.0863, 0.4149] \qquad (22.31)$$

Again, the base period for the prices will be taken to be December 1970. For prices that are not available in the current month, the last available price is carried forward. The resulting Young index with carry forward of missing prices using the modified Turvey data set can be found in the P_Y column of Table 22.23.

22.81 The geometric Laspeyres index is defined in Chapter 19 as follows:

$$P_{GL}(p^0, p^m, s) \equiv \prod_{n=1}^{N} (p_n^m / p_n^0)^{s_n} \qquad (22.32)$$

Thus the geometric Laspeyres index makes use of the same information as the Young index except that a geometric average of the price relatives is taken instead of an arithmetic one. Again, the base year is taken to be 1970 and the base period for prices is taken to be December 1970. The index is illustrated using the modified Turvey data set with carry forward of missing prices; see the P_{GL} column of Table 22.23.

22.82 It is of interest to compare the above three indices that use annual baskets to the fixed base Laspeyres rolling year indices computed earlier. The rolling year index that ends in the current month is centered five-and-a-half months backwards. Hence the above three annual basket type indices will be compared with an arithmetic average of two rolling year indices that have their last month five and six months forward. This latter centered rolling year index is labeled P_{CRY} and is listed in the last column of Table 22.23.[37] Note the zero entries for the last six rows of this column; the data set does not extend six months into 1975, so the centered rolling year indices cannot be calculated for these last six months.

Table 22.23 Lowe, Young, geometric Laspeyres and centered rolling year indices with carry forward prices

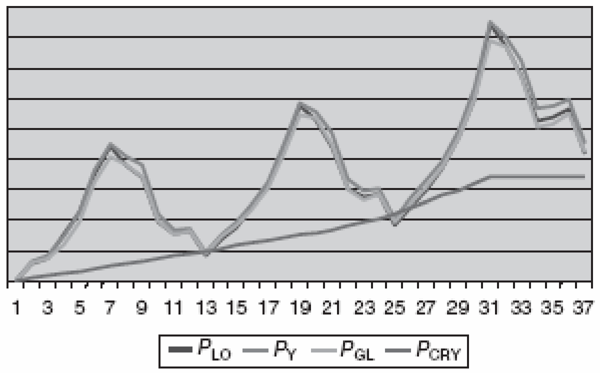
[37] This series is normalized to equal 1 in December 1970, so that it is comparable to the other month-to-month indices.

Year	Month	P_{Lo}	P_Y	P_{GL}	P_{CRY}
1970	12	1.0000	1.0000	1.0000	1.0000
1971	1	1.0554	1.0609	1.0595	1.0091
	2	1.0711	1.0806	1.0730	1.0179
	3	1.1500	1.1452	1.1187	1.0242
	4	1.2251	1.2273	1.1942	1.0298
	5	1.3489	1.3652	1.3249	1.0388
	6	1.4428	1.4487	1.4068	1.0478
	7	1.3789	1.4058	1.3819	1.0547
	8	1.3378	1.3797	1.3409	1.0631
	9	1.1952	1.2187	1.1956	1.0729
	10	1.1543	1.1662	1.1507	1.0814
	11	1.1639	1.1723	1.1648	1.0885
	12	1.0824	1.0932	1.0900	1.0965
1972	1	1.1370	1.1523	1.1465	1.1065
	2	1.1731	1.1897	1.1810	1.1174
	3	1.2455	1.2539	1.2363	1.1254
	4	1.3155	1.3266	1.3018	1.1313
	5	1.4262	1.4508	1.4183	1.1402
	6	1.5790	1.5860	1.5446	1.1502
	7	1.5297	1.5550	1.5349	1.1591
	8	1.4416	1.4851	1.4456	1.1690
	9	1.3038	1.3342	1.2974	1.1806
	10	1.2752	1.2960	1.2668	1.1924
	11	1.2852	1.3034	1.2846	1.2049
	12	1.1844	1.2032	1.1938	1.2203
1973	1	1.2427	1.2710	1.2518	1.2386
	2	1.3003	1.3308	1.3103	1.2608
	3	1.3699	1.3951	1.3735	1.2809
	4	1.4691	1.4924	1.4675	1.2966
	5	1.5972	1.6329	1.5962	1.3176
	6	1.8480	1.8541	1.7904	1.3406
	7	1.7706	1.8010	1.7711	0.0000
	8	1.6779	1.7265	1.6745	0.0000
	9	1.5253	1.5676	1.5072	0.0000

Year	Month	P_{Lo}	P_Y	P_{GL}	P_{CRY}
	10	1.5371	1.5746	1.5155	0.0000
	11	1.5634	1.5987	1.5525	0.0000
	12	1.4181	1.4521	1.4236	0.0000

22.83 It can be seen that the Lowe, Young and geometric Laspeyres indices have a considerable amount of seasonality in them and do not at all approximate their rolling year counterparts listed in the last column of Table 22.23.[38] Hence, without seasonal adjustment, the Lowe, Young and geometric Laspeyres indices are not suitable predictors for their seasonally adjusted rolling year counterparts.[39] The four series, P_{Lo}, P_Y, P_{GL} and P_{CRY}, listed in Table 22.23 are also plotted in Figure 22.4. It can be seen that the Young price index is generally the highest, followed by the Lowe index, while the geometric Laspeyres is the lowest of the three month-to-month indices. The centered rolling year Laspeyres counterpart index, P_{CRY}, is generally below the other three indices (and of course does not have the strong seasonal movements of the other three series), but it moves in a roughly parallel fashion to the other three indices.[40] Note that the seasonal movements of P_{Lo}, P_Y, and P_{GL} are quite regular. This regularity is exploited in paragraphs 22.91 to 22.96 in order to use these month-to-month indices to predict their rolling year counterparts.

Figure 22.4 Lowe, Young, geometric Laspeyres and centered rolling year Laspeyres indices with carry forward prices



22.84 Part of the problem may be the fact that the prices of strongly seasonal goods have been carried forward for the months when the commodities are not available. This will tend to add to the amount of seasonal movements in the indices, particularly when there is high

[38] The sample means of the four indices are 1.2935 (Lowe), 1.3110 (Young), 1.2877 (geometric Laspeyres) and 1.1282 (rolling year). Of course, the geometric Laspeyres indices will always be equal to or less than their Young counterparts, since a weighted geometric mean is always equal to or less than the corresponding weighted arithmetic mean.

[39] In paragraphs 22.91 to 22.96, the Lowe, Young and geometric Laspeyres indices are seasonally adjusted.

[40] In Figure 22.4, P_{CRY} is artificially set equal to the June 1973 value for the index, which is the last month that the centered index can be constructed from the available data.

general inflation. Thus in the following section, the Lowe, Young and geometric Laspeyres indices are computed again using an imputation method for the missing prices rather than simply carrying forward the last available price.

Annual basket indices with imputation of unavailable prices

22.85 Instead of simply carrying forward the last available price of a seasonal commodity that is not sold during a particular month, it is possible to use an imputation method to fill in the missing prices. Alternative imputation methods are discussed by Armknecht and Maitland-Smith (1999) and Feenstra and Diewert (2001). The basic idea is to take the last available price and impute prices for the missing periods, using the trend of another index. This other index could be an index of available prices for the general category of commodity or higher-level components of the CPI. For the purposes of this section, the imputation index is taken to be a price index that grows at the multiplicative rate of 1.008, since the fixed base rolling year Laspeyres indices for the modified Turvey data set grow at approximately 0.8 per cent per month.[41] Using this imputation method to fill in the missing prices, the Lowe, Young and geometric Laspeyres indices defined in the previous section can be recomputed. The resulting indices are listed in Table 22.24, along with the centered rolling year index P_{CRY} for comparison purposes.

Table 22.24: Lowe, Young, geometric Laspeyres and centered rolling year indices with imputed prices

Year	Month	P_{LoI}	P_{YI}	P_{GLI}	P_{CRY}
1970	12	1.0000	1.0000	1.0000	1.0000
1971	1	1.0568	1.0624	1.0611	1.0091
	2	1.0742	1.0836	1.0762	1.0179
	3	1.1545	1.1498	1.1238	1.0242
	4	1.2312	1.2334	1.2014	1.0298
	5	1.3524	1.3682	1.3295	1.0388
	6	1.4405	1.4464	1.4047	1.0478
	7	1.3768	1.4038	1.3798	1.0547
	8	1.3364	1.3789	1.3398	1.0631
	9	1.1949	1.2187	1.1955	1.0729
	10	1.1548	1.1670	1.1514	1.0814
	11	1.1661	1.1747	1.1672	1.0885
	12	1.0863	1.0972	1.0939	1.0965
1972	1	1.1426	1.1580	1.1523	1.1065
	2	1.1803	1.1971	1.1888	1.1174
	3	1.2544	1.2630	1.2463	1.1254

[41] For the last year of data, the imputation index is escalated by an additional monthly growth rate of 1.008.

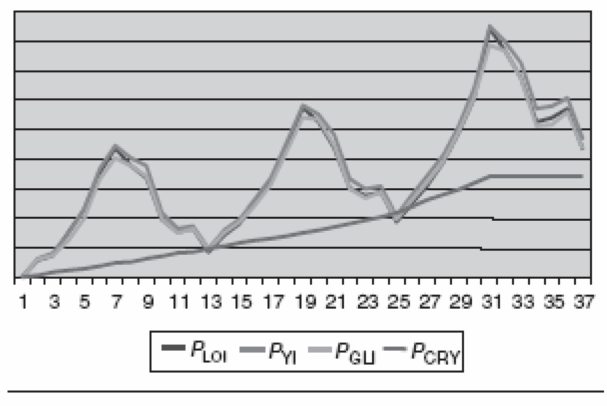
Year	Month	P_{LoI}	P_{YI}	P_{GLI}	P_{CRY}
	4	1.3260	1.3374	1.3143	1.1313
	5	1.4306	1.4545	1.4244	1.1402
	6	1.5765	1.5831	1.5423	1.1502
	7	1.5273	1.5527	1.5326	1.1591
	8	1.4402	1.4841	1.4444	1.1690
	9	1.3034	1.3343	1.2972	1.1806
	10	1.2758	1.2970	1.2675	1.1924
	11	1.2875	1.3062	1.2873	1.2049
	12	1.1888	1.2078	1.1981	1.2203
1973	1	1.2506	1.2791	1.2601	1.2386
	2	1.3119	1.3426	1.3230	1.2608
	3	1.3852	1.4106	1.3909	1.2809
	4	1.4881	1.5115	1.4907	1.2966
	5	1.6064	1.6410	1.6095	1.3176
	6	1.8451	1.8505	1.7877	1.3406
	7	1.7679	1.7981	1.7684	0.0000
	8	1.6773	1.7263	1.6743	0.0000
	9	1.5271	1.5700	1.5090	0.0000
	10	1.5410	1.5792	1.5195	0.0000
	11	1.5715	1.6075	1.5613	0.0000
	12	1.4307	1.4651	1.4359	0.0000

22.86 As could be expected, the Lowe, Young and geometric Laspeyres indices that use imputed prices are on average somewhat higher than their counterparts that use carry forward prices, but the variability of the imputed indices is generally a little lower.[42] The series listed in Table 22.24 are also plotted in Figure 22.5. It can be seen that the Lowe, Young and geometric Laspeyres indices that use imputed prices still have a huge amount of seasonality in them and do not closely approximate their rolling year counterparts listed in the last

[42] For the Lowe indices, the mean for the first 31 observations increases (with imputed prices) from 1.3009 to 1.3047, but the standard deviation decreases from 0.18356 to 0.18319. For the Young indices, the mean for the first 31 observations increases from 1.3186 to 1.3224, but the standard deviation decreases from 0.18781 to 0.18730. For the geometric Laspeyres indices, the mean for the first 31 observations increases from 1.2949 to 1.2994, and the standard deviation also increases slightly from 0.17582 to 0.17599. The imputed indices are preferred to the carry forward indices on general methodological grounds; in high-inflation environments, the carry forward indices will be subject to sudden jumps as the previously unavailable commodities become available.

column of Table 22.24.[43] Hence, without seasonal adjustment, the Lowe, Young and geometric Laspeyres indices using imputed prices are not suitable predictors for their seasonally adjusted rolling year counterparts.[44] As these indices stand, they are not suitable as measures of general inflation going from month to month.

Figure 22.5 Lowe, Young and geometric Laspeyres with imputed prices and centered rolling year indices



Bean and Stine Type C or Rothwell indices

22.87 The final month-to-month index[45] that is considered in this chapter is the *Bean and Stine Type C* (1924; p. 31) or *Rothwell* (1958; p. 72) index.[46] This index makes use of seasonal baskets in the base year, denoted as the vectors $q^{0,m}$ for the months m = 1,2,…,12. The index also makes use of a vector of base year unit value prices, $p^0 \equiv [p_1{}^0,\ldots,p_5{}^0]$, where the nth price in this vector is defined as:

$$p_n^0 \equiv \frac{\sum_{m=1}^{12} p_n^{0,m} q_n^{0,m}}{\sum_{m=1}^{12} q_n^{0,m}} \qquad n = 1,\ldots,5. \tag{22.33}$$

The *Rothwell price index for month m in year t* can now be defined as follows:

$$P_R\left(p^0, p^{t,m}, q^{0,m}\right) \equiv \frac{\sum_{n=1}^{5} p_n^{t,m} q_n^{0,m}}{\sum_{n=1}^{5} p_n^0 q_n^{0,m}} \qquad m = 1,\ldots,12. \tag{22.34}$$

[43] Note also that Figures 22.4 and 22.5 are very similar.

[44] In paragraphs 22.91 to 22.96, the Lowe, Young and geometric Laspeyres indices using imputed prices are seasonally adjusted.

[45] For other suggested month-to-month indices in the seasonal context, see Balk (1980a; 1980b; 1980c; 1981).

[46] This is the index favoured by Baldwin (1990, p. 271) and many other price statisticians in the context of seasonal commodities.

Thus as the month changes, the quantity weights for the index change, and hence the month-to-month movements in this index are a mixture of price and quantity changes.[47]

22.88 Using the modified Turvey data set, the base year is chosen to be 1970 as usual and the index is started off at December 1970. The Rothwell index P_R is compared to the Lowe index with carry forward of missing prices, $P_{LO,}$ in Table 22.25. To make the series slightly more comparable, the *normalized Rothwell index* P_{NR} is also listed in Table 22.25; this index is simply equal to the original Rothwell index divided by its first observation.

Table 22.25 The Lowe with carry forward prices, Rothwell and normalized Rothwell indices

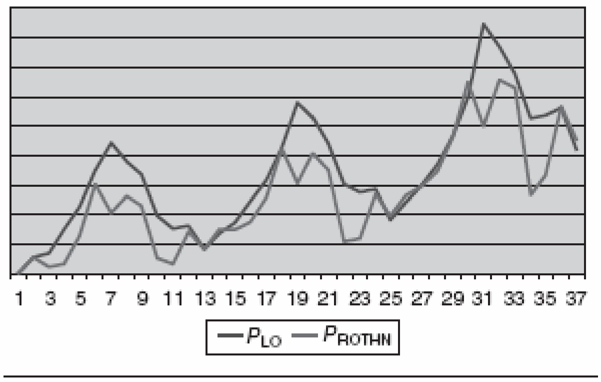
Year	**Month**	P_{Lo}	P_{NR}	P_R
1970	12	1.0000	1.0000	0.9750
1971	1	1.0554	1.0571	1.0306
	2	1.0711	1.0234	0.9978
	3	1.1500	1.0326	1.0068
	4	1.2251	1.1288	1.1006
	5	1.3489	1.3046	1.2720
	6	1.4428	1.2073	1.1771
	7	1.3789	1.2635	1.2319
	8	1.3378	1.2305	1.1997
	9	1.1952	1.0531	1.0268
	10	1.1543	1.0335	1.0077
	11	1.1639	1.1432	1.1146
	12	1.0824	1.0849	1.0577
1972	1	1.1370	1.1500	1.1212
	2	1.1731	1.1504	1.1216
	3	1.2455	1.1752	1.1459
	4	1.3155	1.2561	1.2247
	5	1.4262	1.4245	1.3889
	6	1.5790	1.3064	1.2737
	7	1.5297	1.4071	1.3719
	8	1.4416	1.3495	1.3158
	9	1.3038	1.1090	1.0813
	10	1.2752	1.1197	1.0917
	11	1.2852	1.2714	1.2396

[47] Rothwell (1958, p. 72) showed that the month-to-month movements in the index have the form of an expenditure ratio divided by a quantity index.

Year	Month	P_{Lo}	P_{NR}	P_R
	12	1.1844	1.1960	1.1661
1973	1	1.2427	1.2664	1.2348
	2	1.3003	1.2971	1.2647
	3	1.3699	1.3467	1.3130
	4	1.4691	1.4658	1.4292
	5	1.5972	1.6491	1.6078
	6	1.8480	1.4987	1.4612
	7	1.7706	1.6569	1.6155
	8	1.6779	1.6306	1.5898
	9	1.5253	1.2683	1.2366
	10	1.5371	1.3331	1.2998
	11	1.5634	1.5652	1.5261
	12	1.4181	1.4505	1.4143

22.89 Figure 22.6, which plots the Lowe index with the carry forward of the last price and the normalized Rothwell index, shows that the Rothwell index has smaller seasonal movements than the Lowe index, and is less volatile in general.[48] It is evident that there are still large seasonal movements in the Rothwell index, and it may not be a suitable index for measuring general inflation without some sort of seasonal adjustment.

Figure 22.6 The Lowe and normalized Rothwell price indices



P_{LO} should be P_{Lo}, P_{ROTHN} should be PN_{NR}

22.90 In the following section, the annual basket type indices (with and without imputation) defined in paragraphs 22.78 to 22.86 will be seasonally adjusted using essentially the same method that was used in paragraphs 22.55 to 22.62.

[48] For all 37 observations in Table 22.25, the Lowe index has a mean of 1.3465 and a standard deviation of 0.20313, while the normalized Rothwell index has a mean of 1.2677 and a standard deviation of 0.18271.

Forecasting rolling year indices using month-to-month annual basket indices

22.91 Recall Table 22.23 showing the Lowe, Young, geometric Laspeyres (using carry forward prices) and the centered rolling year indices for the 37 observations running from December 1970 to December 1973, P_{Lo}, P_Y, P_{GL} and P_{CRY}, respectively. For each of the first three series, define a seasonal adjustment factor, *SAF*, as the centered rolling year index, P_{CRY}, divided by P_{Lo}, P_Y and P_{GL}, respectively, for the first 12 observations. Now for each of the three series, repeat these 12 seasonal adjustment factors for observations 13 to 24, and then repeat them again for the remaining observations. These operations will create three *SAF* series for all 37 observations (label them SAF_{Lo}, SAF_Y and SAF_{GL}, respectively). Only the first 12 observations in the P_{Lo}, P_Y, P_{GL} and P_{CRY} series are used to create the three *SAF* series. Finally, define *seasonally adjusted Lowe, Young and geometric Laspeyres indices* by multiplying each unadjusted index by the appropriate seasonal adjustment factor:

$$P_{LoSA} \equiv P_{Lo}\, SAF_{Lo} \qquad P_{YSA} \equiv P_Y\, SAF_Y \qquad P_{GLSA} \equiv P_{GL}\, SAF_{GL} \qquad (22.35)$$

These three seasonally adjusted annual basket type indices are listed in Table 22.26 along with the target index, the centered rolling year index, P_{CRY}.

Table 22.26 Seasonally adjusted Lowe, Young and geometric Laspeyres indices with carry forward prices and the centered rolling year index

Year	**Month**	P_{LoSA}	P_{YSA}	P_{GLSA}	P_{CRY}
1970	12	1.0000	1.0000	1.0000	1.0000
1971	1	1.0091	1.0091	1.0091	1.0091
	2	1.0179	1.0179	1.0179	1.0179
	3	1.0242	1.0242	1.0242	1.0242
	4	1.0298	1.0298	1.0298	1.0298
	5	1.0388	1.0388	1.0388	1.0388
	6	1.0478	1.0478	1.0478	1.0478
	7	1.0547	1.0547	1.0547	1.0547
	8	1.0631	1.0631	1.0631	1.0631
	9	1.0729	1.0729	1.0729	1.0729
	10	1.0814	1.0814	1.0814	1.0814
	11	1.0885	1.0885	1.0885	1.0885
	12	1.0824	1.0932	1.0900	1.0965
1972	1	1.0871	1.0960	1.0919	1.1065
	2	1.1148	1.1207	1.1204	1.1174
	3	1.1093	1.1214	1.1318	1.1254
	4	1.1057	1.1132	1.1226	1.1313
	5	1.0983	1.1039	1.1120	1.1402
	6	1.1467	1.1471	1.1505	1.1502
	7	1.1701	1.1667	1.1715	1.1591
	8	1.1456	1.1443	1.1461	1.1690

Year	Month	P_{LoSA}	P_{YSA}	P_{GLSA}	P_{CRY}
	9	1.1703	1.1746	1.1642	1.1806
	10	1.1946	1.2017	1.1905	1.1924
	11	1.2019	1.2102	1.2005	1.2049
	12	1.1844	1.2032	1.1938	1.2203
1973	1	1.1882	1.2089	1.1922	1.2386
	2	1.2357	1.2536	1.2431	1.2608
	3	1.2201	1.2477	1.2575	1.2809
	4	1.2349	1.2523	1.2656	1.2966
	5	1.2299	1.2425	1.2514	1.3176
	6	1.3421	1.3410	1.3335	1.3406
	7	1.3543	1.3512	1.3518	0.0000
	8	1.3334	1.3302	1.3276	0.0000
	9	1.3692	1.3800	1.3524	0.0000
	10	1.4400	1.4601	1.4242	0.0000
	11	1.4621	1.4844	1.4508	0.0000
	12	1.4181	1.4521	1.4236	0.0000

22.92 The four series in Table 22.26 coincide for their first 12 observations, which follows from the way the seasonally adjusted series were defined. Also, the last six observations are missing for the centered rolling year series, P_{CRY}, since data for the first six months of 1974 would be required in order to calculate all these index values. Note that from December 1971 to December 1973, the three seasonally adjusted annual basket type indices can be used to predict the corresponding centered rolling year entries; see Figure 22.7 for plots of these predictions. What is remarkable in Table 22.26 and Figure 22.7 is that the predicted values of these seasonally adjusted series are fairly close to the corresponding target index values.[49] This result is somewhat unexpected since the annual basket indices use price information for only two consecutive months, whereas the corresponding centered rolling year index uses price information for some 25 months.[50] It should be noted that the seasonally adjusted

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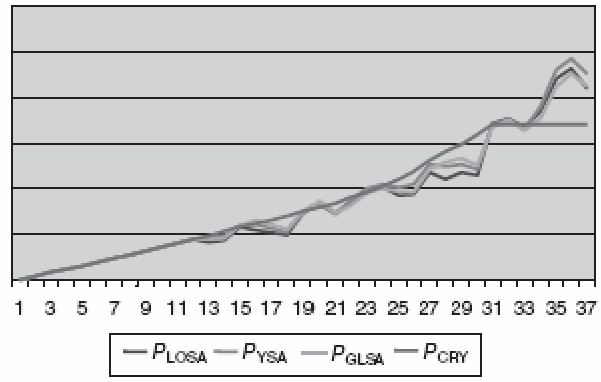
[49] For observations 13 to 31, the seasonally adjusted series can be regressed on the centered rolling year series. For the seasonally adjusted Lowe index, an R^2 of 0.8816 is obtained; for the seasonally adjusted Young index, an R^2 of 0.9212 is obtained; and for the seasonally adjusted geometric Laspeyres index, an R^2 of 0.9423 is obtained. These fits are not as good as the fit obtained in paragraphs 22.55 to 22.62, where the seasonally adjusted approximate rolling year index is used to predict the fixed base Laspeyres rolling year index. This R^2 is 0.9662; recall the discussion of Table 22.20.

[50] For seasonal data sets that are not as regular as the modified Turvey data set, the predictive power of the seasonally adjusted annual basket type indices may be considerably less; i.e., if there are abrupt changes in the seasonal pattern of prices, these month-to-month indices cannot be expected to accurately predict a rolling year index.

geometric Laspeyres index is generally the best predictor of the corresponding rolling year index for this data set. It can be seen from Figure 22.7 that for the first few months of 1973, the three month-to-month indices underestimate the centered rolling year inflation rate, but by the middle of 1973, the month-to-month indices are right on target.[51]

Figure 22.7 Seasonally adjusted Lowe, Young and geometric Laspeyres indices with carry forward prices and the centered rolling year index



P_{LOSA} should be P_{LoSA}

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22.93 The above manipulations can be repeated, replacing the carry forward annual basket indices by their imputed counterparts; i.e., use the information in Table 22.24 (instead of Table 22.23) and in Table 22.27 (instead of Table 22.26). A seasonally adjusted version of the Rothwell index presented in the previous section may also be found in Table 22.27.[52] The five series in Table 22.27 are also represented graphically in Figure 22.8.

Table 22.27 Seasonally adjusted Lowe, Young and geometric Laspeyres indices with imputed prices, seasonally adjusted Rothwell and centered rolling year indices

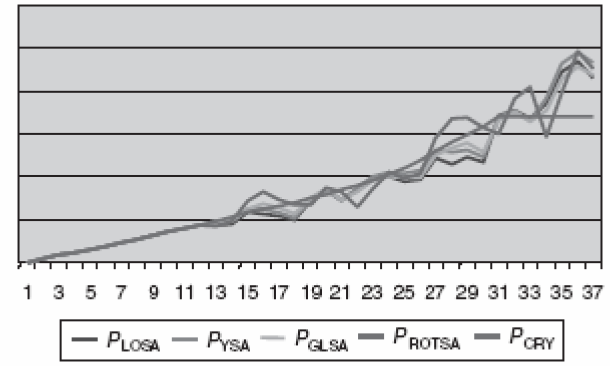
Year	**Month**	P_{LoSA}	P_{YSA}	P_{GLSA}	P_{ROTHSA}	P_{CRY}
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0091	1.0091	1.0091	1.0091	1.0091
	2	1.0179	1.0179	1.0179	1.0179	1.0179
	3	1.0242	1.0242	1.0242	1.0242	1.0242
	4	1.0298	1.0298	1.0298	1.0298	1.0298
	5	1.0388	1.0388	1.0388	1.0388	1.0388
	6	1.0478	1.0478	1.0478	1.0478	1.0478
	7	1.0547	1.0547	1.0547	1.0547	1.0547
	8	1.0631	1.0631	1.0631	1.0631	1.0631

[51] Recall that the last six months of P_{CRY} have been artificially held constant; six months of data for 1974 would be required to evaluate these centered rolling year index values, but these data are not available.

[52] The same seasonal adjustment technique as was defined by equations (22.35) was used.

Year	Month	P_{LoSA}	P_{YSA}	P_{GLSA}	P_{ROTHSA}	P_{CRY}
	9	1.0729	1.0729	1.0729	1.0729	1.0729
	10	1.0814	1.0814	1.0814	1.0814	1.0814
	11	1.0885	1.0885	1.0885	1.0885	1.0885
	12	1.0863	1.0972	1.0939	1.0849	1.0965
1972	1	1.0909	1.0999	1.0958	1.0978	1.1065
	2	1.1185	1.1245	1.1244	1.1442	1.1174
	3	1.1129	1.1250	1.1359	1.1657	1.1254
	4	1.1091	1.1167	1.1266	1.1460	1.1313
	5	1.0988	1.1043	1.1129	1.1342	1.1402
	6	1.1467	1.1469	1.1505	1.1339	1.1502
	7	1.1701	1.1666	1.1715	1.1746	1.1591
	8	1.1457	1.1442	1.1461	1.1659	1.1690
	9	1.1703	1.1746	1.1642	1.1298	1.1806
	10	1.1947	1.2019	1.1905	1.1715	1.1924
	11	1.2019	1.2103	1.2005	1.2106	1.2049
	12	1.1888	1.2078	1.1981	1.1960	1.2203
1973	1	1.1941	1.2149	1.1983	1.2089	1.2386
	2	1.2431	1.2611	1.2513	1.2901	1.2608
	3	1.2289	1.2565	1.2677	1.3358	1.2809
	4	1.2447	1.2621	1.2778	1.3373	1.2966
	5	1.2338	1.2459	1.2576	1.3131	1.3176
	6	1.3421	1.3406	1.3335	1.3007	1.3406
	7	1.3543	1.3510	1.3518	1.3831	0.0000
	8	1.3343	1.3309	1.3285	1.4087	0.0000
	9	1.3712	1.3821	1.3543	1.2921	0.0000
	10	1.4430	1.4634	1.4271	1.3949	0.0000
	11	1.4669	1.4895	1.4560	1.4903	0.0000
	12	1.4307	1.4651	1.4359	1.4505	0.0000

Figure 22.8 Seasonally adjusted Lowe, Young and geometric Laspeyres indices with imputed prices, seasonally adjusted Rothwell and centered rolling year indices



P_{LOSA} should be P_{LoSA}

22.94 Again, the seasonally adjusted annual basket type indices listed in the P_{LoSA}, P_{YSA} and P_{GLSA} columns of Table 22.27 (using imputations for the missing prices) are reasonably close to the corresponding centered rolling year index listed in the last column of Table 22.27.[53] The seasonally adjusted geometric Laspeyres index is the closest to the centered rolling year index, and the seasonally adjusted Rothwell index is the furthest away. The three seasonally adjusted month-to-month indices that use annual weights, P_{LoSA}, P_{YSA} and P_{GLSA}**,** dip below the corresponding centered rolling year index, P_{CRY}, for the first few months of 1973 when the rate of month-to-month inflation suddenly increases, but by the middle of 1973, all four indices are fairly close to each other. The seasonally adjusted Rothwell does not do a very good job of approximating P_{CRY} for this particular data set, although this could be a function of the rather simple method of seasonal adjustment that is used.

22.95 Comparing the results in Tables 22.26 and 22.7, it can be seen that, for the modified Turvey data set, it did not make a great deal of difference whether missing prices are carried forward or imputed; the seasonal adjustment factors picked up the lumpiness in the unadjusted indices that occurs if the carry forward method is used. Nevertheless, the three month-to-month indices that used annual weights and imputed prices did predict the corresponding centered rolling year indices somewhat better than the three indices that used carry forward prices. Hence, the use of imputed prices over carry forward prices is recommended.

22.96 The conclusions that emerge from this section are rather encouraging for statistical agencies that wish to use an annual basket type index as their flagship index.[54] It appears that for commodity groups that have strong seasonality, an annual basket type index for this group

[53] For observations 13 to 31, the seasonally adjusted series can be regressed on the centered rolling year series. For the seasonally adjusted Lowe index, an R^2 of 0.8994 is obtained; for the seasonally adjusted Young index, an R^2 of 0.9294 is obtained; and for the seasonally adjusted geometric Laspeyres index, an R^2 of 0.9495 is obtained. For the seasonally adjusted Rothwell index, an R^2 of 0.8704 is obtained, which is lower than the other three fits. For the Lowe, Young and geometric Laspeyres indices using imputed prices, these R^2 are higher than those obtained using carry forward prices.

[54] Taking into account the results of previous chapters, the use of the annual basket Young index is not encouraged because of its failure of the time reversal test and the resulting upward bias.

can be seasonally adjusted[55] and the resulting seasonally adjusted index value can be used as a price relative for the group at higher stages of aggregation. The preferred type of annual basket type index appears to be the geometric Laspeyres index rather than the Lowe index, but the differences between the two are not large for this data set.

Conclusion

22.97 A number of tentative conclusions can be drawn from the results of the previous sections in this chapter:

- The inclusion of seasonal commodities in maximum overlap month-to-month indices will frequently lead to substantial biases. Hence, unless the maximum overlap month-to-month indices using seasonal commodities cumulated for a year are close to their year-over-year counterparts, the seasonal commodities should be excluded from the month-to-month index or the seasonal adjustment procedures suggested in paragraphs 22.91 to 22.96 should be used.
- Year-over-year monthly indices can always be constructed even if there are strongly seasonal commodities.[56] Many users will be interested in these indices; moreover, these indices are the building blocks for annual indices and for rolling year indices. Statistical agencies should compute these indices, which may be labeled “analytic series” in order to prevent user confusion with the primary month-to-month CPI.
- Rolling year indices should also be made available as analytic series. These indices will give the most reliable indicator of annual inflation at a monthly frequency. This type of index can be regarded as a seasonally adjusted CPI, and is the most natural to use as a central bank inflation target. It has the disadvantage of measuring year-over-year inflation with a lag of six months; hence it cannot be used as a short-run indicator of month-to-month inflation. Nevertheless, the techniques suggested in paragraphs 22.55 to 22.62 and 22.91 to 22.96 could be used, so that timely forecasts of these rolling year indices can be made using current price information.
- Annual basket indices can also be successfully used in the context of seasonal commodities. Most users of the CPI will, however, want to use seasonally adjusted versions of these annual basket type indices. The seasonal adjustment can be done using the index number methods explained in paragraphs 22.91 to 22.96, or traditional statistical agency seasonal adjustment procedures could be used.[57]

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[55] It is not necessary to use rolling year indices in the seasonal adjustment process, but their use is recommended because they increase the objectivity and reproducibility of the seasonally adjusted indices.

[56] There may be problems with the year-over-year indices if shifting holidays or abnormal weather changes “normal” seasonal patterns. In general, choosing a longer time period will mitigate these types of problems; i.e., quarterly seasonal patterns will be more stable than monthly patterns, which in turn will be more stable than weekly patterns.

[57] There is, however, a problem with using traditional X-11 type seasonal adjustment procedures for seasonally adjusting the main CPI because “final” seasonal adjustment factors are generally not available until data have been collected for two or three more years. Since the main CPI cannot be revised, this may preclude using X-11 type seasonal adjustment procedures on it. Note that the index number method of seasonal adjustment explained in this chapter does not suffer from this problem.

- From an a priori point of view, when making a price comparison between any two periods, the Paasche and Laspeyres indices are of equal importance. Under normal circumstances, the spread between the Laspeyres and Paasche indices will be reduced by using chained indices rather than fixed base indices. Hence, it is suggested that when constructing year-over-year monthly or annual indices, the chained Fisher index (or the chained Törnqvist–Theil index, which closely approximates the chained Fisher) be chosen as the target index that a statistical agency should aim to approximate. When constructing month-to-month indices, however, chained indices should always be checked against their year-over-year counterparts to check for chain drift. If substantial drift is found, the chained month-to-month indices must be replaced by fixed base indices or seasonally adjusted annual basket type indices.[58]
- If current period expenditure shares are not all that different from base year expenditure shares, approximate chained Fisher indices will normally provide a very close practical approximation to the chained Fisher target indices. Approximate Laspeyres, Paasche and Fisher indices use base period expenditure shares whenever they occur in the index number formula in place of current period (or lagged current period) expenditure shares. Approximate Laspeyres, Paasche and Fisher indices can be computed by statistical agencies using their normal information sets.
- The geometric Laspeyres index is an alternative to the approximate Fisher index; it uses the same information and will normally be close to the approximate Fisher index.

It is evident that more research needs to be carried out on the problems associated with the index number treatment of seasonal commodities. There is, as yet, no consensus on what is best practice in this area.

[58] Alternatively, some sort of multilateral index number formula could be used; see, for example, Caves, Christensen and Diewert (1982a) or Feenstra and Shapiro (2003).