9. CALCULATING CONSUMER PRICE INDICES IN PRACTICE

Introduction

9.1 The purpose of this chapter is to provide a general description of the ways in which consumer price indices (CPIs) are calculated in practice. The methods used in different countries are not exactly the same, but they have much in common. There is clearly interest from both compilers and users of CPIs in knowing how most statistical offices actually set about calculating their CPIs.

9.2 As a result of the greater insights into the properties and behaviour of price indices that have been achieved in recent years, it is now recognized that some traditional methods may not necessarily be optimal from a conceptual and theoretical viewpoint. Concerns have also been voiced in a number of countries about possible biases that may be affecting CPIs. These issues and concerns need to be considered in this manual. Of course, the methods used to compile CPIs are inevitably constrained by the resources available, not merely for collecting and processing prices, but also for gathering the expenditure data needed for weighting purposes. In some countries, the methods used may be severely constrained by lack of resources.

9.3 The calculation of CPIs usually proceeds in two stages. First, price indices are estimated for the elementary expenditure aggregates, or simply elementary aggregates. Then these elementary price indices are averaged to obtain higher-level indices using the relative values of the elementary expenditure aggregates as weights. This chapter starts by explaining how the elementary aggregates are constructed, and what economic and statistical criteria need to be taken into consideration in defining the aggregates. The index number formulae most commonly used to calculate the elementary indices are then presented, and their properties and behaviour illustrated using numerical examples. The pros and cons of the various formulae are considered, together with some alternative formulae that might be used instead. The problems created by disappearing and new items are also explained, as well as the different ways of imputing values for missing prices.

9.4 The second part of the chapter is concerned with the calculation of higher-level indices. The focus is on the ongoing production of a monthly price index in which the elementary price indices are averaged, or aggregated, to obtain higher-level indices. Price-updating of weights, chain linking and reweighting are discussed, with examples being provided. The problems associated with introduction of new elementary price indices and new higher-level indices into the CPI are also dealt with. It is explained how it is possible to decompose the change in the overall index into its component parts. Finally, the possibility of using some alternative and rather more complex index formulae is considered.

9.5 The chapter concludes with a section on data editing procedures, as these are an integral part of the process of compiling CPIs. It is essential to ensure that the right data are entered into the various formulae. There may be errors resulting from the inclusion of incorrect data or from entering correct data inappropriately, and errors resulting from the exclusion of correct data that are mistakenly believed to be wrong. The section examines data editing procedures which try to minimize both types of errors.

The calculation of price indices for elementary aggregates

9.6 CPIs are typically calculated in two steps. In the first step, the elementary price indices for the elementary aggregates are calculated. In the second step, higher-level indices are calculated by averaging the elementary price indices. The elementary aggregates and their price indices are the basic building blocks of the CPI.

Construction of elementary aggregates

9.7 Elementary aggregates are groups of relatively homogeneous goods and services. They may cover the whole country or separate regions within the country. Likewise, elementary aggregates may be distinguished for different types of outlets. The nature of the elementary aggregates depends on circumstances and the availability of information. Elementary aggregates may therefore be defined differently in different countries. Some key points, however, should be noted:

- Elementary aggregates should consist of groups of goods or services that are as similar as possible, and preferably fairly homogeneous.
- They should also consist of items that may be expected to have similar price movements. The objective should be to try to minimize the dispersion of price movements within the aggregate.
- The elementary aggregates should be appropriate to serve as strata for sampling purposes in the light of the sampling regime planned for the data collection.

9.8 Each elementary aggregate, whether relating to the whole country or an individual region or group of outlets, will typically contain a very large number of individual goods or services, or items. In practice, only a small number can be selected for pricing. When selecting the items, the following considerations need to be taken into account:

- The items selected should be ones for which price movements are believed to be representative of all the products within the elementary aggregate.
- The number of items within each elementary aggregate for which prices are collected should be large enough for the estimated price index to be statistically reliable. The minimum number required will vary between elementary aggregates depending on the nature of the products and their price behaviour.
- The object is to try to track the price of the same item over time for as long as the item continues to be representative. The items selected should therefore be ones that are expected to remain on the market for some time, so that like can be compared with like, and problems associated with replacement of items be reduced.

The aggregation structure

9.9 The aggregation structure for a CPI is illustrated in Figure 9.1. Using a classification of consumers’ expenditures such as the Classification of Individual Consumption according to Purpose (COICOP), the entire set of consumption goods and services covered by the overall CPI can be divided into groups, such as “food and non-alcoholic beverages”. Each group is further divided into classes, such as “food”. For CPI purposes, each class can then be further divided into more homogeneous sub-classes, such as “rice”. The sub-classes are the equivalent of the basic headings used in the International Comparison Program (ICP), which calculates purchasing power parities (PPPs) between countries. Finally, the sub-class may be further subdivided to obtain the elementary aggregates, by dividing according to region or type of outlet, as in Figure 9.1. In some cases, a particular sub-class cannot be, or does not need to be, further subdivided, in which case the sub-class becomes the elementary aggregate. Within each elementary aggregate, one or more items are selected to represent all the items in the elementary aggregate. For example, the elementary aggregate consisting of rice sold in supermarkets in the northern region covers all types of rice, from which parboiled white rice and brown rice with over 50 per cent broken grains are selected as representative items. Of course, more representative items might be selected in practice. Finally, for each representative item, a number of specific products can be selected for price collection, such as particular brands of parboiled rice. Again, the number of sampled products selected may vary depending on the nature of the representative product.

9.10 Methods used to calculate the elementary indices from the individual price observations are discussed below. Working upwards from the elementary price indices, all indices above the elementary aggregate level are higher-level indices that can be calculated from the elementary price indices using the elementary expenditure aggregates as weights. The aggregation structure is consistent, so that the weight at each level above the elementary aggregate is always equal to the sum of its components. The price index at each higher level of aggregation can be calculated on the basis of the weights and price indices for its components, that is, the lower-level or elementary indices. The individual elementary price indices are not necessarily sufficiently reliable to be published separately, but they remain the basic building blocks of all higher-level indices.

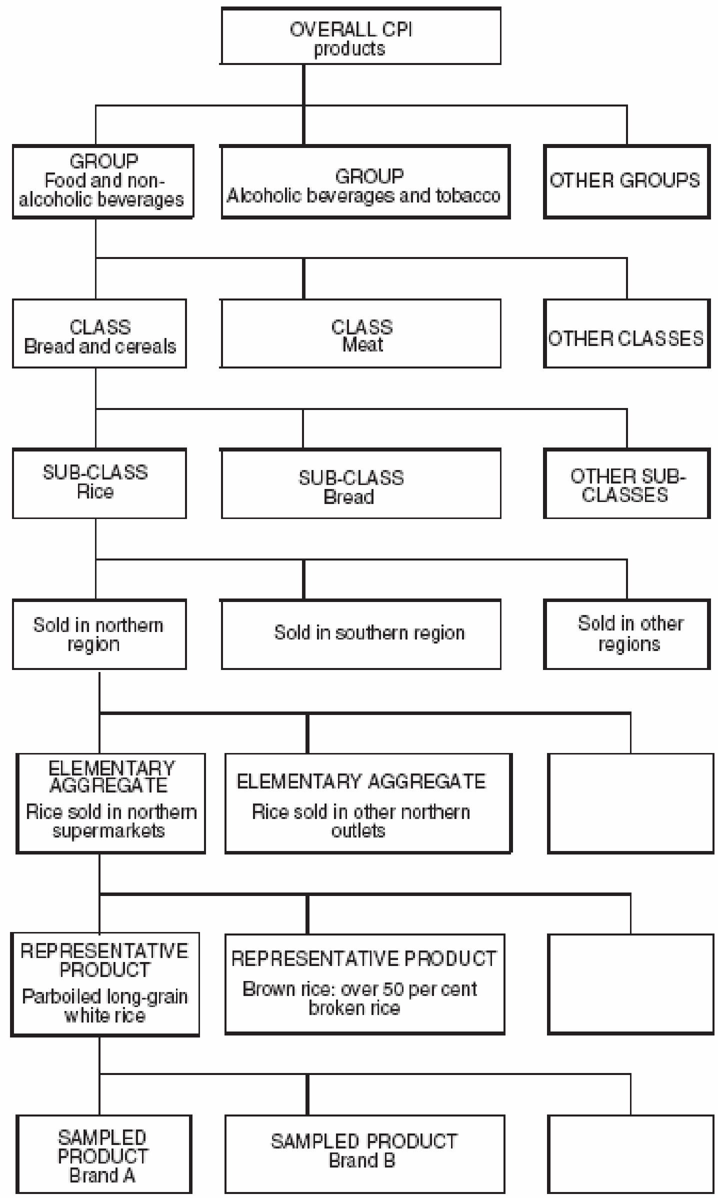
Weights within elementary aggregates

9.11 In most cases, the price indices for elementary aggregates are calculated without the use of explicit expenditure weights. Often, the elementary aggregate is simply the lowest level at which reliable weighting information is available. In this case, the elementary index has to be calculated as an unweighted average of the prices of which it consists. Even in this case, however, it should be noted that when the items are selected with probabilities proportional to the size of some relevant variable such as sales, weights are implicitly introduced by the sampling selection procedure.

9.12 For certain elementary aggregates, information about sales of particular items, market shares and regional weights may be used as explicit weights within an elementary aggregate. When possible, weights should be used that reflect the relative importance of the sampled items, even if the weights are only approximate..

9.13 For example, assume that the number of suppliers of a certain product such as fuel for cars is limited. The market shares of the suppliers may be known from business survey statistics and can be used as weights in the calculation of an elementary aggregate price index for car fuel. Alternatively, prices for water may be collected from a number of local water supply services where the population in each local region is known. The relative size of the population in each region may then be used as a proxy for the relative consumption expenditures to weight the price in each region to obtain the elementary aggregate price index for water. The calculation of weighted elementary indices is discussed in more detail later in the chapter.

Figure 9.1 Typical aggregation structure of a consumer price index



Calculation of elementary price indices

9.14 An elementary price index is the price index for an elementary aggregate. Various different methods and formulae may be used to calculate elementary price indices. This section provides a summary of the methods that have been most commonly used and the pros and cons that statistical offices must evaluate when choosing a formula at the elementary level. Chapter 20 provides a more detailed discussion

9.15 The methods most common in use are illustrated in a numerical example in Table 9.1. In the example an elementary aggregate consists of four items, and it is assumed that prices are collected for all four items in all months, so that there is a complete set of prices. There are no disappearing items, no missing prices and no replacement items. This is quite a strong assumption since many of the problems encountered in practice are attributable to breaks in the continuity of the price series for the individual items for one reason or another. The treatment of disappearing and replacement items is taken up later. It is also assumed that there are no explicit weights available.

Table 9.1 Calculation of price indices for an elementary aggregate

	January	February	March	April	May	June	July
				Prices			
Item A	6.00	6.00	7.00	6.00	6.00	6.00	6.60
Item B	7.00	7.00	6.00	7.00	7.00	7.20	7.70
Item C	2.00	3.00	4.00	5.00	2.00	3.00	2.20
Item D	5.00	5.00	5.00	4.00	5.00	5.00	5.50
Arithmetic mean prices	5.00	5.25	5.50	5.50	5.00	5.30	5.50
Geometric mean prices	4.53	5.01	5.38	5.38	4.53	5.05	4.98
				Month-to-month price ratios			
Item A	1.00	1.00	1.17	0.86	1.00	1.00	1.10
Item B	1.00	1.00	0.86	1.17	1.00	1.03	1.07
Item C	1.00	1.50	1.33	1.25	0.40	1.50	0.73
Item D	1.00	1.00	1.00	0.80	1.25	1.00	1.10
				Current-to-reference month (January) price ratios			
Item A	1.00	1.00	1.17	1.00	1.00	1.00	1.10
Item B	1.00	1.00	0.86	1.00	1.00	1.03	1.10
Item C	1.00	1.50	2.00	2.50	1.00	1.50	1.10
Item D	1.00	1.00	1.00	0.80	1.00	1.00	1.10
Carli index – the arithmetic mean of price ratios							
Month-to-month index	100.00	112.50	108.93	101.85	91.25	113.21	100.07
Chained m/m index	100.00	112.50	122.54	124.81	113.89	128.93	129.02
Direct index on January	100.00	112.50	125.60	132.50	100.00	113.21	110.00
Dutot index – the ratio of arithmetic mean prices							
Month-to-month index	100.00	105.00	104.76	100.00	90.91	106.00	103.77
Chained m/m index	100.00	105.00	110.00	110.00	100.00	106.00	110.00
Direct index on January	100.00	105.00	110.00	110.00	100.00	106.00	110.00
Jevons index – the ratio of geometric mean prices = geometric mean of price ratios							
Month-to-month index	100.00	110.67	107.46	100.00	84.09	111.45	98.70
Chained m/m index	100.00	110.67	118.92	118.92	100.00	111.45	110.00
Direct index on January	100.00	110.67	118.92	118.92	100.00	111.45	110.00

Note: All price indices have been calculated using unrounded figures.

9.16 The properties of the three indices are examined and explained in some detail in Chapter 20. Here, the purpose is to illustrate how they perform in practice, to compare the results obtained by using the different formulae and to summarize their strengths and weaknesses. Three widely used formulae that have been, or still are, in use by statistical offices to calculate elementary price indices are illustrated in Table 9.1. It should be noted, however, that these are not the only possibilities and some alternative formulae are considered later. The first is the Carli index for i = 1,…., n items. It is defined as the simple, or unweighted, arithmetic mean of the price relatives, or price ratios, for the two periods, 0 and t, to be compared:

$$P_C^{0:t} = \frac{1}{n}\sum\left(\frac{p_i^t}{p_i^0}\right) \qquad (9.1)$$

The second is the Dutot index, defined as the ratio of the unweighted arithmetic mean prices:

$$P_D^{0:t} = \frac{\frac{1}{n}\sum p_i^t}{\frac{1}{n}\sum p_i^0} \qquad (9.2)$$

The third is the Jevons index, defined as the unweighted geometric mean of the price ratios, which is identical to the ratio of the unweighted geometric mean prices:

$$P_J^{0:t} = \prod\left(\frac{p_i^t}{p_i^0}\right)^{1/n} = \frac{\prod\left(p_i^t\right)^{1/n}}{\prod\left(p_i^0\right)^{1/n}} \qquad (9.3)$$

9.17 Each *month-to-month* index shows the change in the index from one month to the next. The *chained month-to-month* (m/m) indices link together these monthly changes by successive multiplication. The *direct* indices compare the prices in each successive month directly with those of the reference month, January. By simple inspection of the various indices, it is clear that the choice of formula and method can make a substantial difference to the results obtained. Some results are striking, in particular the large difference between the chained Carli index for July and each of the direct indices for July, including the direct Carli.

9.18 The properties and behaviour of the different indices are summarized in the following paragraphs (see also Chapter 20). First, the differences between the results obtained by using the different formulae tend to increase as the variance of the price relatives, or ratios, increases. The greater the dispersion of the price movements, the more critical the choice of index formula, and method, becomes. If the elementary aggregates are defined in such a way that the price movements within the aggregate are minimized, the results obtained become less sensitive to the choice of formula and method.

9.19 Certain features displayed by the data in Table 9.1 are systematic and predictable; they follow from the mathematical properties of the indices. For example, it is well known that an arithmetic mean is always greater than, or equal to, the corresponding geometric mean, the equality holding only in the trivial case in which the numbers being averaged are all the same. The direct Carli indices are therefore all greater than the Jevons indices, except in May and July when the four price relatives based on January are all equal. In general, the Dutot may be greater or less than the Jevons, but tends to be less than the Carli.

9.20 The Carli and Jevons indices depend only on the price ratios and are unaffected by the price level. The Dutot index, in contrast, is influenced by the price level. In the Dutot index, price changes are implicitly weighted by the price in the base period, so that price changes on more expensive products are assigned a higher weight than similar price changes for cheaper products (this can be seen from equation (9.4)). In Table 9.1 this is illustrated in the development from January to February where all prices are unchanged except for item D, which increase by 50 per cent. Because of the relative low price of this item, its price increase is down-weighted in the Dutot index, which shows only about half the increase compared to Carli and Jevons.

9.21 Another important property of the indices is that the Dutot and the Jevons indices are transitive, whereas the Carli is not. Transitivity means that the chained monthly indices are identical to the corresponding direct indices. This property is important in practice, because many elementary price indices are in fact calculated as chain indices which link together the month-on-month indices. The intransitivity of the Carli index is illustrated dramatically in Table 9.1 when each of the four individual prices in May returns to the same level as it was in January, but the chain Carli registers an increase of almost 14 per cent over January. Similarly, in July, although each individual price is exactly 10 per cent higher than in January, the chain Carli registers an increase of 29 per cent. These results would be regarded as perverse and unacceptable in the case of a direct index, but even in the case of a chain index the results seems so intuitively unreasonable as to undermine the credibility of the chain Carli. The price changes between March and April illustrate the effects of "price bouncing" in which the same four prices are observed in both periods but they are switched between the different items. The monthly Carli index from March to April increases whereas both the Dutot and the Jevons indices are unchanged.

9.22 One general property of geometric means should be noted when using the Jevons index. If any one observation out of a set of observations is zero, their geometric mean is zero, whatever the values of the other observations. The Jevons index is sensitive to extreme falls in prices and it may be necessary to impose upper and lower bounds on the individual price ratios of say 10 and 0.1, respectively, when using the Jevons. Of course, extreme observations often result from errors of one kind or another, so extreme price movements should be carefully checked anyway.

9.23 The message emerging from this brief illustration of the behaviour of just three possible formulae is that different index numbers and methods can deliver very different results. Knowledge of these interrelationships is nevertheless not sufficient to determine which formula should be used, even though it makes it possible to make a more informed and reasoned choice. It is necessary to appeal to other criteria in order to settle the choice of formula. There are two main approaches that may be used, the axiomatic and the economic approaches, which presented below. First, however, it is useful to consider the sampling properties of the elementary indices.

Sampling properties of elementary price indices

9.24 The interpretation of the elementary price indices is related to the way in which the sample of goods and services is drawn. Hence, if goods and services in the sample are selected with probabilities proportional to the population expenditure shares in the price reference period,

- the sample (unweighted) Carli index provides an unbiased estimate of the population Laspeyres price index, and
- the sample (unweighted) Jevons index provides an unbiased estimate of the population Geometric Laspeyres price index (see equation (9.6))

9.25 If goods and services are sampled with probabilities proportional to population quantity shares in the price reference period, the sample (unweighted) Dutot index would provide an estimate of the population Laspeyres price index. However, if the basket for the Laspeyres index contains different kinds of products whose quantities are not additive, the quantity shares, and hence the probabilities, are undefined.

Axiomatic approach to elementary price indices

9.26 As explained in Chapters 16 and 20, one way in which to decide upon an appropriate index formula is to require it to satisfy certain specified axioms or tests. The tests throw light on the properties possessed by different kinds of indices, some of which may not be intuitively obvious. Four basic tests will be cited here to illustrate the axiomatic approach:

- *Proportionality test* – if all prices are λ times the prices in the price reference period, the index should equal λ. The data for July, when every price is 10 per cent higher than in January, show that all three direct indices satisfy this test. A special case of this test is the *identity test*, which requires that if the price of every item is the same as in the reference period, the index should be equal to unity, as in May in the example.
- *Changes in the units of measurement test (commensurability test)* – the price index should not change if the quantity units in which the products are measured are changed, for example, if the prices are expressed per litre rather than per pint. The Dutot index fails this test, as explained below, but the Carli and Jevons indices satisfy the test.
- *Time reversal test* – if all the data for the two periods are interchanged, the resulting price index should equal the reciprocal of the original price index. The Carli index fails this test, but the Dutot and the Jevons indices both satisfy the test. The failure of the Carli to satisfy the test is not immediately obvious from the example, but can easily be verified by interchanging the prices in January and April, for example, in which case the backwards Carli for January based on April is equal to 91.3 whereas the reciprocal of the forwards Carli is 1/132.5 or 75.5.
- *Transitivity test* – the chain index between two periods should equal the direct index between the same two periods. It can be seen from the example that the Jevons and the Dutot indices both satisfy this test, whereas the Carli index does not. For example, although the prices in May have returned to the same levels as in January, the chain Carli registers 113.9. This illustrates the fact that the Carli may have a significant built-in upward bias.

9.27 Many other axioms or tests can be devised, but the above are sufficient to illustrate the approach and also to throw light on some important features of the elementary indices under consideration here.

9.28 The sets of products covered by elementary aggregates are meant to be as homogeneous as possible. If they are not fairly homogeneous, the failure of the Dutot index to satisfy the units of measurement or commensurability test can be a serious disadvantage. Although defined as the ratio of the unweighted arithmetic average prices, the Dutot index may also be interpreted as a weighted arithmetic average of the price ratios in which each ratio is weighted by its price in the base period. This can be seen by rewriting formula (9.2) above as

$$P_D^{0:t} = \frac{\frac{1}{n}\sum p_i^0 \left(p_i^t / p_i^0 \right)}{\frac{1}{n}\sum p_i^0} \qquad (9.4)$$

However, if the products are not homogeneous, the relative prices of the different items may depend quite arbitrarily on the quantity units in which they are measured.

9.29 Consider, for example, salt and pepper, which are found within the same sub-class of COICOP. Suppose the unit of measurement for pepper is changed from grams to ounces, while leaving the units in which salt is measured (say kilos) unchanged. As an ounce of pepper is equal to 28.35 grams, the "price" of pepper increases by over 28 times, which effectively increases the weight given to pepper in the Dutot index by over 28 times. The price of pepper relative to salt is inherently arbitrary, depending entirely on the choice of units in which to measure the two goods. In general, when there are different kinds of products within the elementary aggregate, the Dutot index is not acceptable.

9.30 The Dutot index is acceptable only when the set of items covered is homogeneous, or at least nearly homogeneous. For example, it may be acceptable for a set of apple prices even though the apples may be of different varieties, but not for the prices of a number of different kinds of fruits, such as apples, pineapples and bananas, some of which may be much more expensive per item or per kilo than others. Even when the items are fairly homogeneous and measured in the same units, the Dutot's implicit weights may still not be satisfactory. More weight is given to the price changes for the more expensive items, but in practice they may well account for only small shares of the total expenditure within the aggregate. Consumers are unlikely to buy items at high prices if the same items are available at lower prices.

9.31 It may be concluded that from an axiomatic viewpoint, both the Carli and the Dutot indices, although they have been, and still are, widely used by statistical offices, have serious disadvantages. The Carli index fails the time reversal and transitivity tests. In principle, it should not matter whether we choose to measure price changes forwards or backwards in time. We would expect the same answer, but this is not the case for the Carli. Chained Carli indices may be subject to a significant upward bias. The Dutot index is meaningful for a set of homogeneous items but becomes increasingly arbitrary as the set of products becomes more diverse. On the other hand, the Jevons index satisfies all the tests listed above and also emerges as the preferred index when the set of tests is enlarged, as shown in Chapter 20. From an axiomatic point of view, the Jevons index is clearly the index with the best properties, even though it may not have been used much until recently. There seems to be an increasing tendency for statistical offices to switch from using Carli or Dutot indices to the Jevons index.

Economic approach to elementary price indices

9.32 In the economic approach, the objective is to estimate an economic index **–** that is, a *cost of living index* for the elementary aggregate (see Chapter 20). The items for which prices are collected are treated as if they constituted a basket of goods and services purchased by consumers, from which the consumers derive utility. A cost of living index measures the minimum amount by which consumers would have to change their expenditures in order to keep their utility level unchanged, allowing consumers to make substitutions between the items in response to changes in the relative prices of items.

9.33 The economic approach is based on a number of assumptions about consumer behaviour and market conditions, and the representativity of the sample. All of these assumptions often do not hold in reality. At the detailed level of elementary aggregates special conditions will often prevail and change over time and the information available about establishments, products and market conditions may be incomplete. Thus, although the economic approach may be useful in providing a possible economic interpretation of the index, conclusions should be made with caution. In general, in the decision of how to calculate the elementary indices one should be careful not to put too much weight on a strict the economic interpretation of the index formula on the expense of the statistical considerations.

9.34 In the absence of information about quantities or expenditures within an elementary aggregate, an economic index can only be estimated when certain special conditions are assumed to prevail. There are two special cases of some interest. The first case is when consumers continue to consume the same *relative* quantities whatever the relative prices. Consumers prefer not to make any substitutions in response to changes in relative prices. The cross-elasticities of demand are zero. The underlying preferences are described in the economics literature as “Leontief”. With these preferences, a Laspeyres index would provide an exact measure of the cost of living index. In this first case, the Carli index calculated for a random sample would provide an estimate of the cost of living index provided that the items are selected with probabilities proportional to the population expenditure shares. If the items were selected with probabilities proportional to the population quantity shares (assuming the quantities are additive), the sample Dutot would provide an estimate of the population Laspeyres.

9.35 The second case occurs when consumers are assumed to vary the quantities they consume in inverse proportion to the changes in relative prices. The cross-elasticities of demand between the different items are all unity, the expenditure shares being the same in both periods. The underlying preferences are described as “Cobb-Douglas”. With these preferences, the *geometric Laspeyres* would provide an exact measure of the cost of living index. The geometric Laspeyres is a weighted geometric average of the price relatives, using the expenditure shares in the earlier period as weights (the expenditure shares in the second period would be the same in the particular case under consideration). In this second case, the Jevons index calculated for a random sample would provide an unbiased estimate of the cost of living index, provided that the items are selected with probabilities proportional to the population expenditure shares.

9.36 On the basis of the economic approach, the choice between the sample Jevons and the sample Carli rests on which is likely to approximate the more closely to the underlying cost of living index: in other words, on whether the (unknown) cross-elasticities are likely to be closer to unity or zero, on average. In practice, the cross-elasticities could take on any value ranging up to plus infinity for an elementary aggregate consisting of a set of strictly homogeneous items, i.e., perfect substitutes. It should be noted that in the limit when the products really are homogeneous, there is no index number problem, and the price "index" is given by the ratio of the unit values in the two periods, as explained later. It may be conjectured that the average cross-elasticity is likely to be closer to unity than zero for most elementary aggregates, especially since these should be constructed in such a way as to group together similar items that are close substitutes for each other. Thus, in general, the Jevons index is likely to provide a closer approximation to the cost of living index than the Carli. In this case, the Carli index must be viewed as having an upward bias.

9.37 In the Economic approach, the Jevons index is strictly speaking not a fixed basket index, since the quantities are assumed to vary over time in response to changes in relative prices. As a result of the inverse relation of movements in prices and quantities the expenditure shares are constant over time. Carli and Dutot, on the other hand, keep the quantities fixed while the expenditure shares vary in response to change in relative prices.

9.38 The Jevons index does not imply that expenditure shares remain constant. Obviously, the Jevons can be calculated whatever changes do, or do not occur in the expenditure shares in practice. What the economic approach shows is that if the expenditure shares remain constant (or roughly constant), then the Jevons index can be expected to provide a good estimate of the underlying cost of living index. Similarly, if the relative quantities remain constant, then the Carli index can be expected to provide a good estimate, but the Carli does not actually imply that quantities remain fixed.

9.39 It may be concluded that, on the basis of the economic approach as well as the axiomatic approach, the Jevons emerges as the preferred index in general, although there may be cases in which little or no substitution takes place within the elementary aggregate and the direct Carli might be preferred. The Dutot index may be preferred provided the elementary aggregate consists of homogenous products. The index compiler must make a judgement on the basis of the nature of the products actually included in the elementary aggregate.

Chain versus direct indices for elementary aggregates

9.40 In a direct elementary index, the prices of the current period are compared directly with those of the price reference period. In a chain index, prices in each period are compared with those in the previous period, the resulting short-term indices being chained together to obtain the long-term index, as illustrated in Table 9.1.

9.41 Provided that prices are recorded for the same set of items in every period, as in Table 9.1, any index formula defined as the ratio of the average prices will be transitive: that is, the same result is obtained whether the index is calculated as a direct index or as a chain index. In a chain index, successive numerators and denominators will cancel out, leaving only the average price in the last period divided by the average price in the reference period, which is the same as the direct index. Both the Dutot and the Jevons indices are therefore transitive. As already noted, however, a chain Carli index is not transitive and should not be used because of its upward bias. Nevertheless, the direct Carli remains an option.

9.42 Although the chain and direct versions of the Dutot and Jevons indices are identical when there are no breaks in the series for the individual items, they offer different ways of dealing with new and disappearing items, missing prices and quality adjustments. In practice, products continually have to be dropped from the index and new ones included, in which case the direct and the chain indices may differ if the imputations for missing prices are made differently.

9.43 When a replacement item has to be included in a direct index, it will often be necessary to estimate the price of the new item in the price reference period, which may be some time in the past. The same happens if, as a result of an update of the sample, new items have to be linked into the index. Assuming that no information exists on the price of the replacement item in the price reference period, it will be necessary to estimate it using price ratios calculated for the items that remain in the elementary aggregate, a subset of these items or some other indicator. However, the direct approach should only be used for a limited period of time. Otherwise, most of the reference prices would end up being imputed, which would be an undesirable outcome. This effectively rules out the use of the Carli index over a long period of time, as the Carli can only be used in its direct form anyway, being unacceptable when chained. This implies that, in practice, the direct Carli may be used only if the overall index is chain linked annually, or at intervals of two or three years.

9.44 In a chain index, if an item becomes permanently missing, a replacement item can be linked into the index as part of the ongoing index calculation by including the item in the monthly index as soon as prices for two successive months are obtained. Similarly, if the sample is updated and new products have to be linked into the index, this will require successive old and new prices for the present and the preceding months. For a chain index, however, the missing observation will have an impact on the index for two months, since the missing observation is part of two links in the chain. This is not the case for a direct index, where a single, non-estimated missing observation will only have an impact on the index in the current period. For example, for a comparison between periods 0 and 3, a missing price of an item in period 2 means that the chain index excludes the item for the last link of the index in periods 2 and 3, while the direct index includes it in period 3 since a direct index will be based on items whose prices are available in periods 0 and 3. In general, however, the use of a chain index can make the estimation of missing prices and the introduction of replacements easier from a computational point of view, whereas it may be inferred that a direct index will limit the usefulness of overlap methods for dealing with missing observations.

9.45 The direct and the chain approaches also produce different by-products that may be used for monitoring price data. For each elementary aggregate, a chain index approach gives the latest monthly price change, which can be useful for both data editing and imputation of missing prices. By the same token, however, a direct index derives average price levels for each elementary aggregate in each period, and this information may be a useful by-product. Nevertheless, because the availability of cheap computing power and of spreadsheets allows such by-products to be calculated whether a direct or a chained approach is applied, the choice of formula should not be dictated by considerations regarding by-products.

Consistency in aggregation

9.46 Consistency in aggregation means that if an index is calculated stepwise by aggregating lower-level indices to obtain indices at progressively higher levels of aggregation, the same overall result should be obtained as if the calculation had been made in one step. For presentational purposes this is an advantage. If the elementary aggregates are calculated using one formula and the elementary aggregates are averaged to obtain the higher-level indices using another formula, the resulting CPI is not consistent in aggregation. It may be argued, however, that consistency in aggregation is not necessarily an important or even appropriate criterion, or that it is unachievable when the amount of information available on quantities and expenditures is not the same at the different levels of aggregation. In addition, there may be different degrees of substitution within elementary aggregates as compared to the degree of substitution between products in different elementary aggregates.

9.47 The Carli index would be consistent in aggregation with the Laspeyres index if the items were to be selected with probabilities proportional to expenditures in the reference period. This is typically not the case. The Dutot and the Jevons indices are also not consistent in aggregation with a higher-level Laspeyres. As explained below, however, the CPIs actually calculated by statistical offices are usually not true Laspeyres indices anyway, even though they may be based on fixed baskets of goods and services. If the higher-level index were to be defined as a geometric Laspeyres, consistency in aggregation could be achieved by using the Jevons index for the elementary indices at the lower level, provided that the individual items are sampled with probabilities proportional to expenditures. Although unfamiliar, a geometric Laspeyres has desirable properties from an economic point of view and is considered again later.

Missing price observations

9.48 The price of an item may fail to be collected in some period either because the item is missing temporarily or because it has permanently disappeared. The two classes of missing prices require different treatment. Temporary unavailability may occur for seasonal items (particularly for fruit, vegetables and clothing), because of supply shortages or possibly because of some collection difficulty (say, an outlet was closed or a price collector was ill). The treatment of seasonal items raises a number of particular problems. These are dealt with in Chapter 22 and will not be discussed here.

Treatment of temporarily missing prices

9.49 In the case of temporarily missing observations for non-seasonal items, one of four actions may be taken:

- Omit the item for which the price is missing so that a matched sample is maintained (like is compared with like) even though the sample is depleted
- Carry forward the last observed price
- Impute the missing price by the average price change for the prices that are available in the elementary aggregate
- Impute the missing price by the price change for a particular comparable item from another similar outlet

9.50 Omitting an observation from the calculation of an elementary index is equivalent to assuming that the price would have moved in the same way as the average of the prices of the items that remain included in the index. Omitting an observation changes the implicit weights attached to the other prices in the elementary aggregate.

9.51 Carrying forward the last observed price should be avoided wherever possible and is acceptable only for a very limited number of periods. Special care needs to be taken in periods of high inflation or when markets are changing rapidly as a result of a high rate of innovation and product turnover. While simple to apply, carrying forward the last observed price biases the resulting index towards zero change. In addition, when the price of the missing item is recorded again, there is likely to be a compensating step-change in the index to return to its proper value. The adverse effect on the index will be increasingly severe if the item remains unpriced for some length of time. In general, to carry forward is not an acceptable procedure or solution to the problem.

9.52 Imputation of the missing price by the average change of the available prices may be applied for elementary aggregates where the prices can be expected to move in the same direction. The imputation can be made using all of the remaining prices in the elementary aggregate. As already noted, this is numerically equivalent to omitting the item for the immediate period, but it is useful to make the imputation so that if the price becomes available again in a later period the sample size is not reduced in that period. In some cases, depending on the homogeneity of the elementary aggregate, it may be preferable to use only a subset of items from the elementary aggregate to estimate the missing price. In some instances, this may even be a single comparable item from a similar type of outlet whose price change can be expected to be similar to the missing one.

9.53 Table 9.2 illustrates the calculation of the price index for an elementary aggregate consisting of three items where one of the prices is missing in March. Section (a) of Table 9.2 shows the indices where the missing price has been omitted from the calculation. The direct indices are therefore calculated on the basis of items A, B and C for all months except March, where they are calculated on the basis of items B and C only. The chained indices are calculated on the basis of all three prices from January to February and from April to May. From February to March and from March to April the monthly indices are calculated on the basis of items B and C only.

9.54 For both the Dutot and the Jevons indices, the direct and chain indices now differ from March onwards. The first link in the chain index (January to February) is the same as the direct index, so the two indices are identical numerically. The direct index for March completely ignores the price decrease of item A between January and February, while this is taken into account in the chain index. As a result, the direct index is higher than the chain index for March. On the other hand, in April and May, when all prices are again available, the direct index captures the price development, whereas the chain index fails to track the development in the prices.

9.55 In section (b) of Table 9.2 the missing price for item A in March is imputed by the average price change of the remaining items from February to March. While the index may be calculated as a direct index, comparing the prices of the present period with the reference period prices, the imputation of missing prices should be made on the basis of the average price change from the preceding to the present period, as shown in the table. Imputation on the basis of the average price change from the base period to the present period should not be used as it ignores the information about the price change of the missing item that has already been included in the index. The treatment of imputations is discussed in more detail in Chapter 7.

Table 9.2 Imputation of temporarily missing prices

	January	February	March	April	May
			Prices		
Item A	6.00	5.00		7.00	6.60
Item B	7.00	8.00	9.00	8.00	7.70
Item C	2.00	3.00	4.00	3.00	2.20
(a) Omit missing item from the index calculation					
Carli index – the arithmetic mean of price ratios					
Direct index	100.00	115.87	164.29	126.98	110.00
Dutot index – the ratio of arithmetic mean prices					
Month-to-month index	100.00	106.67	118.18	84.62	91.67
Chained m/m index	100.00	106.67	126.06	106.67	97.78
Direct index	100.00	106.67	144.44	120.00	110.00
Jevons index – the ratio of geometric mean prices = geometric mean of price ratios					
Month-to-month index	100.00	112.62	122.47	81.65	87.31
Chained m/m index	100.00	112.62	137.94	112.62	98.33
Direct index	100.00	112.62	160.36	125.99	110.00
(b) Imputation					
Carli index – the arithmetic mean of price ratios					
Impute price for item A in March as 5×(9/8+4/3)/2 = 6.15					
Direct index	100.00	115.87	143.67	126.98	110.00
Dutot index – the ratio of arithmetic mean prices					
Impute price for item A in March as 5×((9+4)/(8+3)) = 5.91					
Month-to-month index	100.00	106.67	118.18	95.19	91.67
Chained m/m index	100.00	106.67	126.06	120.00	110.00
Direct index	100.00	106.67	126.06	120.00	110.00
Jevons index – the ratio of geometric mean prices = geometric mean of price ratios					
Impute price for item A in March as $5\times(9/8\times4/3)^{0.5} = 6.12$					
Month-to-month index	100.00	112.62	122.47	91.34	87.31
Chained m/m index	100.00	112.62	137.94	125.99	110.00
Direct index	100.00	112.62	137.94	125.99	110.00

Treatment of permanently disappeared items

9.56 Items may disappear permanently for a variety of reasons. The item may disappear from the market because new items have been introduced or the outlets from which the price has been collected have stopped selling the product. Where products disappear permanently, a replacement product has to be sampled and included in the index. The replacement product should ideally be one that accounts for a significant proportion of sales, is likely to continue to be sold for some time, and is likely to be representative of the sampled price changes of the market that the old product covered.

9.57 The timing of the introduction of replacement items is important. Many new products are initially sold at high prices which then gradually drop over time, especially as the volume of sales increases. Alternatively, some products may be introduced at artificially low prices to stimulate demand. In such cases, delaying the introduction of a new or replacement item until a large volume of sales is achieved may miss some systematic price changes that ought to be captured by CPIs. It may be desirable to try to avoid forced replacements caused when products disappear completely from the market, and to try to introduce replacements when sales of the items they replace are falling away, but before they cease altogether.

9.58 Table 9.3 shows an example where item A disappears after March and item D is included as a replacement from April onwards. Items A and D are not available on the market at the same time and their price series do not overlap.

Table 9.3 Disappearing items and their replacements with no overlapping prices

	January	February	March	April	May
			Prices		
Item A	6.00	7.00	5.00		
Item B	3.00	2.00	4.00	5.00	6.00
Item C	7.00	8.00	9.00	10.00	9.00
Item D				9.00	8.00
(a) Imputation					
Carli index – the arithmetic mean of price ratios					
Impute price for item D in January as 9/((5/3+10/7)×0.5) = 5.82					
Direct index	100.00	99.21	115.08	154.76	155.38
Dutot index – the ratio of arithmetic mean prices					
Impute price for item D in March as 9/((5+10)/(4+9)) = 7.80					
Month-to-month index	100.00	106.25	105.88	115.38	95.83
Chained m/m index	100.00	106.25	112.50	129.81	124.40
Impute price for item D in January as 9/((5+10)/(3+7)) = 6.00					
Direct index	100.00	106.25	112.50	150.00	143.75
Jevons index – the ratio of geometric mean prices = geometric mean of price ratios					
Impute price for item D in March as $9/((5/4\times10/9)^{0,5}) = 7.64$					
Month-to-month index	100.00	96.15	117.13	117.85	98.65
Chained m/m index	100.00	96.15	112.62	132.73	130.94
Impute price for item D in January as $9/((5/3\times10/7)^{0.5}) = 5.83$					
Direct index	100.00	96.15	112.62	154.30	152.22
(b) Omit missing prices					
Dutot index – the ratio of arithmetic mean prices					
Month-to-month index	100.00	106.25	105.88	115.38	95.83
Chained m/m index	100.00	106.25	112.50	129.81	124.40
Jevons index – the ratio of geometric mean prices = geometric mean of price ratios					
Monthly index	100.00	96.15	117.13	117.85	98.65
Chained m/m index	100.00	96.15	112.62	132.73	130.94

9.59 To include the new item in the index from April onwards, an imputed price needs to be calculated either for the base period (January) if a direct index is being calculated or for the preceding period (March) if a chain index is calculated. In both cases, the imputation method ensures that the inclusion of the new item does not, in itself, affect the index. In the case of a chain index, imputing the missing price by the average change of the available prices gives the same result as if the item is simply omitted from the index calculation until it has been priced in two successive periods. This allows the chain index to be compiled by simply chaining the month-to-month index between periods t-1 and t, based on the matched set of prices in those two periods, onto the value of the chain index for period t-1. In the example, no further imputation is required after April, and the subsequent movement of the index is unaffected by the imputed price change between March and April.

9.60 In the case of a direct index, however, an imputed price is always required for the reference period in order to include a new item. In the example, the price of the new item in each month after April still has to be compared with the imputed price for January. As already noted, to prevent a situation in which most of the reference period prices end up being imputed, the direct approach should only be used for a limited period of time.

9.61 The situation is somewhat simpler when there is an overlap month in which prices are collected for both the disappearing and the replacement item. In this case, it is possible to link the price series for the new item to the price series for the old item that it replaces. Linking with overlapping prices involves making an implicit adjustment for the difference in quality between the two items, as it assumes that the relative prices of the new and old item reflect their relative qualities. For perfect or nearly perfect markets this may be a valid assumption, but for certain markets and products it may not be so reasonable. The question of when to use overlapping prices is dealt with in detail in Chapter 7. The overlap method is illustrated in Table 9.4.

Table 9.4 Disappearing and replacement items with overlapping prices

	January	February	March	April	May
			Prices		
Item A	6.00	7.00	5.00		
Item B	3.00	2.00	4.00	5.00	6.00
Item C	7.00	8.00	9.00	10.00	9.00
Item D			10.00	9.00	8.00
Carli index – the arithmetic mean of price ratios					
Impute price for item D in January as 6/(5/10) = 12.00					
Direct index	100.00	99.21	115.08	128.17	131.75
Dutot index – the ratio of arithmetic mean prices					
Chain the monthly indices based on matched prices					
Month-to-month index	100.00	106.25	105.88	104.35	95.83
Chained m/m index	100.00	106.25	112.50	117.39	112.50
Divide item D's price in April and May by 10/5 = 2 and use item A's price in January as base price					
Direct index	100.00	106.25	112.50	121.88	118.75
Impute price for item D in January as 6/(5/10) = 12.00					
Direct index	100.00	106.25	112.50	109.09	104.55
Jevons index – the ratio of geometric mean prices = geometric mean of price ratios					
Chain the monthly indices based on matched prices					
Month-to-month index	100.00	96.15	117.13	107.72	98.65
Chained m/m index	100.00	96.15	112.62	121.32	119.68
Divide item D's price in April and May with 10/5 = 2 and use item A's price in January as base price					
Direct index	100.00	96.15	112.62	121.32	119.68
Impute price for item D in January as 6/(5/10) = 12.00					
Direct index	100.00	96.15	112.62	121.32	119.68

9.62 In the example in Table 9.4, overlapping prices are obtained for items A and D in March. Their relative prices suggest that one unit of item D is worth two units of item A. If the index is calculated as a direct Carli, the January base period price for item D can be imputed by dividing the price of item A in January by the price ratio of items A and D in March.

9.63 A monthly chain index of arithmetic mean prices will be based on the prices of items A, B and C until March, and from April onwards on the prices of items B, C and D. The replacement item is not included until prices for two successive periods are obtained. Thus, the monthly chain index has the advantage that it is not necessary to carry out any explicit imputation of a reference price for the new item.

9.64 If a direct index is calculated defined as the ratio of the arithmetic mean prices, the price of the new item needs to be adjusted by the price ratio of A and D in March in every subsequent month, which complicates computation. Alternatively, a reference period price of item D for January may be imputed. This, however, results in a different index because the price ratios are implicitly weighted by the relative base period prices in the Dutot index, which is not the case for the Carli or the Jevons indices. For the Jevons index, all three methods give the same result, which is an additional advantage of this approach.

Calculation of elementary price indices using weights

9.65 The Carli, Dutot and Jevons indices are all calculated without the use of explicit weights. However, as already mentioned, in certain cases weighting information may be available that could be exploited in the calculation of the elementary price indices. Weights within elementary aggregates may be updated independently and possibly more often than the elementary aggregate weights themselves.

9.66 A special situation occurs in the case of tariff prices. A tariff is a list of prices for the purchase of a particular kind of good or service under different terms and conditions. One example is electricity, where one price is charged during daytime while a lower price is charged at night. Similarly, a telephone company may charge a lower price for a call at the weekend than in the rest of the week. Another example may be bus tickets sold at one price to ordinary passengers and at lower prices to children or old age pensioners. In such cases, it is appropriate to assign weights to the different tariffs or prices in order to calculate the price index for the elementary aggregate.

9.67 The increasing use of electronic points of sale in many countries, in which both prices and quantities are scanned as the purchases are made, means that valuable new sources of information may become increasingly available to statistical offices. This could lead to significant changes in the ways in which price data are collected and processed for CPI purposes. The treatment of scanner data is examined in Chapters 7, 8 and 21.

9.68 If the reference period expenditures for all the individual items within an elementary aggregate, or estimates thereof, were to be available, the elementary price index could itself be calculated as a Laspeyres price index, or as a geometric Laspeyres.

9.69 The Laspeyres price index is defined as:

$$P_L^{0:t} = \frac{\sum p_i^t \cdot q_i^0}{\sum p_i^0 \cdot q_i^0} = \sum w_i^0 \cdot \left(\frac{p_i^t}{p_i^0}\right), \quad w_i^0 = \frac{p_i^0 \cdot q_i^0}{\sum p_i^0 \cdot q_i^0} \qquad (9.5)$$

w_i^0 indicates the expenditure shares for the individual items in the reference period. As the quantities are often unknown, the index usually will have to be calculated by weighting together the individual price ratios by their expenditure share in the price reference period,

w_i^0. The available weighting data may refer to an earlier period than the price reference period, but may still provide a good estimate. A more general version of (9.5) would be that of a Lowe or a Young index, where the weights are not necessarily those of the price reference period. These two indices are discussed in more details later in this chapter. Note that if all shares are equal, equation (9.5) reduces to the Carli index. If the shares are proportional to the prices in the reference period, it reduces to the Dutot index.

9.70 The geometric version of the Laspeyres index is defined as:

$$P_{GL}^{0:t} = \prod \left(\frac{p_i^t}{p_i^0} \right)^{w_0^i} = \frac{\prod \left(p_i^t \right)^{w_i^0}}{\prod \left(p_i^0 \right)^{w_i^0}} \;, \quad \sum w_i^0 = 1 \qquad (9.6)$$

where the weights, w_i^0, are again the expenditure shares in the reference period. When the weights are all equal, equation (9.6) reduces to the Jevons index. If the expenditure shares do not change much between the weight reference period and the current period, then the geometric Laspeyres index approximates a Törnqvist index. A more general version of (9.6) would be that of a Geometric Young index, where the weights are not necessarily those of the price reference period.

9.71 The weights may be attached to the individual price observations or to groups of price observations. For example two outlets may both report, say, 5 prices that enter into the calculation of an elementary aggregate price index. However, the only weighting information may refer to the overall relative market share of the two establishments rather than to the individual products. Thus, if the relative market shares are 40/60, the two groups of prices may be weighted according to the 40/60 shares of the establishments.

9.72 Table 9.5 provides an example of calculation of an elementary index using weights. The elementary aggregate consists of three items for which prices are collected monthly. The expenditure shares are estimated to 0.80, 0.17 and 0.03.

Table 9.5 Calculation of a weighted elementary index

	Weight	December	January	February	Pct. Change Dec. – Feb.
Item *A*	0.80	7	7	9	28.6
Item *B*	0.17	20	20	10	-50.0
Item *C*	0.03	28	28	12	-57.1
Weighted arithmetic mean of price ratios (Laspeyres)					**Index**
((9/7)*0.8 + (10/20)*0.17 + (12/28)*0.03) * 100 =					112.64
Weighted geometric mean of price ratios (geometric Laspeyres)					
$((9/7)^{0.8} * (10/20)^{0.17} * (12/28)^{0.03}) * 100 =$					105.95
Ratio of weighted arithmetic mean prices					
Weighted arithmetic mean prices		9.84	9.84	9.26	
(9.26/9.84) * 100 =					94.11

9.73 One option is to calculate the index as the weighted arithmetic mean of the price ratios, which gives an index of 112.64. The individual price changes are weighted according to their explicit weights, irrespective of the price levels. This corresponds to the calculation of a Laspeyres price index, where the price ratios and the weights refer to the same reference month. The index may also be calculated as the weighted geometric mean of the price ratios, the geometric Laspeyres index, which gives an index of 105.95.

9.74 A third option would be to calculate the index as the ratio of the weighted arithmetic mean prices. As already noted, an elementary index should only be based on arithmetic mean prices if it includes homogenous products measured in the same unit; otherwise it is not meaningful to calculate an average price. In practice, this will also mean that the price level of the products should be more or less the same. Secondly, this approach weights the price changes according to the relative price level in the reference period. Hence, the increase of 28.6% on item A that accounts for 80% of the market is down weighted because of its relative low price, resulting in an index of 94.11. This calculation is misleading, however.

9.75 The difference between the two arithmetic methods can be illustrated by an example: Assume an elementary aggregate with two commodities, X and Y of equal weights (50/50). The price of X is constant 90, the price of Y increases from 10 to 12. The weighted arithmetic mean of the price ratios gives 90/90 + 12/10 = 1.10. The ratio of arithmetic weighted prices gives (90+12)/(90+10) = 1.02. In the first approach, the price increases of the two commodities are equally weighted, which gives an increase of 10%. The problem in the second approach is that it weights the 0% price increase on X by 90/100, and the 20% increase of Y by only 10/100, which gives an overall increase of 2%. This can only be justified, if the weights are proportional to the relative price level in the reference period. That is, if the weight of X is 90 and that of Y is 10, which, however, contradicts the assumption of 50/50 weights. Because of the calculation method the weights are twisted according to the relative price levels resulting in a misleading index.

9.76 Weighting information at the very detailed level is resource demanding to obtain and update. This has to be balanced against the possible gains in terms of a more accurate price index, and in some cases, it may be the better option to use an unweighted approach.

Other formulae for elementary price indices

9.77 Another type of average is the harmonic mean. In the present context, there are two possible versions: either the harmonic mean of price ratios or the ratio of harmonic mean prices. The harmonic mean of price ratios is defined as:

$$P_{HR}^{0:t} = \frac{1}{\frac{1}{n}\sum \frac{p_i^0}{p_i^t}} \qquad (9.7)$$

The ratio of harmonic mean prices is defined as:

$$P_{RH}^{0:t} = \frac{\sum \frac{n}{p_i^0}}{\sum \frac{n}{p_i^t}} \tag{9.8}$$

Formula (9.8), like the Dutot index, fails the commensurability test and would only be an acceptable possibility when the items are all fairly homogeneous. Neither formula appears to be used much in practice, perhaps because the harmonic mean is not a familiar concept and would not be easy to explain to users. Nevertheless, at an aggregate level, the widely used Paasche index is a weighted harmonic average.

9.78 The ranking of the three common types of mean is always arithmetic ≥ geometric ≥ harmonic. It is shown in Chapter 20 that, in practice, the Carli index (the arithmetic mean of the price ratios) is likely to exceed the Jevons index (the geometric mean) by roughly the same amount that the Jevons exceeds the harmonic mean. The harmonic mean of the price relatives has the same kinds of axiomatic properties as the Carli index, but with opposite tendencies and biases. It fails the transitivity, time reversal and price bouncing tests.

9.79 In recent years, attention has focused on formulae that can take account of the substitution that may take place within an elementary aggregate. As already explained, the Carli and the Jevons indices may be expected to approximate a cost of living index if the cross-elasticities of substitution are close to 0 and 1, respectively, on average. A more flexible formula that allows for different elasticities of substitution is the unweighted Lloyd-Moulton (LM) index:

$$P_{LM}^{0:t} = \left[\sum \frac{1}{n} \left(\frac{P_i^t}{P_i^0} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{9.9}$$

where σ is the elasticity of substitution. The Carli and the Jevons indices can be viewed as special cases of the LM in which $\sigma = 0$ and $\sigma = 1$. The advantage of the LM formula is that σ is unrestricted. Provided a satisfactory estimate can be made of σ, the resulting elementary price index is likely to approximate to the underlying cost of living index. The LM index reduces "substitution bias" when the objective is to estimate the cost of living index. The difficulty is the need to estimate elasticities of substitution, a task that will require substantial development and maintenance work. The formula is described in more detail in Chapter 17.

Unit value indices

9.80 The unit value index is simple in form. The unit value in each period is calculated by dividing total expenditure on some product by the related total quantity. It is clear that the quantities must be strictly additive in an economic sense, which implies that they should relate to a single homogeneous product. The unit value index is then defined as the ratio of unit values in the current period to that in the reference period. It is not a price index as normally understood, as it is essentially a measure of the change in the average price of a *single* product when that product is sold at different prices to different consumers, perhaps at different times within the same period. Unit values, and unit value indices, should not be calculated for sets of heterogeneous products.

9.81 Unit values do play an important part in the process of calculating an elementary price index, as they are the appropriate average prices that need to be entered into an elementary price index. Usually, prices are sampled at a particular time or period each month, and each price is assumed to be representative of the average price of that item in that period. In practice, this assumption may not hold. In this case, it is necessary to estimate the unit value for each item, even though this will inevitably be more costly. Thus, having specified the item to be priced in a particular store, data should be collected on both the value of the total sales in a particular month and the total quantities sold in order to derive a unit value to be used as the price input into an elementary aggregate formula. It is particularly important to do this if the item is sold at a sale price for part of the period and at the regular price in the rest of the period. Under these conditions, neither the sale price nor the regular price is likely to be representative of the average price at which the item has been sold or the price change between periods. The unit value over the whole month should be used. With the possibility of collecting more and more data from electronic points of sale, such procedures may be increasingly used. It should be stressed, however, that the item specifications must remain constant through time. Changes in the item specifications could lead to unit value changes that reflect quantity or quality changes, and should not be part of price changes.

Formulae applicable to scanner data

9.82 Scanner data obtained from electronic points of sale are becoming an increasingly important source of data for CPI compilation. Their main advantage is that the number of price observations can be enormously increased and that both price and quantity information is available in real time. There are, however, many practical considerations to be taken into account, which are discussed in other chapters of this manual.

9.83 Access to detailed and comprehensive quantity and expenditure information within an elementary aggregate means that there are no constraints on the type of index number that may be employed. Not only Laspeyres and Paasche but superlative indices such as Fisher and Törnqvist may be envisaged. As noted at the beginning of this chapter, it is preferable to introduce weighting information as it becomes available rather than continuing to rely on simple unweighted indices such as Carli and Jevons. Advances in technology, both in the retail outlets themselves and in the computing power available to statistical offices, suggest that traditional elementary price indices may eventually be replaced by superlative indices, at least for some elementary aggregates in some countries. The methodology must be kept under review in the light of the resources available.

The calculation of higher-level indices

9.84 A statistical office must have some target index at which to aim. Statistical offices have to consider what kind of index they would choose to calculate in the ideal hypothetical situation in which they had complete information about prices and quantities in both time periods compared. If the CPI is meant to be a *cost of living index*, then a superlative index such as a Fisher, Walsh or Tornqvist-Theil would have to serve as the theoretical target, as a superlative index may be expected to approximate to the underlying cost of living index.

9.85 Many countries do not aim to calculate a cost of living index and prefer the concept of a fixed *basket index*, sometimes also referred to as a *pure price index* or an *inflation index*. A basket index is one that measures the change in the total value of a given basket of goods and services between two time periods. This general category of index is described here as a *Lowe index* (see Chapter 15). It should be noted that, in general, there is no necessity for the basket to be the actual basket in one or other of the two periods compared. If the target index is to be a basket or Lowe index, the preferred basket might be one that attaches equal importance to the baskets in both periods; for example, the Walsh index. Thus, the same index may emerge as the preferred target in both the basket and the cost of living approaches.

9.86 In Chapters 15-17 the superlative indices Walsh, Fisher and Törnqvist show up as being "best" in all the approaches to index number theory. These three indices, and the Marshall-Edgeworth price index, while not superlative, give very similar results so that for any practical reason it will not make any difference which one is chosen as the preferred target index. In practice, a statistical office may prefer to designate a basket index that uses the actual basket in the earlier of the two periods as its target index on grounds of simplicity and practicality. In other words, the Laspeyres index may be the target index.

9.87 The theoretical target index is a matter of choice. In practice, it is likely to be either a Laspeyres or some superlative index. Even when the target index is the Laspeyres, there may be a considerable gap between what is actually calculated and what the statistical office considers to be its target. It is now necessary to consider what statistical offices tend to do in practice.

Consumer price indices as weighted averages of elementary indices

9.88 A higher-level index is an index for some expenditure aggregate above the level of an elementary aggregate, including the overall CPI itself. The inputs into the calculation of the higher-level indices are:.

- The elementary aggregate price indices
- The expenditure shares of the elementary aggregates

9.89 The higher-level indices are calculated simply as weighted arithmetic averages of the elementary price indices. The weights typically remain fixed for a sequence of at least 12 months. Some countries revise their weights at the beginning of each year in order to try to approximate as closely as possible to current consumption patterns, but many countries continue to use the same weights for several years. The weights may be changed only every five years or so. The use of fixed weights has the considerable practical advantage that the index can make repeated use of the same weights. This saves both time and money. Revising the weights can be both time-consuming and costly, especially if it requires new household expenditure surveys to be carried out.

9.90 The second stage of calculating a CPI does not involve individual prices or quantities. Instead, a higher-level index is calculated by averaging the elementary price indices by their pre-determined weights. The formula can be written as follows:

$$P^{0:t} = \sum w_j^b P_j^{0:t} \ , \quad \sum w_j^b = 1 \qquad (9.10)$$

where $P^{0:t}$ denotes the overall CPI, or any higher-level index, from period 0 to t, w_j^b is the weight attached to each of the elementary price indices, and $P_j^{0:t}$ is the corresponding elementary price index. The elementary indices are identified by the subscript j, whereas the higher-level index carries no subscript. As already noted, a higher-level index is any index, including the overall CPI, above the elementary aggregate level. The weights are derived from expenditures in period *b*, which in practice has to precede period 0, the price reference period. This general category of index is described here as a *Young index* after another nineteenth-century index number pioneer who advocated this type of index.

9.91 Provided the elementary aggregate indices are calculated using a transitive formula such as Jevons or Dutot, but not Carli, and provided that there are no new or disappearing items from period *0* to *t*, equation (9.10) is equivalent to:

$$P^{0:t} = \sum w_j^b P_j^{0:t-1} P_j^{t-1:t} \ , \quad \sum w_j^b = 1 \qquad (9.11)$$

The difference is that equation (9.10) is based on the direct elementary indices from *0* to *t*, while (9.11) uses the chained elementary indices. The advantage of the latter is that it allows the sampled products within the elementary price index from *t*-1 to *t* to differ from the sampled products in the periods from *0* to *t*-1. Hence, it allows replacement items and new items to be linked into the index from period *t*-1 without the need to estimate a price for period *0*. For example, if one of the sampled items in periods *0* and *t*-1 is no longer available in period *t*, and the price of a replacement product is available for *t*-1 at *t*, the new replacement product can be included in the index using the overlap method.

9.92 Equations (9.10) and (9.11) are additive and apply at each level of aggregation. That is, a higher-level index is the same whether calculated on the basis of the elementary price indices or on the basis of the intermediate higher-level indices. The additivity also facilitates the presentation of the index.

9.93 It is useful to recall that three kinds of reference period may be distinguished:

- *Weight reference period*. The period covered by the expenditure statistics used to calculate the weights. Usually, the weight reference period is a year.
- *Price reference period*. The period whose prices are used as denominators in the index calculation.
- *Index reference period*. The period for which the index is set to 100.

9.94 The three periods are generally different. For example, a CPI might have 1998 as the weight reference year, December 2002 as the price reference month and the year 2000 as the index reference period. The weights typically refer to a whole year, or even two or three years, whereas the periods for which prices are compared are typically months or quarters. The weights are usually estimated on the basis of an expenditure survey that was conducted some time before the price reference period. For these reasons, the weight reference period and the price reference period are invariably separate periods in practice.

9.95 The index reference period is often a year; but it could be a month or some other period. An index series may also be re-referenced to another period by simply dividing the series by the value of the index in that period, without changing the rate of change of the index. The expression “base period” can mean any of the three reference periods and is ambiguous. The expression “base period” should only be used when it is absolutely clear in context exactly which period is referred to.

A numerical example

9.96 Table 9.5 illustrates the calculation of higher-level indices where the weight and the price reference periods are identical, i.e. b = 0. The index consists of five elementary aggregate indices and two intermediate higher-level indices, G and H. The overall index and the higher-level indices are all calculated using (9.10). Thus, for example, the overall index for April can be calculated from the two intermediate higher-level indices for April as:

$$P^{Jan:apr} = 0,6 \times 103,92 + 0,4 \times 101,79 = 103,06$$

or directly from the five elementary indices as:

$$P^{Jan:apr} = 0,2 \times 108,75 + 0,25 \times 100 + 0,15 \times 104 + 0,1 \times 107,14 + 0,3 \times 100 = 103,06$$

Table 9.6 The aggregation of elementary price indices

	Weight	January	February	March	April	May	June
Month-to-month elementary price indices							
A	0.20	100.00	102.50	104.88	101.16	101.15	100.00
B	0.25	100.00	100.00	91.67	109.09	101.67	108.20
C	0.15	100.00	104.00	96.15	104.00	101.92	103.77
D	0.10	100.00	92.86	107.69	107.14	100.00	102.67
E	0.30	100.00	101.67	100.00	98.36	103.33	106.45
Direct or chained monthly elementary price indices with January = 100							
A	0.20	100.00	102.50	107.50	108.75	110.00	110.00
B	0.25	100.00	100.00	91.67	100.00	101.67	110.00
C	0.15	100.00	104.00	100.00	104.00	106.00	110.00
D	0.10	100.00	92.86	100.00	107.14	107.14	110.00
E	0.30	100.00	101.67	101.67	100.00	103.33	110.00
Total		100.00	100.89	99.92	103.06	105.03	110.00
Higher-level indices							
G=A+B+C	0.60	100.00	101.83	99.03	103.92	105.53	110.00
H=D+E	0.40	100.00	99.46	101.25	101.79	104.29	110.00
Total		100.00	100.89	99.92	103.06	105.03	110.00

Factoring the Young index

9.97 It is possible to calculate the change in a higher-level Young index between two consecutive periods, such as t-1 and t, as a weighted average of the individual price indices between t-1 and t provided that the weights are updated to take account of the price changes between the price reference period 0 and the previous period, t-1. This makes it possible to factor equation (9.10) into the product of two component indices in the following way:

$$\begin{aligned} P^{0:t} &= \sum w_j^b \cdot P_j^{0:t} = \sum w_j^b \cdot P_j^{0:t-1} \frac{\sum w_j^b \cdot P_j^{0:t-1} \cdot P_j^{t-1:t}}{\sum w_j^b \cdot P_j^{0:t-1}} \\ &= P^{0:t-1} \sum \frac{w_j^b \cdot P_j^{0:t-1}}{\sum w_j^b \cdot P_j^{0:t-1}} P_j^{t-1:t} \qquad (9.12) \\ &= P^{0:t-1} \sum w_j^{b(t-1)} P_j^{t-1:t} \ , \\ &\textit{where} \quad w_j^{b(t-1)} = w_j^b P_j^{0:t-1} \Big/ \sum w_j^b P_j^{0:t-1} \end{aligned}$$

$P^{0:t-1}$ is the Young index for period t-1. The weight $w_j^{b(t-1)}$ is the original weight for elementary aggregate j price-updated by multiplying it by the elementary price index for j from *0* to t-1, the adjusted weights being rescaled to sum to unity. The price-updated weights are hybrid weights because they implicitly revalue the quantities of b at the prices of t-1 instead of at the average prices of b. Such hybrid weights do not measure the actual expenditure shares of any period.

9.98 The index for period t can thus be calculated by multiplying the already calculated index for t-1 by a separate Young index between t-1 and t with hybrid price-updated weights. In effect, the higher-level index is calculated as a chain index in which the index is moved forward period by period. Equation (9.12) gives the same flexibility as (9.11) to introduce replacement items and makes it easier to monitor the movements of the recorded prices for errors, as month-to-month movements are smaller and less variable than the long-term price changes since the base period.

9.99 Price-updating of weights in this case does not change the index, because the elementary indices are re-scaled to the same period to which the weights are price-updated. Hence, equation (9.12) is just another way of calculating the index, which may be preferred because of practical reasons. However, the expenditure weights may also be price-updated from the weight reference period to the price reference period, when the weights are applied in the CPI. This type of price-updating will affect the CPI, as explained in the next section.

9.100 Note from equation (9.12) that

$$P^{0:t} / P^{0:t-1} \neq \sum w_j^b P_j^{t-1:t} \qquad (9.12a)$$

The essence of equation (9.12a) is that monthly changes in lower-level indices cannot be aggregated to the monthly change of the overall index using the weights w_i^b. As seen from equation (9.12) the weights need to be updated to reflect the effects of the price changes that have taken place from period 0 to t.

Price-updating of expenditure weights

9.101 As already noted most, if not all, statistical offices calculate the higher-level indices by use of equation (9.10) or the equivalent (9.11) or (9.12). However, for the practical calculation of a CPI the situation is complicated by the fact that the weight reference period usually precede the price reference period and the duration of the weight reference period is typically much longer than the period to which the prices refer. The weights usually refer to the expenditure data over a year, or longer, while the price reference period is usually a month in some later year. For example, a monthly CPI may be compiled from January 2003 onward with December 2002 as the price reference month, but the latest available weights may refer to the year 2000, or even some earlier year.

9.102 This means that the statistical office has to decide if the weights should be re-referenced, or *price-updated*, from the weight reference period to the price reference period, or if the weights should be applied as they stand without price-updating.

9.103 By price-updating the weights are aligned to the same reference period as the prices. If the statistical office decides to price-update the weights, the resulting index will be a *Lowe* *index*. The Lowe index is a *fixed basket index*, which from period to period measures the value of the same (annual) basket of goods and service. It is defined as follows:

$$P_{Lo}^{0:t} = \frac{\sum p_i^t q_i^b}{\sum p_i^0 q_i^b} \tag{9.13}$$

The individual quantities (q_i^b) in the weight reference period *b* make up the basket. The index measures the cost of the period *b* basket in period *t* in relation to the cost of the same basket in period *0*. However, to be used in practice it is necessary first to express the index as a function of volume shares rather than individual quantities:

$$\begin{gathered} P_{Lo}^{0:t} = \frac{\sum p_i^t q_i^b}{\sum p_i^0 q_i^b} = \sum \frac{p_i^0 q_i^b}{\sum p_i^0 q_i^b} \frac{p_i^t}{p_i^0} = \sum w_i^{b(0)} \frac{p_i^t}{p_i^0}, \\ w_i^{b(0)} = \frac{w_i^b \left(p_i^0 / p_i^b \right)}{\sum w_i^b \left(p_i^0 / p_i^b \right)} \ , \quad w_i^b = \frac{p_i^b q_i^b}{\sum p_i^b q_i^b} \end{gathered} \tag{9.14}$$

The individual price ratios are weighted together with their *hybrid* value shares, $w_i^{b(0)}$, i.e. the period *b* quantities valued at period 0 prices. The hybrid shares are calculated by price-updating the period *b* expenditure shares from period *b* to 0. By price-updating the expenditure shares the underlying quantities are kept constant while the expenditure shares are changing according to the development in the relative prices. The practical counterpart of (9.14) is:

$$P_{Lo}^{0:t} = \sum w_j^{b(0)} P_j^{0:t}, \quad w_j^{b(0)} = \frac{w_j^b P_j^{b:0}}{\sum w_j^b P_j^{b:0}} \tag{9.14a}$$

9.104 In practice the higher-level indices are calculated by weighting together the elementary indices by their price-updated weights. The price-updated weights are calculated by multiplying the original period *b* expenditure shares by their elementary indices from period *b* to 0 and rescaling to sum to unity. A CPI calculated by (9.14a) using annual weights measures the change from month to month in the total cost of an annual basket of goods and services that may date back several years before the price reference period.

9.105 The Lowe index is not a Laspeyres index, as the weight and price reference periods do not coincide. However, it reduces to the Laspeyres index when $b = 0$ and to the Paasche index when $b = t$. Further, the Lowe index can be expressed as the ratio of two Laspeyres indices, one from *b* to 0 and one from *b* to *t*:

$$P_{Lo}^{0:t} = \frac{\sum p_i^t q_i^b}{\sum p_i^0 q_i^b} = \frac{\sum p_i^t q_i^b}{\sum p_i^b q_i^b} \bigg/ \frac{\sum p_i^0 q_i^b}{\sum p_i^b q_i^b} \tag{9.15}$$

9.106 From equation (9.15) it follows that the Lowe index from period 0 to *t* will show the same rate of change as a Laspeyres price index with period *b* as weight and price reference period. In other words, price-updating the weights from *b* to 0 means that the index will show the same rate of changes as if the weights had been applied from period *b*.

9.107 Because it uses the fixed basket of an earlier period, the Lowe index is sometimes loosely described as a "Laspeyres-type" index, but this description is unwarranted. A true Laspeyres index requires the basket to be that purchased in the price reference month, whereas in most CPIs the basket refers to a period different from the price reference month. When the weights are annual and the prices are monthly, it is not possible, even retrospectively, to calculate a monthly Laspeyres price index.

9.108 As shown in Chapter 15, a Lowe index from 0 to *t* that uses quantities derived from an earlier period than the price reference period is likely to exceed the Laspeyres index from 0 to *t*, where both weights and prices refer to period 0, and by a progressively larger amount, the further back in time the weight reference period. The Lowe index is likely to have an even greater upward bias than the Laspeyres as compared with some target superlative index or underlying cost of living index.

9.109 The statistical office may decide instead to calculate the higher-level indices without price-updating the weights. This corresponds to the calculation of a *Young index*, in terms of the share weighted arithmetic mean of the price ratios:

$$P_{Yo}^{0:t} = \sum w_i^b \left(\frac{p_i^t}{p_i^0} \right), \quad w_i^b = \frac{p_i^b q_i^b}{\sum p_i^b q_i^b} \qquad (9.16)$$

9.110 The Young index is general in the sense that the shares are not restricted to refer to any particular period, but may refer to any period or an average of different periods, for example. The Young index is a *fixed weight index* where focus is that the weights should be as representative as possible for the average value shares of the period covered by the index. A fixed weight index is not necessarily a fixed basket index, i.e. it does not necessarily measure the change in the value of an actual basket such as the Lowe index. The Young index measures the development in the cost of a period 0 set of purchases with period *b* value proportions between the expenditure components. This does not correspond to the changing value of any actual basket, unless the expenditure shares have remained unchanged from *b* to 0. In the special case where *b* equals 0 it reduces to the Laspeyres. The Young index is described in more details in Chapter 15.

9.111 In practice, when the calculation of higher-level indices has to be based on elementary price indices and weights of the elementary aggregates, the Young index is calculated simply by weighting together the elementary indices from 0 to *t* by their volume shares as they stand without price-updating:

$$P_{Yo}^{0:t} = \sum w_j^b P_j^{0:t} \qquad (9.16a)$$

9.112 Note that even if it is decided to price-update the weights and calculate a fixed basket or Lowe index, the index is calculated in the form of a Young index, namely as a share weighted arithmetic average of the elementary indices. Thus, a Lowe index is equal to a Young index in which the weights are *hybrid* value shares obtained by revaluing the period *b* quantities at the prices of the price reference month.

9.113 By keeping the expenditure shares constant from the weight reference period to the price reference period the underlying quantities are assumed to vary in response to changes in relative prices. Hence, if households tend to keep constant expenditure shares by substituting from goods or services with relative price increases to goods or services with relative price decreases, the period *b* expenditure shares will be good estimates of the expenditure shares in the price reference period when the weights are introduced in the index. In turn, if expenditure shares stay unchanged, the Young index will be a good estimate of a target superlative index. However, if quantities tend to remain constant, i.e. the households does not substitute between goods and services in response to relative price changes, the Young index will be biased downwards compared to a superlative target index.

9.114 The difference between the Young and Lowe indices can be illustrated by subtracting the one from the other:

$$\begin{aligned} P_{Lo}^{0:t} - P_{Yo}^{0:t} &= \sum w_j^{b(0)} \cdot P_j^{0:t} - \sum w_j^{b} \cdot P_j^{0:t} \\ &= \sum \left(w_j^{b(0)} - w_j^{b} \right) P_j^{0:t} \end{aligned} \qquad (9.17)$$

9.115 The Lowe index gives more weight to those elementary indices the prices of which have increased by more than average from *b* to 0 and less weights to those where the prices have increased by less than average. Therefore, if there are long-term trends in the prices, so that prices which have increased relatively from *b* to 0 continues to do so from 0 to *t*, and prices which have fallen from *b* to 0 continues to fall, the Lowe index will exceed the Young index. This indicates a long-run tendency for the Lowe index to exceed the Young index. This effect is build into the formulas and is not related to what may or may not take place in reality in terms of households substituting in response to changing relative prices.

9.116 Whether a Young or Lowe index is the better estimate of a superlative target index depends on whether the original (w^i_b) or the price-updated ($w^i_{b(0)}$) weights is the better estimate of the average expenditure shares from 0 to *t*.[1] If, on average, the elasticity of substitution at the elementary aggregate level is closer to one, Young is the better estimate. If the elasticity of substitution is closer to zero, on average, Lowe is the better estimate.

9.117 Normal consumer behaviour suggests that in general some substitution should be expected, so that the Lowe index will tend to be biased upward compared to a superlative target index. As the Young index allow for some substitution from *b* to *0,* while Lowe does not, it may be argued that the traditional Laspeyres bias to some degree is reduced in the Young index as compared to the Lowe index. Thus, to omit price-updating may be one practical way in which to reduce this type of bias.

9.118 It is up to the statistical offices to decide for themselves whether to price-update the expenditure shares or not. If the primary aim is to compile a CPI that measures the price development of an actual fixed basket of goods and services, then the weights should be price-updated. The resulting fixed basket, or Lowe, index will provide a good estimate of the price development if quantities tend to remains constant.

[1] Both Lowe and Young can be expressed in the same form as the Walsh index, as the expenditure share weighted arithmetic average of the price ratios, the only difference being the weighting component.

9.119 If the statistical office consider that the expenditure shares of the weight reference period are the better estimates of the average expenditure shares in the period in which the weights are supposed to be used, this may be an argument for applying the expenditure shares as they stand without price-updating. If the volume shares roughly remain constant, it will provide a good estimate of the price development.

9.120 Price-updating the expenditure shares does not imply that the resulting weights are necessarily more up to date. When there is a strong inverse relation between movements of price and quantities, price-updating on its own could produce perverse results. For example, the price of computers has been declining rapidly in recent years. If the quantities are held fixed while the prices are updated, the resulting expenditure weights on computers would also decline rapidly. In practice, however, the expenditure weight for computers might actually be rising because of a very rapid increase in quantities of computers purchased.

9.121 Both quantities and expenditure shares changes through time and progressively more, the longer the elapse of time between the weight reference period and the period when the weights are introduced in the index. Thus, whether the weights are price-updated or not, they should be reviewed and updated frequently to reduce potential bias. When rapid changes take place in relative quantities as well as relative prices, statistical offices are effectively obliged to change the expenditure weights more often. Price-updating on its own cannot cope with this situation. The weighting basis has to be updated with respect to both quantities as well as prices, which, in effect, implies that new weights have to be included.

Example of price-updating

9.122 In practice, the price-updated weights can be calculated by multiplying the original period *b* expenditure weights (w_j^b) by the corresponding elementary price indices ($P_j^{b:0}$) from period *b* to period *0*, and rescaling to sum to unity. Thus, as shown in equation (9.14a), the weights price-updated from *b* to *0* $\left(w_j^{b(0)}\right)$ can be calculated as:

$$w_j^{b(0)} = \frac{w_j^b \cdot P_j^{b:0}}{\sum w_j^b \cdot P_j^{b:0}} \qquad (9.18)$$

9.123 In Table 9.7, the base period *b* is assumed to be the year 2000, so the weights are the expenditure shares in 2000. In section (a) of the table, 2000 is also used as the price reference period. In practice, however, weights based on 2000 cannot be introduced until some time after 2000 because of the time needed to collect and process the weighting data. In section (b) of Table 9.6, it is assumed that the 2000 weights are introduced in December 2002 and that this is also chosen as the new price reference base.

9.124 Note that it would be possible in December 2002 to calculate the indices based on 2000 shown in section (a) of the table, but it is decided to make December 2002 the price reference base. This does not prevent the index with the December 2002 price reference period from being calculated backwards a few months into 2002, if desired.

Table 9.7 Price-updating of expenditure weights

	Weight	2000	Nov. 02	Dec. 02	Jan. 03	Feb. 03	Mar. 03
(a) Index with 2000 as weight and price reference period							
	w_{00}			*Elementary price indices*			
A	0.20	100.00	98.00	99.00	102.00	101.00	104.00
B	0.25	100.00	106.00	108.00	107.00	109.00	110.00
C	0.15	100.00	104.00	106.00	98.00	100.00	97.00
D	0.10	100.00	101.00	104.00	108.00	112.00	114.00
E	0.30	100.00	102.00	103.00	106.00	105.00	106.00
				Higher-level indices			
G=A+B+C	0.60	100.00	102.83	104.50	103.08	104.08	104.75
H=D+E	0.40	100.00	101.75	103.25	106.50	106.75	108.00
Total		100.00	102.40	104.00	104.45	105.15	106.05
(b) Index re-referenced to December 2002 and weights price-updated to December 2002							
	$w_{00(Dec02)}$			*Elementary price indices*			
A	0.190	101.01	98.99	100.00	103.03	102.02	105.05
B	0.260	92.59	98.15	100.00	99.07	100.93	101.85
C	0.153	94.34	98.11	100.00	92.45	94.34	91.51
D	0.100	96.15	97.12	100.00	103.85	107.69	109.62
E	0.297	97.09	99.03	100.00	102.91	101.94	102.91
				Higher-level indices			
G=A+B+C	0.603	95.69	98.41	100.00	98.64	99.60	100.24
H=D+E	0.397	96.85	98.55	100.00	103.15	103.39	104.60
Total		96.15	98.46	100.00	100.43	101.11	101.97
Rescaled to 2000=100		100.00	102.40	104.00	104.45	105.15	106.05

9.125 If it is decided to preserve the quantities, the resulting index is a basket index, or Lowe index, in which the quantities are those of the year 2000. This implies that the *movements* of the index must be identical with those of the index based on 2000 shown in section (a) of the table. In this case, if the index is to be presented as a weighted average of the elementary price indices with December 2002 as the price reference period, the expenditure weights for 2000 have to be price-updated to December 2002. This is illustrated in section (b) of Table 9.7, where the updated weights are obtained by multiplying the original weights for 2000 in section (a) of the table by the price indices for the elementary aggregates between 2000 and December 2002, and then rescaling the results to sum to unity. These are the weights labelled $w_{00(\mathrm{Dec02})}$ in the table.

9.126 The indices with price-updated weights in section (b) of Table 9.7 are Lowe indices in which b = 2000 and 0 = December 2002. These indices can be expressed as ratios of the indices in the upper part of the table. For example, the overall Lowe index for March 2003 with December 2002 as its price reference base, namely 101.97, is the ratio of the index for March 2003 based on 2000 shown in section (a) of the table, namely 106.05, divided by the index for December 2002 based on 2000, namely 104.00. Thus, the price-updating preserves the movements of the indices in section (a) of the table while shifting the price reference period to December 2002.

9.127 On the other hand, it could be decided to calculate a series of Young indices using the expenditure shares from 2000 as they stand without price-updating. If the expenditure shares were actually to remain constant, the quantities would have had to move inversely with the prices between 2000 and December 2002. As the quantities are kept constant in the price-updated Lowe index the movements of the two indices usually will be different. In the special case where the relative prices remains unchanged from the weight to the price reference period the price-updated weights will be unchanged and the Young and the Lowe index will give the same result.

The introduction of new weights and chain linking

9.128 From time to time, the weights for the elementary aggregates have to be revised to ensure that they reflect current expenditure patterns and consumer behaviour. When new weights are introduced, the price reference period for the new index can be the last period of the old index, the old and the new indices being linked together at this point. The old and the new indices make a chain index.

9.129 The introduction of new weights is often a complex operation because it provides the opportunity to introduce new items, new samples, new data sources, new compilation practices, new elementary aggregates, new higher-level indices or new classifications. These tasks are often undertaken simultaneously at the time of reweighting to minimize overall disruption to the time series and any resulting inconvenience to users of the indices.

9.130 In many countries, reweighting and chaining are carried out about every five years, but some countries introduce new weights each year. Chain indices do not have to be linked annually; the linking may be done less frequently. The real issue is not whether to chain or not but how frequently to chain. Reweighting is inevitable sooner or later, as the same weights cannot continue to be used for ever. Whatever the time frame, statistical offices have to address the issue of chain linking sooner or later. It is inevitable and a major task for index compilers.

Frequency of reweighting

9.131 It is reasonable to continue to use the same set of elementary aggregate weights so long as consumption patterns at the elementary aggregate level remain fairly stable. Over time, consumers will tend to substitute away from products of which the prices have increased relatively. Thus, in general, movements in prices and quantities tend to be inversely related. This kind of substitution behaviour on the part of consumers implies that a Lowe index based on the fixed basket of an earlier period will tend to have an upward bias compared with a basket index using up-to-date weights.

9.132 Another reason why consumption patterns change is that new products are continually being introduced on the market while others drop out. Over the longer term, consumption patterns are also influenced by several other factors. These include rising incomes and standards of living, demographic changes in the structure of the population, changes in technology, and changes in tastes and preferences.

9.133 There is wide consensus that regular updating of weights – at least every five years, and more often if there is evidence of rapid changes in consumption patterns – is a sensible and necessary practice. The question of how often to change the weights and chain link the index is nevertheless not straightforward, as frequent linking can also have some disadvantages. It can be costly to obtain new weights, especially if they require more frequent expenditure surveys. Annual chaining has the advantage that changes (such as the inclusion of new goods) can be introduced on a regular basis, although every index needs some ongoing maintenance, whether annually chained or not.

9.134 Expenditures on certain types of products are strongly influenced by short-term fluctuations in the economy. For example, expenditures on cars, major durables, expensive luxuries, and so on, may change drastically from year to year. In such cases, it may be preferable to base the weight on an average of two or more years of expenditure.

The calculation of a chain index

9.135 Assume that a series of fixed weight Young indices has been calculated with period 0 as the price reference period and that in a subsequent period, *k*, a new set of weights has to be introduced in the index. The new set of weights may, or may not, have been price-updated from the new weight reference period to period *k*. A chain index is then calculated as:

$$\begin{aligned} P^{0:t} &= P^{0:k} \sum w_j^k P_j^{k:t-1} P_j^{t-1:t} \\ &= P^{0:k} \sum w_j^k P_j^{k:t} \\ &= P^{0:k} P^{k:t} \end{aligned} \qquad (9.19)$$

9.136 There are several important features of a chain index:

- The chain index formula allows weights to be updated, and facilitates the introduction of new items and sub-indices and the removal of obsolete ones.
- In order to be able to chain the old and the new series, an overlapping period (*k*) is needed in which the index has to be calculated using both the old and the new set of weights.
- A chain index may have two or more links. In each link, the index may be calculated as a fixed weight index using equation (9.10), or indeed using any other index formula. The chaining period may be a month or a year, provided the weights and indices refer to the same period.
- Chaining is intended to ensure that the individual indices on all levels show the correct development through time.
- Chaining leads to non-additivity so that chained indices at lower-level cannot be aggregated into indices at higher level using the latest set of weights. If, on the other hand, the index reference period is changed and the index series prior to the chaining period is rescaled to the new index reference period, this series cannot be aggregated to higher-level indices by use of the new weights.

9.137 An example of the calculation of a chain index is presented in Table 9.8. From 1998 to December 2002 the index is calculated with the year 1998 as weight and price reference period. From December 2002 onwards, a new set of weights is introduced. The weights may refer to the year 2000, for example, and may or may not have been price-updated to December 2002. A new fixed weight index series is then calculated with December 2002 as the price reference month. Finally, the new index series is linked onto the old index with 1998 = 100 by multiplication to get a continuous index from 1998 to March 2003. The chained higher-level indices in Table 9.8 are calculated as:

$$P^{00:t} = P^{98:Dec02} \sum w_j^{00(Dec02)} P_j^{Dec02:t} \quad (9.20)$$

9.138 Because of the lack of additivity, the overall chain index for March 2003 (129.07), for example, cannot be calculated as the weighted arithmetic mean of the chained higher-level indices G and H using the weights from December 2002.

Table 9.8 Calculation of a chain index

	Weight 1998	1998	Nov. 2002	Dec. 2002	Weight 2000	Dec. 2002	Jan. 2003	Feb. 2003	Mar. 2003
			1998 = 100				*December 2002 = 100*		
Elementary price indices									
A	0.20	100.00	120.00	121.00	0.25	100.00	100.00	100.00	102.00
B	0.25	100.00	115.00	117.00	0.20	100.00	102.00	103.00	104.00
C	0.15	100.00	132.00	133.00	0.10	100.00	98.00	98.00	97.00
D	0.10	100.00	142.00	143.00	0.18	100.00	101.00	104.00	104.00
E	0.30	100.00	110.00	124.00	0.27	100.00	103.00	105.00	106.00
Total		100.00	119.75	124.90		100.00	101.19	102.47	103.34
Higher-level indices									
G=A+B+C	0.60	100.00	120.92	122.33	0.55	100.00	100.36	100.73	101.82
H=D+E	0.40	100.00	118.00	128.75	0.45	100.00	102.20	104.60	105.20
Total		100.00	119.75	124.90		100.00	101.19	102.47	103.34
Chaining of higher-level indices to 1998 = 100									
G=A+B+C	0.60	100.00	120.92	122.33	0.55	122.33	122.78	123.22	124.56
H=D+E	0.40	100.00	118.00	128.75	0.45	128.75	131.58	134.67	135.45
Total		100.00	119.75	124.90		124.90	126.39	127.99	129.07

The introduction of new elementary aggregates

9.139 First, consider the situation in which new weights are introduced and the index is chain linked in December 2002. The overall coverage of the CPI is assumed to remain the same, but certain items have increased sufficiently in importance to merit recognition as new elementary aggregates. Possible examples are the introduction of new elementary aggregates for mobile telephones or Internet access.

9.140 Consider the calculation of the new index from December 2002 onwards, the new price reference period. The calculation of the new index presents no special problems and can be carried out using formula (9.10). However, if the weights are price-updated from, say, 2000 to December 2002, difficulties may arise because the elementary aggregate for mobile telephones did not exist prior to December 2002, so there is no price index with which to price-update the weight for mobile telephones. Prices for mobile telephones may have been recorded prior to December 2002, possibly within another elementary aggregate (communications equipment), so it may be possible to construct a price series which can be used for price-updating. Otherwise, price information from other sources, such as purchasing power parity (PPP) surveys, business statistics, or industry sources, may have to be used. If no information is available, then movements in the price indices for similar elementary aggregates may be used as proxies for price-updating.

9.141 The inclusion of a new elementary aggregate means that the next and successive higher-level indices contain a different number of elementary aggregates before and after the linking. Therefore, the rate of change of the higher-level index whose composition has changed may be difficult to interpret. However, failing to introduce new goods or services for this reason would result in an index that does not reflect the actual dynamic changes taking place in the economy. If it is customary to revise the CPI backwards, then the prices of the new product and their weights might be introduced retrospectively. If the CPI is not revised backwards, which is usually the case, there is little that can be done to improve the quality of the chain index. In many cases, the addition of a single elementary aggregate is unlikely to have a significant effect on the higher-level indices into which it enters. If the addition of an elementary aggregate is believed to have a significant impact on the time series of the higher-level index, it may be necessary to discontinue the old series and commence a new higher-level index. These decisions can only be made on a case-by-case basis.

The introduction of new higher-level indices

9.142 It may be necessary to introduce a new higher-level index in the overall CPI. This situation may occur if the coverage of the CPI is enlarged or the grouping of elementary aggregates is changed. It then needs to be decided what the initial value of the new higher-level index should be when it is included in the calculation of the overall CPI. Take as an example the situation in Table 9.8 and assume that a new higher-level index from January 2003 has to be included in the index. The question is then what should be the December 2002 value to which the new higher-level index is to be linked. There are two options:

- Estimate the value in December 2002 that the new higher-level index would have had with 1998 as the price reference period, and link the new series from January 2003 onwards on to this value. This procedure will prevent any break in the index series.
- Use 100 in December 2002 as the starting point for the new higher-level index. This simplifies the problem from a calculation perspective, although there remains the difficulty of explaining the index break to users.

In any case, major changes such as those just described should, so far as possible, be made in connection with the regular reweighting and chaining in order to minimize disruptions to the index series.

9.143 A final case to consider concerns classification change. For example, a country may decide to change from a national classification to an international one, such as the Classification of Individual Consumption according to Purpose (COICOP). The changes in the composition of the aggregates within the CPI may then be so large that it is not meaningful to link them. In such cases, it is recommended that the CPI with the new classification should be calculated backwards for at least one year so that consistent annual rates of change can be calculated.

Partial reweighting

9.144 The weights for the elementary aggregates may be obtained from a variety of sources over a number of periods. Consequently, it may not be possible to introduce all the new weighting information at the same time. In some cases, it may be preferable to introduce new weights for some elementary aggregates as soon as possible after the information is received. The introduction of new weights for a subset of the overall index is known as partial reweighting.

9.145 Partial reweighting has particular implications for the practice of price-updating the weights. Weighting information may not be available for some elementary aggregates at the time of rebasing. Thus, it may be necessary to consider price-updating not only the new weights, but also the old weights for those elementary aggregates for which no new weights are available. The weights for the latter may have to be price-updated over a long period, which, for reasons given earlier, may give rise to serious problems if relative quantities have changed inversely to the relative price changes. Data on both quantity and price changes should be sought before undertaking such updates. The disadvantage of partial reweighting is that the implicit quantities belong to different periods, so that the composition of the basket is obscure and not well defined.

9.146 It may be concluded that the introduction of new weights and the linking of a new series to an old series is not difficult in principle. The difficulties arise in practice when trying to align weight and price reference periods and when deciding whether higher-level indices comprising different elementary aggregates should be chained over time. It is not possible for this manual to provide specific guidance on decisions such as these, but compilers should consider carefully the economic logic and statistical reliability of the resulting chained series and also the needs of users. In order to facilitate the decision-making process, careful thought should be given to these issues in advance during the planning of a reweighting exercise, paying particular attention to which of the indices are to be published.

9.147 *Long-term and short-term links*. Consider a long-term chain index in which the weights are changed annually. In any given year, the current monthly indices are first calculated using the latest set of available weights, which cannot be those of the current year. However, when the weights for the year in question become available subsequently, the monthly indices can then be recalculated on the basis of the weights for that same year. The resulting series can then be used in the long-term chain index, rather than the original indices first published. Thus, the movements of the long-term chain index from, say, any one December to the following December are based on weights of that same year, the weights being changed each December. This method has been developed by the Central Statistical Office of Sweden, where it is applied in the calculation of the CPI. It is described in *Swedish price index, A handbook of Methods* (Statistics Sweden, 2001).

9.148 Assume that each link runs from December to December. The long-term index for month m of year Y with December of year 0 as index reference period is then calculated using the formula:

$$P^{Dec0:mY} = \left(\prod_{y=1}^{Y-1} P^{Decy-1:Decy}\right) P^{DecY-1:mY} = P^{Dec0:Dec1} P^{Dec1:Dec2} \; \dots \; P^{DecY-2:DecY-1} P^{DecY-1:mY} \tag{9.21}$$

9.149 In the actual Swedish practice, a factor scaling the index from December year 0 to the average of year 0 is multiplied onto the right-hand side of equation (9.21) to have a full year as the reference period. The long-term movement of the index depends on the long-term links only, as the short-term links are successively replaced by their long-term counterparts. For example, let the short-term indices for January to December 2001 be calculated as:

$$P^{Dec00:m01} = \sum w_j^{00(Dec00)} P_j^{Dec00:m01} \tag{9.22}$$

where $w_j^{00(Dec00)}$ are the weights from 2000 price-updated to December 2000. When weights for 2001 become available, this is replaced by the long-term link:

$$P^{Dec00:Dec01} = \sum w_j^{01(Dec00)} P_j^{Dec00:Dec01} \quad (9.23)$$

where $w_j^{01(Dec00)}$ are the weights from 2001 price-backdated to December 2000. The same set of weights from 2001 price-updated to December 2001 is used in the new short-term link for 2002:

$$P^{Dec01:m02} = \sum w_j^{01(Dec01)} P_j^{Dec01:m02} \quad (9.24)$$

9.150 Using this method, the movement of the long-term index is determined by contemporaneous weights. The method is conceptually attractive because the weights that are most relevant for most users are those based on consumption patterns at the time the price changes actually take place. The method takes the process of chaining to its logical conclusion, at least assuming the indices are not chained more frequently than once a year. As the method uses weights that are continually revised to ensure that they are representative of current consumer behaviour, the resulting index also largely avoids the substitution bias that occurs when the weights are based on the consumption patterns of some period in the past. The method may therefore appeal to statistical offices whose objective is to estimate a cost of living index.

9.151 Finally, it may be noted that the method involves some revision of the index first published. In some countries, there is opposition to revising a CPI once it has been first published, although it is standard practice for other economic statistics, including the national accounts, to be revised as more information and more up-to-date information become available. This point is considered further below.

Decomposition of index changes

9.152 Users of the index are often interested in how much of the change in the overall index is attributable to the change in the price of some particular good or group of products, such as oil or food. Alternatively, there may be interest in what the index would be if housing or energy were left out. Questions of this kind can be answered by decomposing the change in the overall index into its constituent parts.

9.153 Assume that the index is calculated as in equation (9.10) or (9.11). The relative change of the index from *t-m* to *t* can then be written as:

$$\frac{P^{0:t}}{P^{0:t-m}} - 1 = \frac{\sum w_j^b P_j^{0:t-m} P_j^{t-m:t}}{\sum w_j^b P_j^{0:t-m}} - 1 \quad (9.25)$$

Hence, a sub-index, *j*, from *t-m* to *t* enters the higher-level index with a weight of:

$$\frac{w_j^b P_j^{0:t-m}}{\sum w_j^b P_j^{0:t-m}} = \frac{w_j^b P_j^{0:t-m}}{P^{0:t-m}} \tag{9.26}$$

The effect on the higher-level index of a change in a sub-index can then be calculated as:

$$Effect = \frac{w_j^b P_j^{0:t-m}}{P^{0:t-m}} \left(\frac{P_j^{0:t}}{P_j^{0:t-m}} - 1 \right) = \frac{w_j^b}{P^{0:t-m}} \left(P_j^{0:t} - P_j^{0:t-m} \right) \tag{9.27}$$

With $m = 1$, the formula (9.23) gives the effect of a monthly change; with $m = 12$, it gives the effect of the change over the past 12 months.

9.154 If the index is calculated as a chain index, as in equation (9.19), then a sub-index, j, from t-m enters the higher-level index with a weight of:

$$\frac{w_j^k P_j^{k:t-m}}{P^{k:t-m}} = \frac{w_j^k \left(P_j^{0:t-m} / P_j^{0:k} \right)}{\left(P^{0:t-m} / P^{0:k} \right)} \tag{9.28}$$

9.155 The effect on the higher-level index of a change in a sub-index then is:

$$Effect = \frac{w_j^k}{P^{k:t-m}} \left(P_j^{k:t} - P_j^{k:t-m} \right) = \frac{w_j^k}{\left(P^{0:t-m} / P^{0:k} \right)} \left(\frac{P_j^{0:t} - P_j^{0:t-m}}{P_j^{0:k}} \right) \tag{9.29}$$

It is assumed that t-m lies in the same link (i.e. t-m refers to a period later than k). If the effect of a sub-index on a higher-level index is to be calculated across a chain, the calculation needs to be carried out in two steps: one with the old series up to the link period, and one from the link period to period t.

9.156 The calculation of the effect of a change in a sub-index on a higher-level index is illustrated in Table 9.9. The index is calculated in one link so that equation (9.27) may be applied for the decomposition. For instance, the effect in percentage points of the increase for housing from January 2002 to January 2003 can be calculated as 0.25/118.6×(120.0 – 110.0) = 2.11 percentage points. This means that, of the increase of 10.03 per cent in the all-items index, 2.11 percentage points can be attributed to the increase in the index for housing.

Table 9.9 Decomposition of index changes

	Weight	Index			Change in % from Jan. 02 to Jan. 03	Effect (contribution)	
		2000	Jan. 02	Jan. 03		% points of total change	% of total change
Food	0.30	100.0	120.0	130.0	8.33	2.53	25.21
Clothing	0.10	100.0	130.0	145.0	11.54	1.26	12.61
Housing	0.25	100.0	110.0	120.0	9.09	2.11	21.01
Transport	0.20	100.0	125.0	130.0	4.00	0.84	8.40
Miscellaneous	0.15	100.0	114.0	140.0	22.81	3.29	32.77
All items	1.00	100.0	118.6	130.5	10.03	10.03	100.00

Some alternatives to fixed weight indices

9.157 Monthly CPIs are, typically, arithmetic weighted averages of the price indices for the elementary aggregates, in which the weights are kept fixed over a number of periods – which may range from 12 months to many years. The repeated use of the same weights relating to some past period *b* simplifies calculation procedures and reduces data collection requirements. It is also cheaper to keep using the results from an old expenditure survey than to conduct an expensive new one. Moreover, when the weights are known in advance of the price collection, the index can be calculated immediately after the prices have been collected and processed.

9.158 The longer the same weights are used, however, the less representative they become of current consumption patterns, especially in periods of rapid technical change when new kinds of goods and services are continually appearing on the market and old ones disappearing. This may undermine the credibility of an index that purports to measure the rate of change in the total cost of a basket of goods and services typically consumed by households. Such a basket needs to be representative not only of the households covered by the index, but also of expenditure patterns at the time the price changes occur.

9.159 Similarly, if the objective is to compile a cost of living index, the continuing use of the same fixed basket is likely to become increasingly unsatisfactory the longer the same basket is used. The longer the same basket is used, the greater the upward bias in the index is likely to become. It is well known that the Laspeyres index has an upward bias compared with a cost of living index. However, a Lowe index between periods *0* and *t* with weights from an earlier period *b* will tend to exceed the Laspeyres between *0* and *t* by an amount that increases the further back in time period *b* is (see Chapter 15).

9.160 There are several possible ways of minimizing or avoiding the potential biases from the use of fixed weight indices. These are outlined below.

Annual chaining

9.161 One way in which to minimize the potential biases from the use of fixed-weight indices is obviously to keep the weights and the base period as up to date as possible by frequent rebasing and chaining. Quite a number of countries have adopted this strategy and revise their weights annually. In any case, as noted earlier, it would be impossible to deal with the changing universe of products without some chaining of the price series within the elementary aggregates, even if the weights attached to the elementary aggregates remain fixed. Annual chaining eliminates the need to choose a base period, as the weight reference period is always the previous year, or possibly the preceding year.

9.162 *Annual chaining with current weights*. When the weights are changed annually, it is possible to replace the original weights based on the previous year, or years, by those of the current year, if the index is revised retrospectively as soon as information on the current year's expenditures becomes available. The long-term movements in the CPI are then based on the revised series. This is the method adopted by the Swedish Statistical Office, as explained above. This method could provide unbiased results.

Other index formulae

9.163 When the weights are revised less frequently, say every five years, another possibility would be to use a different index formula for the higher-level indices instead of an arithmetic average of the elementary price indices. One possibility would be a weighted geometric average. This is not subject to the same potential upward bias as the arithmetic average. More generally, a weighted version of the Lloyd-Moulton formula might be considered. This formula takes account of the substitutions that consumers make in response to changes in relative prices, and should be less subject to bias for this reason. It reduces to the geometric average when the elasticity of substitution is unity, on average. It is unlikely that such a formula could replace the arithmetic average in the foreseeable future and gain general acceptance, if only because it cannot be interpreted as measuring changes in the value of a fixed basket. It could, however, be compiled on an experimental basis and might well provide a useful supplement to the main index. It could at least flag the extent to which the main index is liable to be biased and throw light on its properties.

Retrospective superlative indices

9.164 Finally, it is possible to calculate a superlative index retrospectively. Superlative indices, such as Fisher and Törnqvist indices, treat both periods compared symmetrically and require expenditure data for both periods. Although the CPI may have to be some kind of Lowe index when it is first published, it may be possible to estimate a superlative index later when much more information becomes available about consumers' expenditures period by period. At least one office, the United States Bureau of Labor Statistics, publishes such an index. The publication of revised or supplementary CPIs raises matters of statistical policy, although users readily accept revisions in other fields of economic statistics. Moreover, users are already confronted with more than one CPI in the European Union (EU) where the harmonized index for EU purposes may differ from the national CPI. Thus the publication of supplementary indices which throw light on the properties of the main index and which may be of considerable interest to some users seems justified and acceptable.

Data editing

9.165 This chapter has been concerned with the methods used by statistical offices to calculate their CPIs. This concluding section considers the data editing carried out by statistical offices, a process that is very closely linked to the calculation of the price indices for the elementary aggregates. Data collection, recording and coding – the data capture processes – are dealt with in Chapters 5 to 7. The next step in the production of price indices is data editing. Data editing is here meant to comprise two steps:

- Detecting of possible errors and outliers
- Verifying and correction of data

9.166 Logically, the purpose of detection errors and outliers is to exclude errors or outliers from the index calculation. Errors may be falsely reported prices, or they may be caused by recording or coding mistakes. Also, missing prices because of non-response may be dealt with as errors. Possible errors and outliers are usually identified as observations that fall outside some pre-specified acceptance interval or are judged to be unrealistic by the analyst on some other ground. It may also be the case, however, that even if an observation is not identified as a potential error, it may actually show up to be false. Such observations are sometimes referred to as inliers. Sometimes, by chance, the sampling may have captured an exceptional price change, which falls outside the acceptance interval but has been verified as correct. In some discussions of survey data, any extreme value is described as an outlier. The term is reserved here for extreme values that have been verified as being correct.

9.167 When a possible error has been identified, it needs to be verified whether it is in fact an error or not. This clarification can usually be made by asking the respondent to verify the price, or by comparison with the price change of comparable items. If the value is in fact an error, it needs to be corrected. This can be done easily if the respondent can provide the correct price or, where this is not possible, by imputation or omitting the price from the index calculation. If the value proves to be correct, it should be included in the index. If it proves to be an outlier, it can be accepted or corrected according to a predefined practice, e.g. omitting or imputation.

9.168 Although the power of computers provides obvious benefits, not all of these activities have to be computerized. There should be a complete set of procedures and records that controls the processing of data, even though some or all of it may be undertaken without the use of computers. It is not always necessary for all of one step to be completed before the next is started. If the process uses spreadsheets, for example, with default imputations pre-defined for any missing data, the index can be estimated and re-estimated whenever a new observation is added or modified. The ability to examine the impact of individual price observations on elementary aggregate indices and the impact of elementary indices on various higher-level aggregates is a useful aid in all aspects of the computation and analytical processes.

9.169 It is neither necessary nor desirable to apply the same degree of scrutiny to all reported prices. The price changes recorded by some respondents carry more weight than others, and statistical analysts should be aware of this. For example, one elementary aggregate with a weight of 2 per cent, say, may contain 10 prices, while another elementary aggregate of equal weight may contain 100 prices. Obviously, an error in a reported price will have a much smaller effect in the latter, where it may be negligible, while in the former it may cause a significant error in the elementary aggregate index and even influence higher-level indices.

9.170 There may be an interest in the individual elementary indices, as well as in the aggregates built from them. Since the sample sizes used at the elementary level may often be small, any price collected, and error in it, may have a significant impact on the results for individual products or industries. The verification of reported data usually has to be done on an index-by-index basis, using the statistical analysts' experience. Analysts will also need the cooperation and support of the respondents to the survey to help explain unusual price movements.

9.171 Obviously, the design of the survey and questionnaires also influences the occurrence of errors. Hence, price reports and questionnaires should be as clear and unambiguous as possible to prevent misunderstandings and errors. Whatever the design of the survey, it is important to verify that the data collected are those that were requested initially. The survey questionnaire should prompt the respondent to indicate if the requested data could not be provided. If, for example, a product is not produced any more and thus is not priced in the current month, a possible replacement would be requested along with details as to the extent of its comparability with the old one. In the event that a respondent cannot supply a replacement, there are a number of procedures for dealing with missing data (also discussed in Chapter 7).

Identifying possible errors and outliers

9.172 One of the ways in which price surveys are different from other economic surveys is that, although prices are recorded, the measurement concern is with price *changes*. As the index calculations consist of comparing the prices of matching observations from one period to another, editing checks should focus on the price changes calculated from pairs of observations, rather than on the reported prices themselves.

9.173 Identification of unusual price changes can be accomplished by:

- Non-statistical checking of input data
- Statistical checking of input data
- Output checking

These will be described in turn.

Non-statistical checking of input data

9.174 Non-statistical checking can be undertaken by manually checking the input data, by inspection of the data presented in comparable tables, or by setting filters.

9.175 When the price reports or questionnaires are received in the statistical office, the reported prices can be checked manually by comparing these to the previously reported prices of the same items or by comparing them to prices of similar items from other outlets. While this procedure may detect obvious unusual price changes, it is far from certain that all possible errors will be detected. It is also extremely time-consuming and, of course, it does not identify coding errors.

9.176 After the price data have been coded, the statistical system can be programmed to present the data in a comparable tabular form. For example, a table showing the percentage change for all reported prices from the previous to the current month may be produced and used for detection of possible errors. Such tables may also include, for comparison, the percentage changes of previous periods and 12-month changes. Most computer programs and spreadsheets can easily sort the observations according to, say, the size of the latest monthly rate of change, so that extreme values can easily be identified. It is also possible to group the observations by elementary aggregates.

9.177 The advantage of grouping observations is that it highlights potential errors so that the analyst does not have to look through all observations. A hierarchical strategy whereby all extreme price changes are first identified and then examined in context may save time, though the price changes in elementary aggregate indices, which have relatively high weights, should also be examined in context.

9.178 Filtering is a method by which possible errors or outliers are identified according to whether the price changes fall outside some pre-defined limits, such as plus or minus 20 per cent or even 50 per cent. This test should capture any serious errors of data coding, as well as some of the cases where a respondent has erroneously reported on a different product. It is usually possible to identify these errors without reference to any other observations in the survey, so this check can be carried out at the data capture stage. The advantage of filtering is that it avoids the analyst having to look through a lot of individual observations. The upper and lower limits may be set for the latest monthly change, or change over some other period. Again, they should take account of the context of the price change, in that they may be specified by item or elementary aggregates or higher-level indices. Larger changes for items with prices that are known to be volatile might be accepted without question. For example, for monthly changes, limits of plus or minus 10 per cent might be set for oil prices, while for professional services the limits might be zero per cent to plus 5 per cent (as any price that falls is suspect), and for computers it might be –5 per cent to zero per cent (as any price that rises is suspect). The limits can also be changed over time. If it is known that oil prices are rising, the limits could be 10 per cent to 20 per cent, while if they are falling, the limits might be –10 per cent to –20 per cent. The count of failures should be monitored regularly to examine the limits. If too many observations are being identified for review, the limits will need to be adjusted, or the scope refined.

9.179 The use of automatic deletion systems is not advised, however. It is a well-recorded phenomenon in pricing that price changes for many products, especially durables, are not undertaken smoothly over time, but saved up to avoid what are termed "menu costs" associated with making a price change. These relatively substantial increases may take place at different times for different models of products and have the appearance of extreme, incorrect values. To delete a price change for each model of the product as being "extreme" at the time it occurs is to ignore all price changes for the industry.

Statistical checking of input data

9.180 Statistical checking of input data compares, for some time period, each price change with the change in prices in the same or a similar sample. Two examples of such filtering are given here, the first based on non-parametric summary measures and the second on the log normal distribution of price changes.

9.181 The first method involves tests based on the median and quartiles of price changes, so they are unaffected by the impact of any single "extreme" observation. Define the median, first quartile and third quartile price ratios as $R_{M,}$ R_{Q1} and R_{Q3}, respectively. Then any observation with a price ratio that is more than a certain multiple C of the distance between the median and the quartile is identified as a potential error. The basic approach assumes that price changes are normally distributed. Under this assumption, it is possible to estimate the proportion of price changes that are likely to fall outside given bounds expressed as multiples of C. Under a normal distribution, $R_{Q1,}$ and R_{Q3} are equidistant from $R_{M.}$ Thus, if C is measured as $R_M - (R_{Q1} + R_{Q3})/2$, then 50 per cent of observations would be expected to lie within plus or minus C from the median. From the tables of the standardized normal distribution this is equivalent to about 0.7 times the standard deviation (σ). If, for example, C was set to 6, the distance implied is about 4σ of the sample, so about 0.17 per cent of observations would be identified this way. With C = 4, the corresponding figures are 2.7σ, or about 0.7 per cent of observations. If C = 3, the distance is 2.02σ, so about 4 per cent of observations would be identified.

9.182 In practice, most prices may not change each month and the share of observations identified as possible errors as a percentage of all changes would be unduly high. Some experimentation with alternative values of C for different industries or sectors may be appropriate. If this test is to be used to identify possible errors for further investigation, a relatively low value of *C* should be used.

9.183 To use this approach in practice, three modifications should be made:

- First, to make the calculation of the distance from the centre the same for extreme changes on the low side as well as on the high side, a transformation of the ratios should be made. The transformed distance for the ratio of one price observation *i*, S_i, should be:

$$S_i = 1 - R_M/R_i \quad \text{if } 0 < R_i < R_M$$
$$S_i = R_i/R_M - 1 \quad \text{if } R_i \geq R_M$$

- Second, if the price changes are grouped closely together, the distances between the median and quartiles may be very small, so that many observations would be identified that had quite small price changes. To avoid this, some minimum distance, say 5 per cent for monthly changes should also be set.
- Third, with small samples the impact of one observation on the distances between the median and quartiles may be too great. Because sample sizes for some elementary indices are small, samples for similar elementary indices may need to be grouped together.

9.184 For a detailed presentation of this method, see Hidiroglou and Berthelot (1986). The method can be expanded to also take into account the level of the prices. Thus, for example, a price increase from 100 to 110 will be attributed a different weight from the weight attributed to a price increase from 10 to 11.

9.185 An alternative method can be used if it is thought that the price changes may be distributed log normally. To apply this method, the standard deviation of the log of all price changes in the sample (excluding unchanged observations) is calculated and a goodness of fit test (χ^2) is undertaken to identify whether the distribution is log normal. If the distribution satisfies the test, all price changes outside two times the exponential of the standard deviation are highlighted for further checking. If the test rejects the log normal hypothesis, all the price changes outside three times the exponential of the standard deviation are highlighted. The same caveats mentioned before about clustered changes and small samples apply.

9.186 The second example is based on the Tukey algorithm. The set of price ratios is sorted and the highest and lowest 5 per cent flagged for further attention. In addition, having excluded the top and bottom 5 per cent, exclude the price ratios that are equal to 1 (no change). The arithmetic (trimmed) mean (AM) of the remaining price ratios is calculated. This mean is used to separate the price ratios into two sets, an upper and a lower one. The upper and lower "mid-means", that is, the means of each of these sets (AM_L, AM_U), are then calculated. Upper and lower Tukey limits (T_L, T_U) are then established as the mean plus (minus) 2.5 times the difference between the mean and the mid-means:

$$T_U = AM + 2.5(AM_U - AM)$$
$$T_L = AM - 2.5(AM - AM_L)$$

Then all those observations that fall above T_U and below T_L are flagged for attention.

9.187 This is a simpler method similar to that based on the normal distribution. Since it excludes all cases of no change from the calculation of the mean, it is unlikely to produce limits that are very close to the mean, so there is no need to set a minimum difference. Its success will also depend on there being a large number of observations on the set of changes being analysed. Again, it will often be necessary to group observations from similar elementary indices. For any of these algorithms, the comparisons can be made for any time periods, including the latest month's changes or longer periods, in particular, 12-month changes.

9.188 The advantage of these two models of filtering compared to the simple method of filtering is that for each period the upper and lower limits are determined by the data and hence are allowed to vary over the year, given that the analyst has decided on the value of the parameters entering the models. A disadvantage is that, unless the analyst is prepared to use approximations from earlier experience, all the data have to be collected before the filtering can be undertaken. Filters should be set tightly enough so that the percentage of potential errors that turn out to be real errors is high. As with all automatic methods, the flagging of an unusual observation is for further investigation, as opposed to automatic deletion.

Checking by impact or data output checking

9.189 Filtering by impact, or output editing, is based on calculating the impact that an individual price change has on an index to which it contributes. This index can be an elementary aggregate index, the total index, or some other aggregate index. The impact that a price change has on an index is its percentage change times its effective weight. However, the exact calculation of the impact will depend on which formula has been applied for the elementary indices. It is possible to set a maximum value for this impact, so that all price changes that cause an impact greater than this can be flagged for review. The impact of a price change on a higher-level index will also depend on the weight of the elementary index in the aggregate.

9.190 At the lowest level, the appearance and disappearance of products in the sample cause the *effective weight* of an individual price to change substantially. The effective weight is also affected if a price observation is used as an imputation for other missing observations. The evaluation of effective weights in each period is possible, though complicated. As an aid to highlighting potential errors, the nominal weights, as a percentage of their sum, will usually provide a reasonable approximation. If the impact of 12-month changes is required to highlight potential errors, approximations are the only feasible filters to use, as the effective weights will vary over the period.

9.191 One advantage of identifying potential errors in this way is that it focuses on the results. Another advantage is that this form of filtering also helps the analyst to describe the contributions to change in the price indices. In fact, much of this kind of analysis is done after the indices have been calculated, as the analyst often wishes to highlight those indices that have contributed the most to overall index changes. Sometimes the analysis results in a finding that particular industries have a relatively high contribution to the overall price change, and that is considered unrealistic. The change is traced back to an error, but it may be late in the production cycle and jeopardize the schedule release date. There is thus a case for identifying such unusual contributions as part of the data editing procedures. The disadvantage of this method is that an elementary index's change may be rejected at that stage. It may be necessary to over-ride the calculated index, though this should be only a stopgap measure until the index sample is redesigned.

Verifying and correcting data

9.192 Some errors, such as data coding errors, can be identified and corrected easily. Ideally, these errors are caught at the first stage of checking, before they need to be viewed in the context of other price changes. Dealing with other potential errors is more difficult. There may be errors that are not identified in the data checking procedure and observations that have been identified as potential errors may prove to be correct, especially if the data checking limits are rather narrow. Some potential failures may only be resolved by checking the data with the respondent.

9.193 If a satisfactory explanation can be obtained from the respondent, the data can be verified or corrected. If not, procedures may differ. Rules may be established that if a satisfactory explanation is not obtained, then the reported price is omitted from the index calculation. Alternatively, it may be left to the analyst to make the best judgement as to the price change. If an analyst makes a correction to some reported data without verifying it with the respondent, the change may subsequently cause problems with the respondent. If the respondent is not told of the correction, the same error may persist in the future. The correct action depends on a combination of confidence in the analysts, the revision policy of the survey, and the degree of communication with respondents. Most statistical organizations do not want to burden respondents unduly.

9.194 In many organizations, a disproportionate share of activity is devoted to identifying and following up potential errors. If this practice leads to little change in the results, as a result of most reports finally being accepted, then the “bounds” on what are considered to be extreme values should be relaxed. More errors are likely to be introduced by respondents failing to report changes that occur than from wrongly reporting changes, and the good will of respondents should not be unduly undermined.

9.195 Generally, the effort spent on identifying potential errors should not be excessive. Obvious mistakes should be caught at the data capture stage. The time spent in identifying observations to query, unless they are highly weighted and excessive, is often better spent treating those cases in the production cycle where things have changed – quality changes or unavailable prices – and reorganizing activities towards maintaining the relevance of the sample, and checking for errors of omission.

9.196 If the price observations are collected in a way that prompts the respondent with the previously reported price, the respondent may report the same price as a matter of convenience. This can happen even though the price may have changed, or even when the particular product being surveyed is no longer available. As prices for many items do not change frequently, this kind of error is unlikely to be spotted by normal checks. Often the situation comes to light when the contact at the responding outlet changes and the new contact has difficulty in finding something that corresponds to the price previously reported. It is advisable, therefore, to keep a record of the last time a particular respondent reported a price change. When that time has become suspiciously long, the analyst should verify with the respondent that the price observation is still valid. What constitutes too long will vary from product to product and the level of overall price inflation, but, in general, any price that has remained constant for more than a year is suspect.

Treatment of outliers

9.197 Detection and treatment of outliers (extreme values that have been verified as being correct) is an insurance policy. It is based on the fear that a particular data point collected is exceptional by chance, and that if there were a larger survey, or even a different one, the results would be less extreme. The treatment, therefore, is to reduce the impact of the exceptional observation, though not to ignore it as, after all, it did occur. The methods to test for outliers are the same as those used to identify potential errors by statistical filtering, described above. For example, upper and lower bounds of distances from the median price change are determined. In this case, however, when observations are found outside those bounds, they may be changed to be at the bounds or imputed by the rate of change of a comparable set of prices. This outlier adjustment is sometimes made automatically, on the grounds that the analyst by definition has no additional information on which to base a better estimate. While such automatic adjustment methods are employed, this manual proposes caution in their use. If an elementary aggregate is relatively highly weighted and has a relatively small sample, an adjustment may be made. The general prescription should be to include verified prices; the exception should be to dampen them.

Treatment of missing price observations

9.198 It is likely that not all the requested data will have been received by the time the index needs to be calculated. It is generally the case that missing data turn out to be delayed. Sometimes, the respondent may report that a price cannot be reported because neither the product, nor any similar substitute is being made any more. Sometimes, of course, what started apparently as a late report becomes a permanent loss to the sample. Different actions need to be taken depending on whether the situation is temporary or permanent.

9.199 For temporarily missing prices, the most appropriate strategy is to minimize the occurrence of missing observations. Survey reports are likely to come in over a period of time before the indices need to be calculated. In many cases, they follow a steady routine; some respondents will tend to file quickly, others typically will be later in the processing cycle. An analyst should become familiar with these patterns. A computerized data capture system can flag those reports that appear to be later than usual, well before the processing deadline. Also, some data are more important than others. Depending on the weighting system, some respondents may be particularly important, and important products should be flagged as requiring particular scrutiny.

9.200 For those reports for which no estimate can be made, two basic alternatives are considered here (see Chapter 7 for a full range of approaches): imputation, preferably targeted, in which the missing price change is assumed to be the same as some other set of price changes; or an assumption of no change, as the preceding period's price is used. This latter procedure ignores the fact that some prices will prove to have changed, and if prices are generally moving in one direction, this will mean that the change in the index will be understated. It is not advised. However, if the index is periodically revised, this approach will lead to fewer subsequent revisions than imputations, since for most products, prices do not generally change in any given period. Standard imputation is to base the estimate of the missing price observation on the change of some similar group of observations.

9.201 There will be situations where the price is permanently missing because the product no longer exists. As there is no replacement for the missing price, an imputation will have to be made for each period until either the sample is redesigned or until a replacement can be found. It is, therefore, more important than in the case of temporarily missing reports, and requires closer attention.

9.202 The missing price can be imputed by using the change in the remaining price observations in the elementary aggregate, which has the same effect as removing the missing observation from the sample, or by the change in a subset of other price observations for comparable items. The series should be flagged as being based on imputed values.

9.203 Samples are designed on the basis that the products chosen for observation are representative of a wider range of products. Imputations for permanently missing prices are indications of weakness in the sample, and their accumulation is a signal that the sample should be redesigned. For indices where there are known to be a large number of disappearances in the sample, the need for replacements should be anticipated.